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Prediction of Compressive Strength of Sawdust Ash- Cement Concrete Using Osadebe's Regression Function

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Abstract: *This work aims at developing mathematical model for predicting the compressive strength of sawdust ash – cement concrete based on Osadebe's five component second- degree polynomial. The model was used to optimize the compressive strength of concrete made from, cement, sawdust ash, sand, granites and water. A total of ninety (90) cubes were cast, comprising of three cubes for each mix ratio of a total of thirty (30) mix ratios. The first fifteen (15) were used to determine the coefficients of the response function, while the other fifteen were used to validate the response function. The results from the response function compared favourably with the experimental results. The response function were tested with the statistical student's t-test and found to be adequate at 95% confidence level. With the response function developed in this work, any desired compressive strength of sawdust ash-cement concrete can be predicted from known mix proportions and vice versa.*
***Keywords**: compressive strength, saw-dust ash, cement, concrete, Osadebe's regression functions.*

1. Introduction

Majority of housing units and infrastructural facilities in Nigeria are constructed using concrete. Amongst the basic constituent of concrete, aggregates and cement are the most expensive, thus, the major determinant factors in the cost of producing concrete are cost of aggregates and cement.

With the global economic recession coupled with the market inflationary trends, the constituent materials used for these structures had led to a very high cost of construction. Hence, researchers in material science and engineering are committed to having local materials to partially or fully replace these costly conventional materials. These local materials could be agricultural or industrial wastes. Some of these wastes include sawdust, pulverized fuel ash, palm kernel shells, slag, fly ash etc, which are produced from milling stations, thermal power station, waste treatment plants, etc.

Numerous achievements have been made in these regards and the subject is attracting attention due to its functional benefit of waste reusability and sustainable development. Reduction in construction costs and the ability to produce light-weight structures, are added advantage.

Sawdust can be defined as loose particles or wood chippings obtained as by-products from sawing of timber into standard useable sizes. Timber is one of the oldest structural materials used by man. Temples and monuments built several years ago, which still remain in excellent condition, show the durability and usefulness of timber (Kullkarni, 2005). Clean Sawdust without a large amount of bark, has proved to be satisfactory. This does not introduce a high content of organic material that may upset the reactions of hydration (Neville, 2000). It has pozzolanic properties and has been shown to react chemically with the calcium hydroxide released from the hydration of Portland cement, to form cement compounds
(Elinwa and Mahmood, 2002).

In this work, a mathematical model for the prediction and optimization of Compressive Strength of concrete is developed for concrete with different percentages of saw dust ash as partial replacement of cement. Compressive test of concrete with different mix ratios and percentages of sawdust ash was carried out in the laboratory. The results were used to develop Osadebe's regression mathematical response function. This response function would be used to predict the compressive strength of concrete given any mix ratio or predict mix ratios given a particular Compressive Strength of concrete.

II. Osadebe's Theory

According to Osadebe(2003), concrete is a four- component composite produced by mixing water, cement, fine aggregate (sand) and coarse aggregate. These ingredients are mixed in reasonable proportions to achieve desired strength of the concrete. In this paper, the fifth component sawdust ash shall be added as one of the component materials of concrete.

Let us consider an arbitrary amount 'S' of a given concrete mixture and let the portion of the i^{th} component of the five constituent materials of the concrete be $S_{i,}$ (where i= 1, 2, 3, 4, 5). This was carried out with the principle of absolute mass. Thus,

$$S_1 + S_2 + S_3 + S_4 + S_5 = S \qquad (1)$$

where $S_1,\ S_2,\ S_3,\ S_4\ and\ S_5$ are the quantities of water, cement, sawdust ash, Sand, and coarse aggregate.

Dividing Eqn(1) through by S, gives:

$$\frac{S_1}{S} + \frac{S_2}{S} + \frac{S_3}{S} + \frac{S_4}{S} + \frac{S_5}{S} = 1 \qquad (2)$$

where $\frac{S_i}{S}$ is the fractional proportion of the i^{th} constituent component of the concrete mixture.

Let $\frac{S_i}{S} = Z_i$, (3)

Substituting Eqn (3) into Eqn (2) yields:

$$Z_1 + Z_2 + Z_3 + Z_4 + Z_5 = 1 \qquad (4)$$

where Z_1, Z_2, $Z_{3,}$ Z_4 and Z_5 are fractional proportions of water, cement, sawdust ash, sand, and coarse aggregate respectively. In general, for any given concrete mixture, exists a vector Z (Z_1, Z_2, $Z_{3,}$ Z_4). In this paper where five component materials are considered, the vector is transformed to Z_i ($Z_1, Z_2,\ Z_{3,}Z_4\ ,Z_5$) whose elements satisfy Eqn (4). Also, for each value of Z_i, the following inequality holds:

$$Z_i > 0 \qquad (5)$$

It is important to note that the proportion of relative constituent ingredient of concrete govern the strength of the concrete at its hardened state. Thus, the compressive strength, Y, of concrete can be expressed mathematically using Eqn (6) as:

$$Y = f(Z_1, Z_2,\ Z_{3,}Z_4\ ,Z_5) \qquad (6)$$

where f (Z_1, Z_2, $Z_{3,}Z_4$ $,Z_5$) is a multi-variate response function whose variables Z_i are subject to the constraints defined in Eqns (4) and (5).

1.1 Osadebe's regression equation

This theory assumed that the response function is continuous and differentiable with respect to its variables, Z_i, hence, it can be expanded using Taylor's series in the neighbourhood of a chosen point $Z^{(0)} = Z_1^{(0)} + Z_2^{(0)} + Z_3^{(0)} + Z_4^{(0)} + Z_5^{(0)}$ as follows:

$$f(Z) = \sum f^m\left(Z^{(0)}\right) + \quad \frac{\left(Z_i - Z^{(0)}\right)}{m!} \qquad (7)$$

for $0 \le m \le \infty$

where m is the degree of polynomial of the response function and $f(Z)$ is the response function. Expanding Eqn (7) to the second order yields:

$$f(Z) = f(Z^{(0)}) + \sum_{i=1}^{5} \frac{\partial f(Z^{(0)})}{\partial Z_i}(Z_i - Z_i^{(0)}) + \frac{1}{2}! \sum_{i=1}^{4}\sum_{j=1}^{5} \frac{\partial^2 f(Z^{(0)})}{\partial Z_i \partial Z_j}(Z_i - Z_i^{(0)})(Z_j - Z_j^{(0)}) + \frac{1}{2}! \sum_{i=1}^{5} \frac{\partial^2 f(Z^{(0)})}{\partial Z_i^2}(Z_i - Z_i^{(0)})^2 + \cdots \quad (8)$$

The point, $Z^{(0)}$ will be chosen as the origin for convenience sake without loss of generality of the formulation. The predictor, Z_i is not the actual portion of the mixture component, rather, it is the ratio of the actual portions to the quantity of concrete. For convenience sake, let Z_i be called the term of "fractional portion". The actual portions of the mixture components are S_i.

Consequently, the origin, $Z^{(0)} = 0$, implies that:

$Z_1^{(0)} = 0$, $Z_2^{(0)} = 0$, $Z_3^{(0)} = 0$, $Z_4^{(0)} = 0$, $Z_5^{(0)} = 0$ (9)

Let: $b_0 = f(0)$, $b_i = \frac{\partial f(0)}{\partial Z_i}$, $b_{ij} = \frac{\partial^2 f(0)}{\partial Z_i \partial Z_j}$ and $b_{ii} = \frac{\partial^2 f(0)}{\partial Z_i^2}$

Then, Eqn (8) can be rewritten as follows:

$$f(0) = b_0 + \sum_{i=1}^{5} b_i Z_i + \sum_{i=1}^{4}\sum_{j=1}^{5} b_{ij} Z_i Z_j + \sum_{i=1}^{5} b_{ii} Z_i^2 + \cdots \quad (10)$$

Multiplying Eqn (4) by b_0, gives the following expression:

$b_0 = b_0Z_1 + b_0Z_2 + b_0 Z_3 + b_0Z_4 + b_0Z_5$ (11)

Similarly, multiplying Eqn (4) by Z_i will yield the following expression:

$Z_1 = Z_1^2 + Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 + Z_1 Z_5$ (12a)

$Z_2 = Z_1 Z_2 + Z_2^2 + Z_2Z_3 + Z_2Z_4 + Z_2Z_5$ (12b)

$Z_3 = Z_1 Z_3 + Z_2Z_3 + Z_3^3 + Z_3Z_4 + Z_3Z_5$ (12c)

$Z_4 = Z_1 Z_4 + Z_2Z_4 + Z_4Z_3 + Z_4^2 + Z_4Z_5$ (12d)

$Z_5 = Z_1 Z_5 + Z_2Z_5 + Z_3Z_5 + Z_4Z_5 + Z_5^2$ (12e)

When Eqns (12a) to (12e), are rearranged, the expression for Z_i^2 becomes;

$Z_1^2 = Z_1 - Z_1Z_2 - Z_1Z_3 - Z_1Z_4 - Z_1Z_5$ (13a)

$Z_2^2 = Z_2 - Z_1Z_2 - Z_2Z_3 - Z_2Z_4 - Z_2Z_5$ (13b)

$Z_3^2 = Z_3 - Z_1Z_3 - Z_2Z_3 - Z_3Z_4 - Z_3Z_5$ (13c)

$Z_4^2 = Z_4 - Z_1Z_4 - Z_2Z_4 - Z_3Z_4 - Z_4Z_5$ (13d)

$Z_5^2 = Z_5 - Z_1Z_5 - Z_2Z_5 - Z_3Z_5 - Z_4Z_5$ (13e)

Substituting Eqn (13a) to (13e) into Eqn (10) and setting $f(0) = Y$ give the expanded form below:

$$Y = b_0Z_1 + b_0Z_2 + b_0Z_3 + b_0Z_4 + b_0Z_5 + b_1Z_1 + b_2Z_2 + b_3Z_3 + b_4Z_4 + b_5Z_5 + b_{12}Z_1Z_2 +$$

$$b_{13}Z_1Z_1Z_3 + b_{14}Z_1Z_4 + b_{15}Z_1Z_5 + b_{23}Z_2Z_3 + b_{24}Z_2Z_4 + b_{25}Z_2Z_5 + b_{34}Z_3Z_4 +$$

$$b_{35}Z_3Z_5 + b_{45}Z_4Z_5 + b_{11}(Z_1 - Z_1Z_2 - Z_1Z_3 - Z_1Z_4 - Z_1Z_5) +$$

$$b_{22}(Z_2 - Z_1Z_2 - Z_2Z_3 - Z_2Z_4 - Z_2Z_5) + b_{33}(Z_3 - Z_1Z_3 - Z_2Z_3 - Z_3Z_4 - Z_3Z_5) +$$

$$b_{44}(Z_4 - Z_1Z_4 - Z_2Z_4 - Z_3Z_4 - Z_4Z_5) - b_{55}(Z_5 - Z_1Z_5 - Z_2Z_5 - Z_3Z_5 - Z_4Z_5) \quad (14a)$$

Factorizing Eqn (14a) gives

$$\begin{aligned} Y = {} & Z_1(b_0 + b_1 + b_{11}) + Z_2(b_0 + b_2 + b_{22}) + Z_3(b_0 + b_3 + b_{33}) + Z_4(b_0 + b_4 + b_{44}) + Z_5(b_0 + b_5 + b_{55}) \\ & + Z_1Z_2(b_{12} - b_{11} - b_{22}) + Z_1Z_3(b_{13} - b_{11} - b_{33}) + Z_1Z_4(b_{14} - b_{11} - b_{44}) \\ & + Z_1Z_5(b_{15} - b_{11} - b_{55}) + Z_2Z_3(b_{23} - b_{22} - b_{33}) + Z_2Z_4(b_{24} - b_{22} - b_{44}) \\ & + Z_2Z_5(b_{25} - b_{22} - b_{55}) + Z_3Z_4(b_{34} - b_{33} - b_{44}) + Z_3Z_5(b_{35} - b_{33} - b_{55}) \\ & + Z_4Z_5(b_{45} - b_{44} - b_{55}) \qquad (14b) \end{aligned}$$

The summation of the constants is equal to a constant thus; let

$$\propto_i = b_0 + b_i + b_{ii} \text{ and } \propto_{ij} = b_{ij} + b_{ii} + b_{jj} \qquad (15)$$

Eqn (14b) becomes:

$$Y = \propto_1Z_1 + \propto_2Z_2 + \propto_3Z_3 + \propto_4Z_4 + \propto_5Z_5 + \propto_{12}Z_1Z_2 + \propto_{13}Z_1Z_3 + \propto_{14}Z_1Z_4 + \propto_{15}Z_1Z_5 + \propto_{23}Z_2Z_3 +$$
$$\propto_{24}Z_2Z_4 + \propto_{25}Z_2Z_5 + \propto_{34}Z_3Z_4 + \propto_{35}Z_3Z_5 + \propto_{45}Z_4Z_5 \qquad (16a)$$

Rewriting Eqn (16a) in a compact form, gives:

$$Y = \sum_{i=1}^{5} \alpha_i Z_i + \sum_{1 \le i \le j}^{5} \alpha_{ij} Z_i Z_j \qquad (16b)$$

And, Y is the response function at any point of observation, Z_i and Z_j are the predictors, and $\propto_i$ and $\propto_{ij}$ are the coefficients of the response equation.

1.2 The Coefficients of the Regression Equation

Let the n^{th} response (compressive strength at n^{th} observation point) be $Y^{(n)}$ and the vector of the corresponding set of variables be as follows:

$Z^{(n)} = (Z_1^{(n)}, Z_2^{(n)}, Z_3^{(n)}, Z_4^{(n)}, Z_5^{(n)})$

Different points of observation will have different predictor at constant coefficient. At n^{th} observation point, the response function, $Y^{(n)}$, will correspond with the predictors $Z_i^{(n)}$.

Thus,

$$Y^{(n)} = \sum_{i=1}^{5} \alpha_i Z_i^{(n)} + \sum_{1 \le i \le j}^{5} \alpha_{ij} Z_i Z_j^{(n)} \qquad (16)$$

where $1 \le i \le j \le 5$ and n= 1, 2, 3,…, 15

Eqn (16) can be rewritten in a matrix form as,

$$[Y^{(n)}] = [Z^{(n)}][\propto] \qquad (17)$$

Expanding Eqn (17) yields:

$$\begin{pmatrix} Y^{(1)} \\ Y^{(2)} \\ Y^{(3)} \\ \cdot \\ \cdot \\ Y^{(15)} \end{pmatrix} = \begin{pmatrix} Z_1^{(1)} & Z_2^{(1)} & Z_3^{(1)} & \ldots & Z_4^{(1)}Z_5^{(1)} \\ Z_1^{(2)} & Z_2^{(2)} & Z_3^{(2)} & \ldots & Z_4^{(2)}Z_5^{(2)} \\ Z_1^{(3)} & Z_2^{(3)} & Z_3^{(3)} & \ldots & Z_4^{(3)}Z_5^{(3)} \\ & & & & \\ Z_1^{(15)} & Z_2^{(15)} & Z_3^{(15)} & \ldots & Z_4^{(15)}Z_5^{(15)} \end{pmatrix} \begin{pmatrix} \propto_1 \\ \propto_2 \\ \propto_3 \\ \\ \\ \propto_{45} \end{pmatrix} \qquad (18)$$

The actual mixture proportions $S_i^{(n)}$ and the corresponding fractional portions, $Z_i^{(n)}$ are shown in Table 1. And the values of the constant coefficient $\propto$ in Eqn (17), are determined with the values of $Y^{(n)}$ and $Z^{(n)}$. Rearranging Eqn (17) gives

$$[\propto] = [Z^{(n)}]^{-1} [Y^{(n)}] \quad (19)$$

Expressing Eqn. (19) in expanded form yields:

$$\begin{pmatrix} \propto_1 \\ \propto_2 \\ \propto_3 \\ \cdot \\ \propto_{45} \end{pmatrix} = \begin{pmatrix} Z_1^{(1)} & Z_2^{(1)} & Z_3^{(1)} & \ldots & Z_4^{(1)}Z_5^{(1)} \\ Z_1^{(2)} & Z_2^{(2)} & Z_3^{(2)} & \ldots & Z_4^{(2)}Z_5^{(2)} \\ Z_1^{(3)} & Z_2^{(3)} & Z_3^{(3)} & \ldots & Z_4^{(3)}Z_5^{(3)} \\ Z_1^{(15)} & Z_2^{(15)} & Z_3^{(15)} & \ldots & Z_4^{(15)}Z_5^{(15)} \end{pmatrix}^{-1} \begin{pmatrix} Y^{(1)} \\ Y^{(2)} \\ Y^{(3)} \\ Y^{(15)} \end{pmatrix} \quad (20)$$

The values of $\propto_1$ to $\propto_{45}$ are obtained from Eqn (20) and substituted into Eqn. (16a) to obtain the regression equation. The values of $Z^{(n)}$] matrix are shown in Table 2 and the values of the inverse of $[Z^{(n)}]$ matrix are presented in Table 3; while the values of $[Y^{(n)}]$ matrix are obtained from the experimental investigation.

III. Materials and Methods

III.1 Materials

The materials used for the laboratory test included:

(i) Water that is good for drinking obtained from a borehole at the premises of Federal University of Technology Owerri, Imo State, Nigeria. The water was clean, fresh and free from dirt, unwanted chemicals or rubbish that may affect the desired quality of concrete.

(ii) Dangote cement, a brand of ordinary Portland cement that conforms to BS 12(1978)

(iii) The fine aggregate, river sand used for this research work were obtained from a flowing river (Otamiri). As at the time of purchase the sharp river sand was wet but free from debris and deleterious matter and clay.

(iv) The coarse aggregate used for this research work was granite chippings quarried from a quarry in Ishiagu, along Enugu-Port Harcourt express way, a town in Ebonyi state, Nigeria. The granite was sun dried for seven days so they can be free from water. They were sieved through a 20mm British test sieve and the materials passing through the sieve were used to produce concrete with various proportions of termite soil.

(v) Sawdust is a by-product from timber, it is a waste product obtained during sawing of timber into standard sizes. The sawdust was obtained from timber milling market (ogbosisi) Owerri. This material was first dried to remove the natural moisture. The waste was burnt in an enclosure (i.e. open drum) at temperature of about 400-500°C to obtain sawdust ash. The ash was allowed to cool; thereafter the ash was sieved with 150μm sieve aperture to obtain the finest particle of material which approximates to the fineness of that of cement used.

The mix ratios used for the laboratory test, were obtained using pentahedron factor space for five – component mixture.

3.2 Compressive Strength Test

Batching of the ingredients was done by mass. A mixture of Cement/ sawdust ash was thoroughly mixed together in the dry state river sand and granite and then, water added. The mixing continued until a uniform and consistent concrete mix is obtained. The entire concrete was cast in concrete mould of sizes 150 x 150x 150 mm. In all, sixty concrete cubes, two from each mix incorporating various proportions of sawdust ash, were cast and cured in a curing water tank for 28 days, and then crushed in a universal testing machine. Thirty of the concrete cubes served as control test. Compressive strength of the cubes was calculated using Eqn (21):

$$\text{Compressive strength} = \frac{compressive\ \ load\ of\ cube\ at\ failure\ \ (N)}{cross\ \ sectional\ \ area\ of\ mould\ \ (mm^2)} \quad (21)$$

The results of the compressive strength test of the concrete cubes are presented in Table 4

Table 1: Selcted mix proportions, S, and their corresponding fractional portions,Z, based on Osadebe's second-degree polynomial

OP	Mix Proportions, S_i					FractionaL Portions, Z_i				
	S_1	S_2	S_3	S_4	S_5	Z_1	Z_2	Z_3	Z_4	Z_5
1	0.5	0.95	0.05	2.25	4.00	0.065	0.123	0.006	0.290	0.516
2	0.55	0.90	0.10	175	3.50	0.0809	0.1323	0.0147	0.2514	0.5147
S3	0.60	0.15	0.15	2.25	4.25	0.0741	0.10493	0.0185	0.2778	0.525
4	0.45	0.80	0.20	1.50	3.00	0.0710	0.1344	0.0336	0.252	0.504
5	0.05	0.75	0.25	2.50	5.00	0.0710	0.081967	0.0273	0.273	0.546
12	0.525	0.925	0.075	2.00	3.75	0.0722	0.0227	0.0103	0.2749	0.5155
13	0.55	0.90	0.10	2.25	4.125	0.069	0.1136	0.0126	0.2839	0.5210
14	0.475	0.875	0.125	1.875	3.50	0.069	0.128	0.0182	0.2737	0.5109
15	0.575	0.85	0.150	2.375	4.50	0.0680	0.1006	0.0178	0.2811	0.5325
23	0.575	0.875	0.125	2.00	3.875	0.0772	0.1175	0.0168	0.2685	0.5200
24	0.50	0.85	0.150	1.625	3.25	0.0784	0.1333	0.0235	0.2549	0.5098
25	0.60	0.825	0.175	2.125	4.25	0.0752	0.1034	0.02194	0.2665	0.5329
34	0.525	0.825	0.175	1.875	3.625	0.0747	0.1174	0.0249	0.2669	0.5160
35	0.625	0.80	0.20	2.375	4.625	0.0725	0.0928	0.0232	0.2754	0.5362
45	0.55	0.775	0.225	2.00	4.00	0.0728	0.1026	0.0298	0.2649	0.5298
CONTROL POINTS										
AC_1	0.550	0.900	0.100	2.083	3.917	0.073	0.119	0.013	0.276	0.519
AC_2	0.517	0.867	0.133	2.000	3.750	0.071	0.119	0.018	0.275	0.516
AC_3	0.533	0.833	0.167	2.083	4.000	0.070	0.109	0.022	0.274	0.525
AC_4	0.525	0.875	0.125	1.938	3.688	0.073	0.122	0.017	0.271	0.516
AC_5	0.550	0.838	0.163	2.125	4.063	0.071	0.108	0.021	0.275	0.525
AC_6	0.575	0.863	0.138	2.188	4.183	0.072	0.109	0.017	0.275	0.526
AC_7	0.538	0.913	0.088	2.125	3.938	0.071	0.120	0.012	0.280	0.518
AC_8	0.600	0.825	0.175	2.375	4.563	0.070	0.097	0.020	0.278	0.534
AC_9	0.520	0.890	0.110	2.000	3.750	0.072	0.122	0.015	0.275	0.516
AC_{10}	0.550	0.850	0.150	2.050	3.950	0.073	0.113	0.020	0.272	0.523
AC_{11}	0.545	0.855	0.145	2.100	4.000	0.071	0.112	0.019	0.275	0.523
AC_{12}	0.545	0.835	0.165	1.975	3.850	0.074	0.113	0.022	0.268	0.522
AC_{13}	0.570	0.868	0.133	2.238	4.238	0.071	0.108	0.016	0.278	0.527
AC_{14}	0.545	0.855	0.145	2.050	3.938	0.072	0.114	0.019	0.272	0.523
AC_{15}	0.538	0.858	0.143	2.150	4.075	0.069	0.110	0.018	0.277	0.525

Legend: OP is Observation Point

Table 3: Inverse of $Z^{(n)}$ Matrix

OP	Z_1	Z_2	Z_3	Z_4	Z_5	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_1Z_5	Z_2Z_3	Z_2Z_4	Z_2Z_5	Z_3Z_4	Z_3Z_5	Z_4Z_5
1	0.0645	0.1226	0.0065	0.2903	0.5161	0.0079	0.0004	0.0187	0.0333	0.0008	0.0356	0.0633	0.0019	0.0033	0.1498
2	0.0809	0.1324	0.0147	0.2574	0.5147	0.0107	0.0012	0.0208	0.0416	0.0019	0.0341	0.0681	0.0038	0.0076	0.1325
3	0.0741	0.1049	0.0185	0.2778	0.5247	0.0078	0.0014	0.0206	0.0389	0.0019	0.0291	0.0551	0.0051	0.0097	0.1457
4	0.0756	0.1345	0.0336	0.2521	0.5042	0.0102	0.0025	0.0191	0.0381	0.0045	0.0339	0.0678	0.0085	0.0169	0.1271
5	0.0710	0.0820	0.0273	0.2732	0.5464	0.0058	0.0019	0.0194	0.0388	0.0022	0.0224	0.0448	0.0075	0.0149	0.1493
12	0.0722	0.1271	0.0103	0.2749	0.5155	0.0092	0.0007	0.0198	0.0372	0.0013	0.0350	0.0655	0.0028	0.0053	0.1417
13	0.0694	0.1136	0.0126	0.2839	0.5205	0.0079	0.0009	0.0197	0.0361	0.0014	0.0322	0.0591	0.0036	0.0066	0.1478
14	0.0693	0.1277	0.0182	0.2737	0.5109	0.0089	0.0013	0.0190	0.0354	0.0023	0.0350	0.0653	0.0050	0.0093	0.1399
15	0.0680	0.1006	0.0178	0.2811	0.5325	0.0068	0.0012	0.0191	0.0362	0.0018	0.0283	0.0536	0.0050	0.0095	0.1497
23	0.0772	0.1174	0.0168	0.2685	0.5201	0.0091	0.0013	0.0207	0.0401	0.0020	0.0315	0.0611	0.0045	0.0087	0.1396
24	0.0784	0.1333	0.0235	0.2549	0.5098	0.0105	0.0018	0.0200	0.0400	0.0031	0.0340	0.0680	0.0060	0.0120	0.1300
25	0.0752	0.1034	0.0219	0.2665	0.5329	0.0078	0.0017	0.0200	0.0401	0.0023	0.0276	0.0551	0.0058	0.0117	0.1420
34	0.0747	0.1174	0.0249	0.2669	0.5160	0.0088	0.0019	0.0199	0.0386	0.0029	0.0313	0.0606	0.0066	0.0129	0.1377
35	0.0725	0.0928	0.0232	0.2754	0.5362	0.0067	0.0017	0.0200	0.0389	0.0022	0.0255	0.0497	0.0064	0.0124	0.1477
45	0.0728	0.1026	0.0298	0.2649	0.5298	0.0075	0.0022	0.0193	0.0386	0.0031	0.0272	0.0544	0.0079	0.0158	0.1403

OP	Z_1	Z_2	Z_3	Z_4	Z_5	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_1Z_5	Z_2Z_3	Z_2Z_4	Z_2Z_5	Z_3Z_4	Z_3Z_5	Z_4Z_5
1	10781.3	102.5	47108.3	706.1	9091.2	-2111.2	-45094.8	5615.1	19937.3	4427.9	-540.4	-1973.2	-11811.3	-41543.4	5305.5
2	338.6	32.7	2246.0	297.7	33.6	368.9	-1905.4	846.4	-146.4	-773.7	336.8	-59.9	-1780.5	305.0	-139.0
3	751.2	5586.6	2391.4	3798.8	400.1	3606.0	-2573.5	-3219.3	850.5	-7563.1	-9272.8	2519.4	6771.8	-1772.2	-2273.8
4	85.2	2988.1	12306.3	78.5	2968.6	1013.4	-2048.8	-166.4	1012.7	-12221.0	-972.7	6088.7	2012.3	-12133.3	-1010.6
5	260.9	803.4	6333.5	50.2	1899.9	919.5	-2572.1	-232.9	1417.8	-4546.0	-403.5	2525.7	1154.9	-6963.5	-646.8
12	-14983.2	-9.6	-69964.3	-1927.6	-7858.6	381.8	65539.1	-11733.5	-23177.1	-800.7	140.5	285.4	24681.3	48294.1	-8867.8
13	-5812.4	-7203.1	-28235.6	-7782.3	-5627.5	13095.6	26123.0	-14530.6	-12464.8	-27466.5	14973.8	13209.9	30565.1	25972.8	-14817.5
14	-8949.8	-4197.2	-11259.5	-1255.3	-1669.8	12310.4	20086.7	-6821.3	-7785.0	-13854.4	4611.2	5412.2	7699.2	8704.3	-3031.7
15	-14396.4	-332.0	-87987.9	-379.7	-19303.0	-4391.2	71216.4	-4758.3	-33570.6	10892.6	-713.3	-5175.4	11837.6	82730.6	-5669.3
23	-2129.6	-6558.1	-9305.4	-1961.4	-185.5	-7506.3	8907.6	4159.3	-1265.9	15743.6	7205.0	-2255.1	-8749.1	2637.7	1263.2
24	-767.2	-3698.6	-25086.3	-68.3	-2278.6	-3885.1	9515.8	-661.3	-3144.9	19833.3	-1350.9	-6606.7	3385.9	15949.7	-1136.9
25	1.5	-484.6	-1022.5	-594.3	-2512.4	205.8	-363.3	-280.8	491.3	1391.1	1053.7	-1896.4	-1902.9	3298.7	2614.9
34	-327.9	-399.0	-3829.4	-2784.7	-1160.8	-554.2	-1841.4	1740.7	1174.9	-2109.4	1954.4	1356.6	6645.0	4442.8	-4307.6
35	-1901.9	-10629.2	-16521.7	-4722.9	-4066.2	-8285.7	10771.3	5906.3	-4999.4	26679.3	14337.9	-12481.2	-19073.1	15992.5	8994.1
45	-644.2	-6890.1	-36296.7	-254.2	-9618.1	-4231.7	9675.9	823.5	-5012.8	31871.0	2658.6	-16642.6	-6220.6	37507.7	3274.2

Legend OP: Observation points

IV. Results And Analysis

The sawdust ash-cement concrete cubes, were crushed on the 28th day, and the results presented in table 4

Table 4: Compressive strength in N/mm^2 of 28th day old concrete cubes

S/No	Replicate 1 Compressive strength(N/mm^2)	Replicate 1 Compressive strength(N/mm^2)	Replicate 1 Compressive strength(N/mm^2)	Average Laboratory Compressive strength(N/mm^2)	Osadebe's Model compressive strength result (N/mm^2)
1	20.69	20.00	20.44	20.34	20.34
2	19.56	19.33	19.11	19.30	19.30
3	18.44	18.22	17.78	18.13	18.13
4	13.33	12.00	15.56	14.45	14.45
5	11.55	9.33	11.11	11.33	11.33
12	18.22	20.00	20.66	19.63	19.63
13	13.78	13.78	13.33	13.63	13.63
14	16.44	10.67	16.89	16.67	16.67
15	8.66	9.33	9.33	9.11	9.11
23	21.33	15.56	16.00	17.66	17.66
24	15.56	16.00	13.30	15.78	15.78
25	14.67	16.00	8.89	15.34	15.34
34	13.78	15.11	11.58	13.48	13.48
35	14.67	14.22	16.44	15.10	15.10
45	20.00	18.22	17.78	18.67	18.67
AC_1	17.47	18.08	16.89	17.48	15.98
AC_2	13.55	15.56	15.11	14.74	13.24
AC_3	15.90	15.33	15.51	15.58	14.51
AC_4	16.89	16.11	16.35	16.45	14.95
AC_5	14.34	13.89	14.28	14.17	13.67
AC_6	14.78	14.56	14.67	14.67	14.17
AC_7	17.22	17.11	17.09	17.14	16.14
AC_8	11.604	11.33	11.51	11.48	11.43
AC_9	16.82	16.36	16.00	16.39	15.39
AC_{10}	14.39	14.29	14.22	14.30	14.80
AC_{11}	13.47	13.31	13.72	13.50	14.00
AC_{12}	14.80	14.88	14.72	14.80	15.50
AC_{13}	12.88	13.03	13.24	13.05	13.20
AC_{14}	14.68	14.68	14.38	14.58	14.76
AC_{15}	13.11	13.04	13.18	13.11	13.81

4.2 The final response function

The solution of Eqn (20),using the responses in Table 4 and the Z matrix, gives the unknown coefficients of the regression equation as follows:

$\alpha_1 = 178559.1$ $\alpha_2 = 15572.47$ $\alpha_3 = -2689.94$ $\alpha_4 = 3939.83$ $\alpha_5 = 1699.999$ $\alpha_{12} = -329399$

$\alpha_{13} = -218425$ $\alpha_{14} = -91088.2$ $\alpha_{15} = -246507$ $\alpha_{23} = -35766.2$ $\alpha_{24} = 649779.6$

$\alpha_{25} = 19521.98$ $\alpha_{34} = -41692.3$ $\alpha_{35} = 32895.84$ $\alpha_{45} = -11530.2$

Substituting these coefficients into Eqn. (16a) yields:

$$Y = 178559.1Z_1 + 15572.47Z_2 - 2689.94Z_3 + 3939.83Z_4 + 1699.999Z_5 - 329399Z_1Z_2 - 1218425Z_1Z_3 - 91088.2Z_1Z_4 - 246507Z_1Z_5 - 35766.2Z_2Z_3 - 64979.6Z_2Z_4 + 19521.98Z_2Z_5 - 41692.3Z_3Z_4 + 32895.84Z_3Z_5 - 11530.20Z_4Z_5 \quad (22)$$

The Eqn (22), is the final response function for the predicting of the compressive strength of sawdust ash-cement concrete based on Osadebe's regression function.

4.3 Test of Adequacy of the response function

The test for adequacy of response function was done using two-tailed statistical student's t-test at 95% accuracy level. The compressive strengths at the control points (i.e. AC_1, AC_2 , AC_3 ,AC_4, AC_5, AC_6 ,AC_7, AC_8 ,AC_9 ,AC_{10}, AC_{11}, AC_{12}, AC_{13}, AC_{14}, AC_{15}) were used for the test . The following two hypotheses were tested using statistical student's t-test.

a) **Null Hypothesis:** There is no significant difference between the laboratory concrete cube strengths and predicted compressive strength results at 95% accuracy level.

b) **Alternative Hypothesis:** There is a significant difference between the laboratory concrete cube strengths and l predicted strength compressive strength results at 95% accuracy level.

The test is carried out and presented in Table 5 using the following equations:
where

Y_E = Responses(compressive strength)from the experiment

Y_M = Responses(compressive strength) from the Second degree polynomial equation

N = Number of observations points

D_i = Difference of Y_E and Y_M

$D_A = \frac{\sum Di}{N}$ =Mean of difference of Y_E and Y_M

$S^2 = \frac{\sum(D_A - Di)^2}{N-1}$ = Variance of difference of Di and DA

$t = \frac{D_A * N^{0.5}}{S}$ = Calculated value of t

Table 5: Statistical Student's t-test for Osadebe's response function

OP	Y_E	Y_M	$D_i = Y_E - Y_M$	$D_A - D_i$	$(D_A - D_i)^2$
AC_1	17.48	15.98	1.50	-1.107	1.2254
AC_2	14.74	13.24	1.50	-1.107	1.2254
AC_3	15.58	14.51	1.07	-0.677	0.4583
AC_4	16.45	14.95	1.50	-1.107	1.2254
AC_5	14.17	13.67	0.50	-0.107	0.0114
AC_6	14.67	14.17	0.50	-0.107	0.0114
AC_7	17.14	16.14	1.00	-0.607	0.3684
AC_8	11.48	11.43	0.05	0.343	0.1176
AC_9	16.39	15.39	1.00	-0.607	0.3684
AC_{10}	14.30	14.80	-0.50	0.893	0.7974
AC_{11}	13.50	14.00	-0.50	0.893	0.7974
AC_{12}	14.80	15.50	-0.70	1.093	1.1946
AC_{13}	13.05	13.20	-0.15	0.543	0.2948
AC_{14}	14.58	14.76	-0.18	0.573	0.3283
AC_{15}	13.11	13.81	-0.70	1.093	1.1946
		□ □ Di =	5.89	□ □ $(DA - Di)^2$ =	9.6191

Legend: OP is the observation point

Here,

□ □ D_i = 5.89

N = 15

$DA = \frac{\sum D_i}{N}$ = 0.393

$S^2 = \frac{\sum(D_A - D_i)^2}{N-1}$ = 0.6871

$S = \sqrt{S^2}$ = 0.8289

The actual value of the total variation in t-test is given by

$t = \frac{DA * N^{0.5}}{S}$ = 1.84

and the allowable value of the total variation in t-test is obtained as follows

Degree of freedom = N-1 = 15-1 = 14

5 % significance for Two-Tailed Test = 2.5 %

1 - 2.5% = 97.5% = 0.975

Therefore allowable total variation in the t-test = $t_{(0.975, N-1)}$ = $t_{(0.975, 14)}$ = 2.14 (Obtained from standard statistical table).

From table 5, the calculated t = 1.84

Thus, $t_{(table)} > t_{(calculated)}$

This implied that difference between the two set of cubes compressive strength is less than allowable difference. Hence null hypothesis is accepted and alternative hypothesis rejected. Hence, Osadebe regression model is adequate.

V. Conclusion

Using Osadebe's second degree polynomial regression equation, mix design model for a five component sawdust ash-cement concrete. This model could predict the compressive strength of concrete cube when the mix ratios are known and vice versa. The predictions from this model were tested at 95% accuracy level using statistical student's t-test and found to be adequate.

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