

## Flood routing by kinematic wave model

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**Abstract:** In this study the kinematic wave equation has been solved numerically using the modified Lax explicit finite difference scheme (MLEFDS) and used for flood routing in a wide prismatic channel and a non-prismatic channel. Two flood waves, one sinusoidal wave and one exponential wave, have been imposed at the upstream boundary of the channel in which the flow is initially uniform. Six different schemes have been introduced and used to compute the routing parameter, the wave celerity  $c$ . Two of these schemes are based on constant depth and use constant celerity throughout the computation process. The rest of the schemes are based on local depths and give celerity dependent on time and space. The effects of the routing parameter  $c$  on the travel time of flood wave, the subsidence of the flood peak and the conservation flood flow volume have been studied. The results seem to indicate that there is a minimal loss/gain of flow volume whatever the scheme is. While it is confirmed that neither of the schemes is 100% volume conservative, it is found that the scheme Kinematic Wave Model-2 (KWM-II) gives the most accurate result giving only 0.1% error in perspective of volume conservation. The results obtained in this study are in good qualitative agreement with those obtained in other similar studies.

**Keywords:** Bangladesh, Channel roughness, Flood peak, The Saint-Venant equation, Modified Lax method, Volume conservation, Wave routing.

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### I. INTRODUCTION

The one-dimensional unsteady gradually varied flow in an open channel is governed by the Saint-Venant (SV) equations [1]. As flood flow in an open channel is unsteady gradually varied flow, the propagation of flood flow can be determined by applying the SV equations. Obtaining the solution of the SV equations is a complex process. However, for slow-rising flood in the upper part of a river where the bed slope is relatively steep, the water depth and the flow velocity change slowly with time so that the inertial force and the pressure force can be neglected compared to the friction force and gravity force. For this case, the SV equations reduce to the well-known KWM.

The KWM has been used widely for the investigation of overland flow and the slow-rising floods. However, the two common difficulties which arise with this model are (i) the selection of a base or reference discharge for evaluating the kinematic wave celerity  $c$ , and (ii) the small volume loss or gain that can occur and affect the peak flood flow [2][3]. The wave celerity has been either kept constant or allowed to vary in time and space. In the constant parameter case, the wave celerity is computed using a single representative flow value and kept constant throughout the whole computation in time and space. In the variable parameter case, the wave celerity is recalculated for each computational step as a function of local values. The way of calculating the wave celerity has a definite effect on the overall accuracy of the flood routing method, particularly with respect to the conservation of volume of flow.

### II. METHODOLOGY

Continuity and momentum equation for unsteady flow are given below.

Continuity equation:  $\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} = 0$  ..... (1)

where  $Q$ ,  $A$ ,  $\delta x$ ,  $\delta t$  are the discharge, cross-sectional area, space derivative and time derivative, respectively.

Momentum equation:  $\frac{\delta V}{\delta t} + V \frac{\delta V}{\delta x} + g \left( \frac{\delta h}{\delta x} - S_o + S_f \right) = 0$  ..... (2)

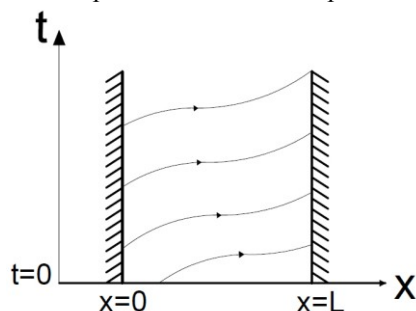
where  $V$ ,  $h$ ,  $S_o$ ,  $S_f$  are the mean velocity, depth of flow, bottom slope and friction slope, respectively.

For KWM (2) is reduced to:  $S_o = S_f$  ..... (3)

The boundary condition represents the influence of the outside world. They are needed because the computations are carried out in a finite region. So, the solution of the Saint-Venant equations require that

boundary conditions be specified at the physical extremities of the system, i.e. along the lines  $x = 0$  and  $x = L$  (Fig. 1), throughout the simulation period.

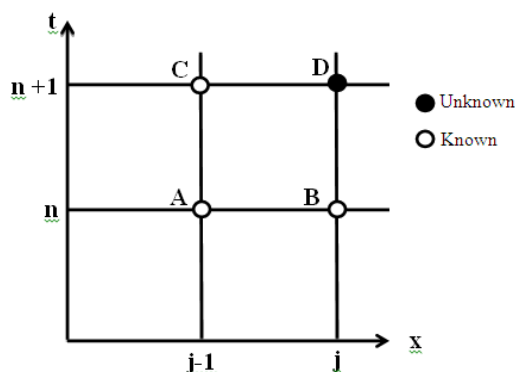
The initial condition represents the history. They are needed because the computations are carried out in a finite time interval. So, the solution of the Saint- Venant equations require that initial values (i.e. values along the initial line  $t=0, 0 \leq x \leq L$ ) of the dependent variables be specified.



**Figure 1: Initial and Boundary conditions**

Either Manning’s or Chezy’s formula can be used to calculate the average flow velocity  $V$ . Based on the formula used the wave celerity  $c$  is calculated by  $1.67V$  and  $1.5V$  for  $V$  calculated by Manning’s or Chezy’s formula, respectively.

Now it is time to present the computational grid for MLEFDS. Figure 2 shows the computational grid, which shows the known and unknown values depending on which six different schemes were developed for flood routing.



**Figure 2: Computational grid for schemes**

Short description of schemes:

KWM-I: Celerity is calculated using the minimum depth of flow.

KWM-II: Celerity is calculated using the maximum depth of flow.

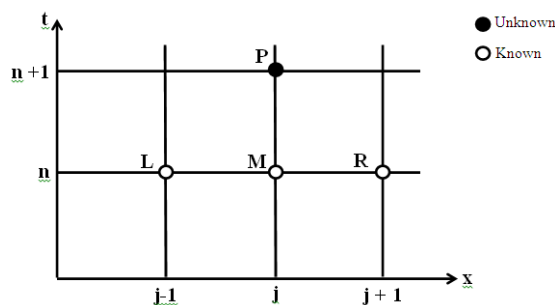
KWM-III: Celerity is calculated using average value of B and C (in Fig. 2).

KWM-IV: Celerity is calculated using average value of A, B and C (in Fig. 2).

KWM-V: Celerity is calculated using average value of B, C and D (in Fig. 2) by trial-and-error method.

KWM-VI: Celerity is calculated using average value of A, B, C and D (in Fig. 2) by trial-and-error method.

All these methods give the value of wave celerity  $c$  which is then used in the MLEFDS to calculate the value of D (in Fig. 2).



**Figure 3: Computational grid for MLEFDS**

$$\text{MLEFDS equation for first condition: } h_j^{n+1} = \frac{1}{2} \alpha (h_{j+1}^n + h_{j-1}^n) + (1 - \alpha) h_j^n - c \frac{(h_{j+1}^n - h_{j-1}^n)}{2\Delta x} \Delta t \dots \dots \dots (4)$$

MLEFDS equation for second condition:  $h_j^{n+1} = h_j^n - c \frac{h_j^n - h_{j-1}^n}{\Delta x} \Delta t$  ..... (5)

Depth/discharge at every grid points except the points on the downstream boundary is calculated by using (4) on the other hand depth/discharge of the points laying the downstream section is calculated using (5).

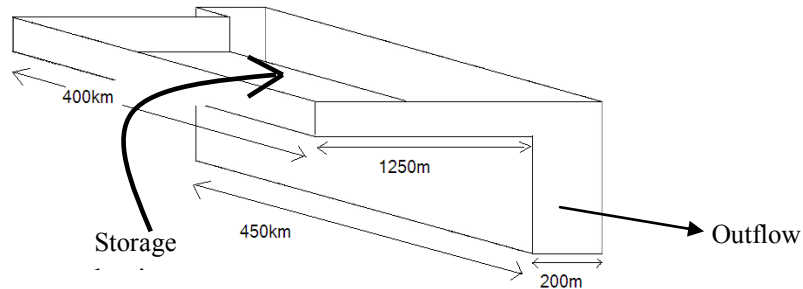
Finally volume (Q and I) at each section is calculated by using Simpson’s rule and volume conservation is calculated by following formula

%Volume conservation =  $\frac{\int_0^T Q dt}{\int_0^T I dt} * 100$  ..... (6)

where Q and I are the outflow and inflow rate, respectively.

**III. THE CHANNEL AND THE WAVES**

The channel considered for the study is shown below



**Figure 4: Schematic diagram of the channel**

The storage becomes active at 50 km downstream and when the depth of flood rises above 6.4m.

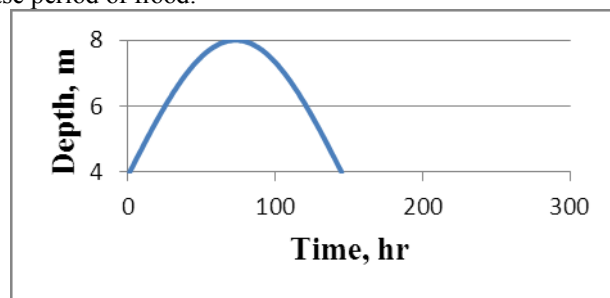
Chosen channel and wave properties are bottom slope  $S_o = .0001$ , Chezy’s  $C = 50 \text{ m}^{1/2}/\text{s}$ , Reach length = 450km, Uniform depth = 4m, Flood peak = 4m,  $\Delta x = 10\text{km}$ ,  $\Delta t = 1.0 \text{ hour}$ ,  $\alpha = 0.85$ .

The functions of the flood waves are given below

The equation of the sinusoidal wave is given by [4]

$f(w_s) = h_o + h_p \sin\left(\frac{i\pi\Delta t}{t_b}\right)$  ..... (7)

where  $f(w_s)$  = Sinusoidal wave function,  $h_o$  = Initial depth of flow,  $h_p$  = Peak depth of flood,  $i = 1, 2, 3, \dots$ ,  $\Delta t$  = Time step,  $t_b$  = Base period of flood.



**Figure 5: The sinusoidal wave [3]**

The exponential wave is given by [3]

$f(w_e) = h_o + h_p \left\{ \frac{t}{T_p} \exp\left(1 - \frac{t}{T_p}\right) \right\}^\psi$  ..... (8)

where  $f(w_e)$  = Exponential wave function,  $h_o$  = Initial depth of flow,  $h_p$  = Peak depth of flood,  $t$  = Time to peak of outflow,  $T_p$  = Time to peak of inflow,  $\psi$  = Curvature parameter.

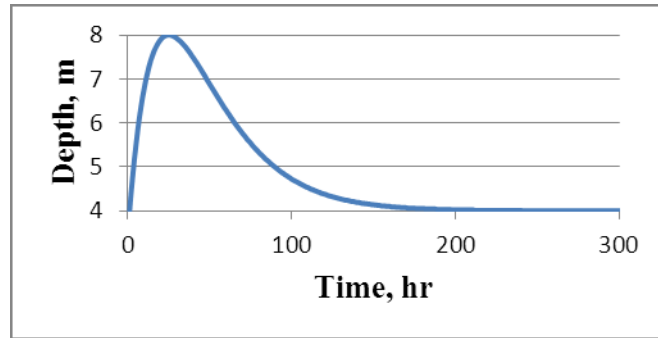


Figure 6: The exponential wave [2]

**IV. RESULTS**

Following is a summary of the results obtained from six different schemes.

**Table 1: Flow volume conservation in percentage at different sections for different schemes**

Section	Distance (km)	Volume conservation (%)					
		KWM-I	KWM-II	KWM-III	KWM-IV	KWM-V	KWM-VI
6	50	99.95	99.99	100.05	100.05	100.04	100.05
11	100	99.89	99.98	100.10	100.10	100.09	100.10
16	150	99.84	99.97	100.14	100.15	100.13	100.14
21	200	99.79	99.95	100.19	100.20	100.17	100.19
26	250	99.75	99.94	100.23	100.25	100.21	100.23
31	300	99.70	99.93	100.28	100.30	100.25	100.28
36	350	99.66	99.92	100.32	100.35	100.30	100.32
41	400	99.61	99.91	100.37	100.40	100.34	100.37
46	450	99.43	99.90	100.40	100.44	100.37	100.40

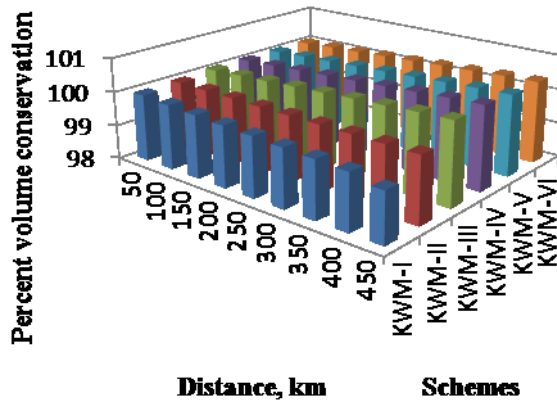


Figure 7: Graphical representation of volume conservation

**Table 2: Peak depths at different sections for different schemes**

Section	Distance (km)	Peak depth (m)					
		KWM-I	KWM-II	KWM-III	KWM-IV	KWM-V	KWM-VI
1	0	8	8	8	8	8	8
6	50	7.9821	7.9979	7.9979	7.9979	7.9979	7.9979
11	100	7.9642	7.9931	7.9931	7.9931	7.9931	7.9931
16	150	7.9462	7.9892	7.9891	7.9891	7.9891	7.9891
21	200	7.9283	7.9857	7.9854	7.9855	7.9854	7.9854
26	250	7.9105	7.982	7.9817	7.9817	7.9817	7.9817
31	300	7.8927	7.9782	7.9778	7.9778	7.9778	7.9778
36	350	7.8752	7.9744	7.9739	7.9739	7.9739	7.9739
41	400	7.8575	7.9707	7.97	7.97	7.97	7.97
46	450	7.8297	7.967	7.9658	7.9659	7.9658	7.9658

While observing the subsidence the storage basin was not taken into account to observe the actual attenuation.

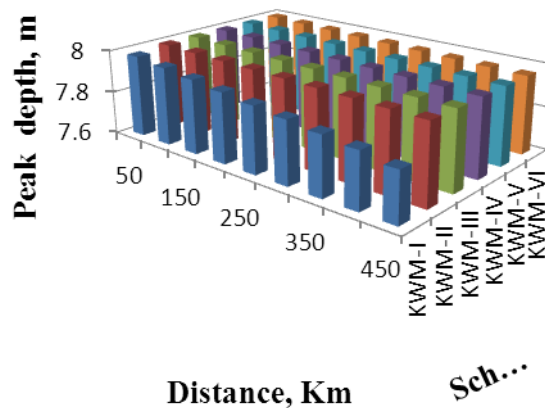


Figure 8: Graphical representation of peak depths

From Fig. 7 and Table 1 it can be summarized that celerity calculated from local depths gives rise to the volume while there is a minimal amount of loss of volume with the celerity  $c$  calculated from constant depths. On the other hand Fig. 8 and Table 2 depicts that in case of scheme KWM-1 the subsidence maximum which is not expected at all. So from the above observations it is obvious that scheme KWM-II gives the most accurate result having only 0.1% volume loss and 0.033m subsidence.

Routed hydrographs for two defined waves is shown below.

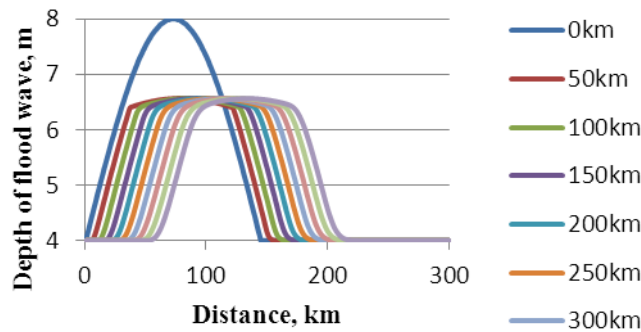


Figure 9: Shape of flood wave at different sections (sinusoidal wave)

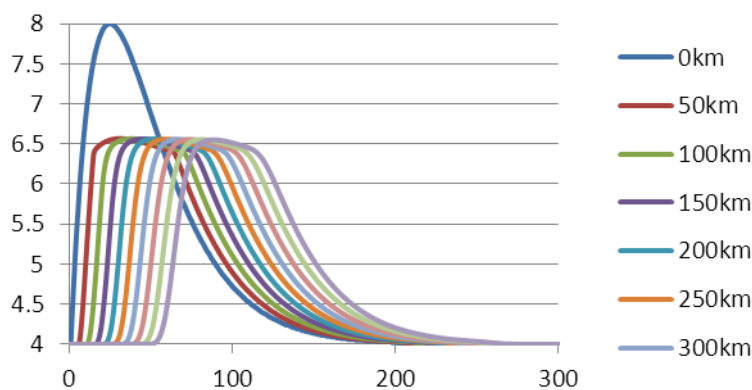


Figure 10: Shape of flood wave at different sections (exponential wave)

## V. CONCLUSIONS AND RECOMMENDATIONS

### 1. Conclusions

The following conclusions may be drawn from this study:

- 1.1 Celerity calculated from lower depth causes lower travel rate and the subsidence is higher than that in case of celerity calculated from higher depths.
- 1.2 There is a linear relationship between inflow and outflow when the depths are calculated from the constant

depths of flow. However, the relation becomes non-linear when celerity is calculated from variable depths. This is evident from the steepening of the rising limb in case of variable depth schemes and remaining unchanged in case of constant depth schemes.

- 1.3 Changed channel section has great effect of attenuating the flood peak. When the channel section expands abruptly, the flood peak falls highly then instead of having a sharp crest there is a flattened crest of flood.

## **2. Recommendations**

- 2.1 This study covers the effect of uniform-shaped flood wave on a wide rectangular channel. It also covers the effect of channel expansion on flood wave propagation. But usually the rivers are not of uniform rectangular shape and the floods are also not of uniform shape. So the further study may be undertaken with an irregular cross-section channel and a nonuniform-shaped flood.
- 2.2 The effect of lateral inflow was neglected here which is very important in flood routing. So, further study may cover the effect of lateral inflow on flood routing.
- 2.3 This study did not cover the effect of wave on a branch of a river. A further study may cover this effect.

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