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Prediction of Compressive Strength of PFA- Cement Concrete

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Abstract: *This work aims at prediction of compressive strength of Pulverised Fuel ash (PFA) – cement concrete based on Osadebe's five component second- degree polynomial. The model was used to optimize the compressive strength of concrete made from, cement, Pulverised Fuel Ash, sand, granites and water. A total of ninety (90) cubes were cast, comprising three cubes for each mix ratio and a total of thirty (30) mix ratios. The first fifteen (15) were used to determine the coefficients of the model, while the other fifteen were used to validate the model. The mathematical model compared favourably with the experimental data and the predictions from the model were tested with the statistical fischer test and found to be adequate at 95% confidence level. With the model developed in this work, any desired compressive strength Pulverised Fuel Ash-cement concrete can be predicted from known mix proportions and vice versa.*
Keywords*: Pulverised Fuel Ash; model; optimization; compressive strength; cement; concrete*

I. Introduction

Construction industries in Nigeria rely heavily on cement for its operation on development of shelter and other infrastructural facilities. The provision of low cost but durable materials is almost universally recognized as one of the set-backs to improve housing conditions in Nigeria and other developing countries. Among the building materials in use today, the Ordinary Portland Cement is a vital element in all types of construction. Cement is very expensive. This has drastically reduced the construction of housing. Consequently, it has become absolutely difficult for majority of Nigeria citizens to own houses.

According to Neville, et al (1990), concrete is a product of water, cement and aggregate, when sufficiently hardened, is used in carrying various loads .However, in Nigeria due to the rapid rising of the cost of ordinary Portland cement (which is an important ingredient in concrete) there is need to develop cheap and replaceable substitute for cement. Consequently, many researches are being carried out on cheap and replaceable or complimentary substitutes for ordinary Portland cement. These researches are also aimed at putting into effective use industrial waste products.

The properties of concrete are controlled by the relative quantities of cement, aggregates and water mixed together both in plastic and hardened states. Also these properties can be improved by the addition of either a chemical admixture or supplementary cementitious material, which will make the number of components of concrete five (that is in addition to water, cement, coarse aggregate and fine aggregate). For this research work the components of concrete are cement, Pulverised Fuel Ash (PFA), fine aggregate, coarse aggregate and water, in this work, a mathematical model for the optimization of Compressive Strength of concrete based on Osadebe's second-degree polynomial is developed with different percentages of PFA as partial replacement of cement. This involves testing concrete from the different mix ratios where cement is partially replaced with PFA and developing a mathematical model that can be used to predict the compressive strength of the concrete given any mix ratio or predict mix ratios given a particular Compressive Strength of concrete.

Osadebe's Concrete Optimization Theory

Osadebe in his theory stated that concrete consists of four- component material. It is produced by mixing water, cement, fine aggregate (sand) and coarse aggregate. These ingredients are mixed in reasonable proportions to achieve desired strength of the concrete. In this paper, the fifth component, Pulverised Fuel Ash shall be added as one of the component materials of concrete.

Let us consider an arbitrary amount 'S' of a given concrete mixture. Let the portion of the i^{th} component of the five constituent materials of the concrete be S_i, i= 1, 2,3,4,5. This was carried out with the principle of absolute mass.

$$S_1 + S_2 + S_3 + S_4 + S_5 = S \quad (1)$$

Or

$$\frac{S_1}{S} + \frac{S_2}{S} + \frac{S_3}{S} + \frac{S_4}{S} + \frac{S_5}{S} = 1 \qquad (2)$$

Where $\frac{S_i}{S}$ is the proportion of the i^{th} constituent component of the concrete mixture.

Let $\frac{S_i}{S} = Z_i$, i = 1, 2, 3, 4, 5 (3)

Substituting equation (3) into equation (2) yields:

$$Z_1 + Z_2 + Z_3 + Z_4 + Z_5 = 1 \qquad (4)$$

Experience has shown that the coefficient of regression when ∑Z=1 is mostly too large that the regression becomes too sensitive. As a result, when values of predictors outside the ones used in formulating the model are used to predict the response, the regression gives outrageous values. To correct this short coming a system of Z that will make ∑ Z=10 will be adopted for convenience.
Multiplying equation (4) by 10 gives equation (4b)

$$10Z_1 + 10Z_2 + 10Z_3 + 10Z_4 + 10Z_5 = 10 \qquad (4b)$$

Let $10Z_i = Z_i$

Thus, $Z_1 + Z_2 + Z_3 + Z_4 + Z_5 = 10$ (4c)

Where Z_1, Z_2, Z_3, Z_4 and Z_5 are proportions of water, cement, Pulverised Fuel Ash, sand, and coarse aggregate respectively. In general, for any given concrete mixture, exists a vector Z (Z_1, Z_2, Z_3, Z_4). In this paper where five component materials are considered, the vector is transformed to Z (Z_1, Z_2, Z_3, Z_4, Z_5) whose elements satisfy equation (4). Also, for each value of Z_i, the following inequality holds:

$$Z_i > 0 \qquad (5)$$

It is important to note that the proportion of relative constituent ingredient of concrete govern the strength of the concrete at its hardened state. Thus, the compressive strength, Y, of concrete can be expressed mathematically using equation (6) as:

$$Y = f(Z_1, Z_2, Z_3, Z_4, Z_5) \qquad (6)$$

Where f (Z_1, Z_2, Z_3, Z_4, Z_5) is a multi-variate response function whose variables Z_i are subject to the constraints as defined in equations (4) and (5).

Osadebe's regression equation

This model assumed that the response function is continuous and differentiable with respect to its variables, Z_i, hence, it can be expanded using Taylor's series in the neighbourhood of a chosen point $Z^{(0)} = Z_1^{(0)} + Z_2^{(0)} + Z_3^{(0)} + Z_4^{(0)} + Z_5^{(0)}$ as follows:

$$f(Z) = \sum f^m\left(Z^{(0)}\right) + \quad \frac{\left(Z_i - Z^{(0)}\right)}{m!} \qquad (7)$$

for $0 \leq m \leq \infty$

Where m is the degree of polynomial of the response function and $f(Z)$ is the response function. Expanding equation (7) to the second order yields:

$$f(Z) = f\left(Z^{(0)}\right) + \sum_{i=1}^{5} \frac{\partial f\left(Z^{(0)}\right)}{\partial Z_i}\left(Z_i - Z_i^{(0)}\right) + \frac{1}{2}! \sum_{i=1}^{4}\sum_{j=1}^{5} \frac{\partial^2 f\left(Z^{(0)}\right)}{\partial Z_i \partial Z_j}\left(Z_i - Z_i^{(0)}\right)\left(Z_j - Z_j^{(0)}\right) + \frac{1}{2}! \sum_{i=1}^{5} \frac{\partial^2 f\left(Z^{(0)}\right)}{\partial Z_i^2}(Z_i - Z_i^{(0)})^2 + \cdots \quad (8)$$

The point $Z^{(0)}$ will be chosen as the origin for convenience sake without loss of generality of the formulation. The predictor Z_i is not the actual portion of the mixture component, rather. It is the ratio of the actual portions to the quantity of concrete. For convenience sake, let Z_i be called the term of "fractional portion". The actual portions of the mixture components are S_i.

Consequently, the origin, $Z^{(0)} = 0$.this implies that:

$Z_1^{(0)} = 0$, $Z_2^{(0)} = 0$, $Z_3^{(0)} = 0$, $Z_4^{(0)} = 0$, $Z_5^{(0)} = 0$ (9)

Let $b_0 = f(0)$, $b_i = \frac{\partial f(0)}{\partial Z_i}$, $b_{ij} = \frac{\partial^2 f(0)}{\partial Z_i \partial Z_j}$ and $b_{ii} = \frac{\partial^2 f(0)}{\partial Z_i^2}$

Equation (8) can then be written as follows:

$$f(0) = b_0 + \sum_{i=1}^{5} b_i Z_i + \sum_{i=1}^{4}\sum_{j=1}^{5} b_{ij} Z_i Z_j + \sum_{i=1}^{5} b_{ii} Z_i^2 + \cdots \quad (10)$$

Multiplying equation (4) by b_0, gives the following expression:

$b_0 = b_0Z_1 + b_0Z_2 + b_0 Z_3 + b_0Z_4 + b_0Z_5$ (11)

Similarly, multiplying equation (4) by Z_i will give the following expression:

$Z_1 = Z_1^2 + Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 + Z_1 Z_5$ (12a)

$Z_2 = Z_1 Z_2 + Z_2^2 + Z_2Z_3 + Z_2Z_4 + Z_2Z_5$ (12b)

$Z_3 = Z_1 Z_3 + Z_2Z_3 + Z_3^3 + Z_3Z_4 + Z_3Z_5$ (12c)

$Z_4 = Z_1 Z_4 + Z_2Z_4 + Z_4Z_3 + Z_4^2 + Z_4Z_5$ (12d)

$Z_5 = Z_1 Z_5 + Z_2Z_5 + Z_3Z_5 + Z_4Z_5 + Z_5^2$ (12e)

Rearranging equations (12a) to (12e), the expression for Z_i^2 becomes;

$Z_1^2 = Z_1 - Z_1Z_2 - Z_1Z_3 - Z_1Z_4 - Z_1Z_5$ (13a)

$Z_2^2 = Z_2 - Z_1Z_2 – Z_2Z_3 – Z_2Z_4 – Z_2Z_5$ (13b)

$Z_3^2 = Z_3 - Z_1Z_3 – Z_2Z_3 – Z_3Z_4 – Z_3Z_5$ (13c)

$Z_4^2 = Z_4 - Z_1Z_4 – Z_2Z_4 – Z_3Z_4 – Z_4Z_5$ (13e)

Substituting equation (13a) to (13e) into equation (10) and setting $f(0) = Y$ will give in the expanded form below:

$Y = b_0Z_1 + b_0Z_2 + b_0Z_3 + b_0Z_4 + b_0Z_5 + b_1Z_1 + b_2Z_2 + b_3Z_3 + b_4Z_4 + b_5Z_5 + b_{12}Z_1Z_2 +$

$b_{13}Z_1Z_1Z_3 + b_{14}Z_1Z_4 + b_{15}Z_1Z_5 + b_{23}Z_2Z_3 + b_{24}Z_2Z_4 + b_{25}Z_2Z_5 + b_{34}Z_3Z_4 +$

$b_{35}Z_3Z_5 + b_{45}Z_4Z_5 + b_{11}(Z_1 - Z_1Z_2 - Z_1Z_3 - Z_1Z_4 - Z_1Z_5) +$

$b_{22}(Z_2 - Z_1Z_2 - Z_2Z_3 - Z_2Z_4 - Z_2Z_5) + b_{33}(Z_3 - Z_1Z_3 - Z_2Z_3 - Z_3Z_4 - Z_3Z_5) +$

$b_{44}(Z_4 - Z_1Z_4 - Z_2Z_4 - Z_3Z_4 - Z_4Z_5) - b_{55}(Z_5 - Z_1Z_5 - Z_2Z_5 - Z_3Z_5 - Z_4Z_5)$

(14a)

Factorizing equation (14a) yields

$$\begin{aligned} Y = {} & Z_1(b_0 + b_1 + b_{11}) + Z_2(b_0 + b_2 + b_{22}) + Z_3(b_0 + b_3 + b_{33}) + Z_4(b_0 + b_4 + b_{44}) + Z_5(b_0 + b_5 + b_{55}) \\ & + Z_1Z_2(b_{12} - b_{11} - b_{22}) + Z_1Z_3(b_{13} - b_{11} - b_{33}) + Z_1Z_4(b_{14} - b_{11} - b_{44}) \\ & + Z_1Z_5(b_{15} - b_{11} - b_{55}) + Z_2Z_3(b_{23} - b_{22} - b_{33}) + Z_2Z_4(b_{24} - b_{22} - b_{44}) \\ & + Z_2Z_5(b_{25} - b_{22} - b_{55}) + Z_3Z_4(b_{34} - b_{33} - b_{44}) + Z_3Z_5(b_{35} - b_{33} - b_{55}) \\ & + Z_4Z_5(b_{45} - b_{44} - b_{55}) \qquad (14b) \end{aligned}$$

The summation of the constants is equal to a constant thus; let

$$\propto_i = b_0 + b_i + b_{ii} \; and \;\; \propto_{ij} = b_{ij} + b_{ii} + b_{jj} \qquad (14c)$$

Equation (14b) becomes:

$Y = \propto_1Z_1 + \propto_2Z_2 + \propto_3Z_3 + \propto_4Z_4 + \propto_5Z_5 + \propto_{12}Z_1Z_2 + \propto_{13}Z_1Z_3 + \propto_{14}Z_1Z_4 + \propto_{15}Z_1Z_5 + \propto_{23}Z_2Z_3 +$

$\propto_{24}Z_2Z_4 + \propto_{25}Z_2Z_5 + \propto_{34}Z_3Z_4 + \propto_{35}Z_3Z_5 + \propto_{45}Z_4Z_5$ (15a)

or

$$Y = \sum_{i=1}^{5} \alpha_i Z_i + \sum_{1 \le i \le j}^{5} \alpha_{ij} Z_i Z_j \qquad (15b)$$

Where, Y is the response function at any point of observation Z_i and Z_j are the predictors; $\propto_i$ and $\propto_{ij}$ are the coefficient of the regression equation.

The Coefficient of the Regression Equation

Let n^{th} response (compressive strength for the serial number n) be $Y^{(n)}$ and the vector of the corresponding set of variables be (see Table 1):

$Z^{(n)} = (Z_1^{(n)}, Z_2^{(n)}, Z_3^{(n)}, Z_4^{(n)}, Z_5^{(n)})$

Different points of observation will have different predictor at constant coefficient. At n^{th} observation point, the response function $Y^{(n)}$ will correspond with the predictors $Z_i^{(n)}$.

Thus,

$$Y^{(n)} = \sum_{i=1}^{5} \alpha_i Z_i^{(n)} + \sum_{1 \le i \le j}^{5} \alpha_{ij} Z_i Z_j^{(n)} \qquad (16)$$

Where $1 \le i \le j \le 5$ and n= 1, 2, 3,…, 15

Equation (16) can be written in a matrix form as,

$[Y^{(n)}] = [Z^{(n)}] [\propto]$ (17)

Expanding equation (17) gives:

$$\begin{pmatrix} Y^{(1)} \\ Y^{(2)} \\ Y^{(3)} \\ . \\ . \\ Y^{(15)} \end{pmatrix} = \begin{pmatrix} Z_1^{(1)} & Z_2^{(1)} & Z_3^{(1)} & \dots & Z_4^{(1)} Z_5^{(1)} \\ Z_1^{(2)} & Z_2^{(2)} & Z_3^{(2)} & \dots & Z_4^{(2)} Z_5^{(2)} \\ Z_1^{(3)} & Z_2^{(3)} & Z_3^{(3)} & \dots & Z_4^{(3)} Z_5^{(3)} \\ & & & & \\ Z_1^{(15)} & Z_2^{(15)} & Z_3^{(15)} & \dots & Z_4^{(15)} Z_5^{(15)} \end{pmatrix} \begin{pmatrix} \propto_1 \\ \propto_2 \\ \propto_3 \\ \\ \propto_{45} \end{pmatrix} . \qquad (18)$$

The actual mixture proportions $S_i^{(n)}$ and the corresponding fractional portions, $Z_i^{(n)}$ is shown in table 1.
The values of the constant coefficient **a** in equation (17) are determined with the values of $Y^{(n)}$ and $Z^{(n)}$. Rearranging equation (17) gives

$$[\propto] = [Z^{(n)}]^{-1} [Y^{(n)}]$$

Expressing equation (19) in expanded form yields:

$$\begin{pmatrix} \propto_1 \\ \propto_2 \\ \propto_3 \\ . \\ \cdot\propto_{45} \end{pmatrix} = \begin{pmatrix} Z_1^{(1)} & Z_2^{(1)} & Z_3^{(1)} & \dots & Z_4^{(1)} Z_5^{(1)} \\ Z_1^{(2)} & Z_2^{(2)} & Z_3^{(2)} & \dots & Z_4^{(2)} Z_5^{(2)} \\ Z_1^{(3)} & Z_2^{(3)} & Z_3^{(3)} & \dots & Z_4^{(3)} Z_5^{(3)} \\ & & & & \\ Z_1^{(15)} & Z_2^{(15)} & Z_3^{(15)} & \dots & Z_4^{(15)} Z_5^{(15)} \end{pmatrix}^{-1} \begin{pmatrix} Y^{(1)} \\ Y^{(2)} \\ Y^{(3)} \\ \\ Y^{(15)} \end{pmatrix} . \qquad (19)$$

The values of $\propto_1$ to $\propto_{45}$ are obtained from equation (19) and substituted into equation (15a) to obtain the regression equation. The values of Z $^{(n)}$] matrix is shown in table 2; the values inverse of [Z $^{(n)}$] matrix is presented in table 3, while the values of [Y $^{(n)}$] matrix are obtained from the experimental investigation.

II. Materials and Methods

The materials used for the laboratory test included:

i. Water used for this work was obtained from the premises of Federal University of Technology, Owerri, Nigeria. The water was clean and good for drinking.
ii. Dangote cement, a brand of ordinary Portland cement that conforms to BS 12(1978)
iii. The fine aggregate, river sand used for this research work were obtained from a Otamiri river, near Federal University of Technology, Owerri
iv. The coarse aggregate, granite chippings used was quarried from crushed rock industries quarry, Ishiagu, along Enugu-Port Harcourt express way, Ebonyi state, Nigeria. The granite has a maximum size of 20mm. They were washed and sun-dried for seven days in the laboratory to ensure that they were free from excessive dust, and organic matter.
v. Pulverised Fuel Ash (PFA) is a pozzolanic material. Pulverised Fuel Ash used as a partial replacement of cement in various mix proportion was obtained from the thermal coal station at Oji River, Enugu state, Nigeria. It was grinded and sieved 212µm sieve to obtain finer particles capable of reacting with cement, fine aggregate, coarse aggregates and water to form Pulverised Fuel Ash-cement concrete

The mix ratios used for the simplex design points were obtained using pentahedron factor space for five – component mixture.

III. Compressive Strength Test

Batching of the ingredients was done by mass. Cement/ Pulverised Fuel Ash were thoroughly mixed together with a mixture of sand and granite. The entire component was cast in concrete mould of size 150 x 150x 150 mm. The concrete cubes were cured in a curing tank for 28 days and were crushed using universal testing machines. Compressive strength of the cubes was calculated using equation 20:

$$\text{Compressive strength} = \frac{compressive \;\; load \; of \; cube \; at \; failure \;\; (N)}{cross \;\; sectional \;\; area \;\; of \; mould \;\; (mm^2)} \qquad (20)$$

The results of the compressive test for laboratory and the model are shown in Table 4

Table 1: Selcted mix proportion and corresponding component fractional based on Osadebe's second-degree polynomial For system of ∑Z =10

S/N	TRIAL MIXES									
	MIX PROPORTIONS					COMPONENT'S FRACTION				
	S_1	S_2	S_3	S_4	S_5	Z_1	Z_2	Z_3	Z_4	Z_5
1	0.57	0.95	0.05	2	4.0	0.75	1.25	0.07	2.64	5.28
2	0.5	0.90	0.10	1.2	2.4	0.98	1.76	0.20	2.35	4.71
3	0.55	0.85	0.15	1.5	2.0	1.09	1.68	0.30	2.97	3.96
4	0.6	0.80	0.20	2.1	4.2	0.76	1.01	0.25	2.66	5.32
5	0.6	0.75	0.25	1.2	4.0	0.88	1.10	0.37	1.76	5.88
12	0.535	0.925	0.75	1.6	3.2	0.84	1.46	0.12	2.53	5.05
13	0.56	0.90	0.10	1.75	3.0	0.89	1.43	0.16	2.77	4.75
14	0.585	0.875	0.125	2.05	4.1	0.76	1.13	0.16	2.65	5.30
15	0.585	0.85	0.15	1.6	4.0	0.81	1.18	0.21	2.23	5.57
23	0.525	0.875	0.125	1.35	2.2	1.03	1.72	0.25	2.66	4.33
24	0.55	0.85	0.15	1.65	3.3	0.85	1.31	0.23	2.54	5.08
25	0.55	0.825	0.175	1.2	3.2	0.92	1.39	0.29	2.02	5.38
34	0.575	0.825	0.175	1.8	3.1	0.89	1.27	0.27	2.78	4.79
35	0.575	0.8	0.20	1.35	3.0	0.97	1.35	0.34	2.28	5.06
45	0.6	0.775	0.225	1.65	4.1	0.82	1.05	0.31	2.24	5.58
CONTROL										
C_1	0.54	0.9	0.1	1.567	2.80	0.914	1.524	0.169	2.653	4.740
C_2	0.573	0.867	0.133	1.866	3.40	2.838	1.265	0.194	2.728	3.971
C_3	0.590	0.833	0.167	1.766	5.066	0.715	1.122	0.225	2.319	5.478
C_4	0.555	0.875	0.125	1.7	3.15	0.867	1.366	0.195	2.654	4.918
C_5	0.580	0.8375	0.1635	1.7	3.55	0.849	1.226	0.239	2.489	5.198
C_6	0.555	0.8625	0.1375	1.475	3.1	0.705	1.407	0.224	2.406	5.051
C_7	0.546	0.925	0.0875	1.675	3.1	0.861	1.443	0.138	2.649	4.703
C_8	0.580	0.825	0.175	1.475	3.5	0.885	1.259	0.267	2.250	5.339
C_9	0.585	0.89	0.11	1.76	3.32	0.841	1.341	0.166	2.691	5.002
C_{10}	0.569	0.85	0.15	1.6	3.32	0.810	1.311	0.231	2.468	5.120
C_{11}	0.571	0.855	0.145	1.68	3.48	0.848	1.270	0.215	2.496	5.170
C_{12}	0.567	0.835	0.165	1.61	3.34	0.870	1.281	0.253	2.470	5.125
C_{13}	0.562	0.8675	0.1325	1.555	3.26	0.881	1.360	0.208	2.438	5.112
C_{14}	0.562	0.855	0.145	1.625	3.42	0.855	1.293	0.2119	2.458	5.174
C_{15}	0.582	0.8573	0.1425	1.74	3.96	0.725	1.178	0.176	2.389	5.438

Table 2: $Z^{(n)}$ Matrix

Z_1	Z_2	Z_3	Z_4	Z_5	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_1Z_5	Z_2Z_3	Z_2Z_4	Z_2Z_5	Z_3Z_4	Z_3Z_5	Z_4Z_5
0.75	1.25	0.07	2.64	5.28	0.938	0.053	1.980	3.96	0.088	3.3	6.6	0.185	0.370	13.939
0.98	1.76	0.20	2.35	4.71	1.725	0.196	2.303	4.616	0.352	4.136	8.290	0.47	0.942	11.069
1.09	1.68	0.30	2.97	3.96	1.831	0.327	3.237	4.316	0.504	4.99	6.653	0.891	1.188	11.761
0.76	1.01	0.25	2.66	5.32	0.768	0.19	2.002	4.043	0.253	2.687	5.373	0.665	1.33	14.151
0.88	1.10	0.37	1.76	5.88	0.968	0.326	1.549	5.174	0.407	1.936	6.468	0.652	2.176	10.349
0.84	1.46	0.12	2.53	5.05	1.226	0.101	2.125	4.242	0.175	3.694	7.373	0.304	0.606	12.777
0.89	1.43	0.16	2.77	4.75	1.273	0.142	2.465	4.228	0.229	3.961	6.793	0.443	0.760	13.158
0.76	1.13	0.16	2.65	5.30	0.859	0.122	2.014	4.028	0.021	2.995	5.989	0.424	0.848	14.045
0.81	1.18	0.21	2.23	5.57	0.956	0.170	1.806	4.512	0.248	2.631	6.573	0.468	1.170	12.421
1.03	1.72	0.25	2.66	4.33	1.772	0.258	2.740	4.460	0.430	4.575	7.448	0.665	1.083	11.518
0.85	1.31	0.23	2.54	5.08	1.114	0.196	2.159	4.318	0.301	3.327	6.655	0.584	1.168	12.903
0.92	1.39	0.29	2.02	5.38	1.279	0.267	1.858	4.950	0.403	2.808	7.478	0.586	1.560	10.868

0.89	1.27	0.27	2.78	4.79	1.130	0.240	2.474	4.264	0.343	3.531	6.083	0.751	1.293	13.316
0.97	1.35	0.34	2.28	5.06	1.310	0.330	2.212	4.108	0.459	3.078	6.831	0.775	1.720	11.537
0.82	1.05	0.31	2.24	5.58	0.861	0.254	1.837	4.576	0.326	2.352	5.859	0.694	1.730	12.499

Table 3: inverse of the $Z^{(n)}$ matrix

Z_1	Z_2	Z_3	Z_4	Z_5	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_1Z_5	Z_2Z_3	Z_2Z_4	Z_2Z_5	Z_3Z_4	Z_3Z_5	Z_4Z_5
-646.173	-338.434	-118.365	-227.864	-221.896	1177.784	-374.486	931.8606	-500.627	609.8273	-1077.1	690.5306	194.6381	-169.875	437.4698
-129.648	21.18276	-25.2242	-338.434	-37.5063	134.9698	-171.024	452.3441	-177.157	42.70483	-252.712	76.93981	249.7454	-105.456	259.5039
2413.256	423.6017	-142.146	1389.092	-8.13604	-2510.62	1089.761	-4040.68	1019.163	-97.585	1689.451	-311.453	-392.897	-47.2449	-472.836
9.979907	12.37629	1.105274	37.06226	20.04836	-19.1537	8.127822	-35.5584	28.19848	-10.5298	20.23001	-21.0455	-3.39347	8.456402	-55.8046
-0.70243	-1.21023	2.260212	-1.23329	-0.661	2.883123	-2.44006	2.725709	-1.794	-0.44122	-2.35277	3.356586	-0.33916	-3.15008	3.199157
139.0292	101.2965	27.81165	166.3936	47.1372	-220.745	118.4014	-299.209	139.1869	-106.941	239.7851	-131.567	-124.554	66.72179	-162.907
-377.35	49.23741	64.51633	-256.673	50.67247	343.061	-195.925	690.3984	-149.966	-112.639	-165.384	-79.6911	51.33869	65.60519	22.52454
49.55029	69.83678	19.51135	-40.4485	-1.95064	-98.9843	26.11717	-21.904	-3.57826	-69.4002	65.61689	-26.0892	4.513161	-10.0775	37.19953
85.16856	86.98385	7.106181	37.8435	34.42751	-154.081	47.67634	-132.674	81.67711	-66.1678	138.788	-100.151	-19.1525	27.25763	-74.7882
-154.381	-44.0953	-2.21373	-120.989	-10.494	141.6209	-71.4494	287.8785	-73.5851	35.67465	-111.037	50.14617	41.22743	-20.8249	52.50349
16.20982	-6.17396	-1.34515	44.80269	9.237299	-15.7313	17.8185	-62.5088	31.65995	3.707483	39.26193	-20.9153	-31.0027	17.11072	-42.1414
4.480164	-8.14336	3.801717	26.28411	-1.06366	1.400644	12.61005	-28.8556	4.92917	-2.98015	12.14311	5.002887	-22.1311	6.388119	-13.8778
-217.843	-37.032	9.710062	-89.2821	12.73257	227.0139	-98.6144	324.6998	-50.5042	11.44769	-134.587	-1.63347	27.82893	21.13685	-5.13301
-258.563	-54.4648	15.37428	-149.431	-7.14505	275.3811	-110.394	432.9805	-126.299	15.0154	-190.351	53.33176	37.376	-3.20157	70.33064
-1.65721	-2.08884	-0.63172	-3.27577	-2.66602	2.88902	-0.57328	4.433649	-3.78464	2.044158	-2.56061	2.406579	-0.94574	0.237125	6.173588

Table 4: Compressive strength in N/mm^2 of 28^{th} day old concrete cubes

Point of observation	Compressive strength of Replication 1 (N/mm^2)	Compressive strength of Replicate 2 (N/mm^2)	Compressive strength of Replicate 3 (N/mm^2)	Mean compressive strength (N/mm^2)	Osadebe model compressive strength result (N/mm^2)
1	42.66	42.66	42.22	42.51	42.51
2	34.22	39.55	40.88	38.21	38.21
3	31.55	34.46	34.22	33.41	33.41
4	41.33	42.66	38.22	40.73	40.73
5	21.33	24.44	20.00	21.92	21.92
12	41.33	39.11	47.55	42.66	42.66
13	40.88	40.00	40.77	40.55	40.55
14	35.11	35.55	32.00	34.22	34.22
15	31.11	28.44	28.88	29.47	29.47
23	30.22	32.88	31.11	31.40	31.40
24	33.77	34.22	33.77	33.92	33.92
25	33.33	31.11	35.55	33.23	33.33
34	26.66	29.33	29.77	28.58	28.58
35	24.88	29.33	27.11	27.10	27.10
45	17.77	18.66	18.66	18.36	18.36
C_1	38.11	42.66	43.55	41.44	41.58
C_2	33.00	33.88	32.44	33.12	30.12
C_3	33.55	20.11	26.22	23.29	23.29
C_4	30.88	34.66	39.55	35.03	31.07
C_5	24.55	24.11	24.55	24.40	23.09
C_6	33.33	32.88	33.33	33.18	33.58
C_7	40.40	40.00	40.77	40.39	43.77
C_8	28.44	29.33	28.88	28.88	25.77
C_9	39.11	34.67	38.22	37.33	34.39
C_{10}	29.33	30.11	29.22	29.55	26.16
C_{11}	29.11	28.66	30.44	29.40	26.58
C_{12}	25.00	25.66	25.88	25.51	23.98
C_{13}	32.00	32.44	33.77	32.77	32.85
C_{14}	27.11	28.11	29.11	28.11	26.80
C_{15}	28.00	28.88	28.88	28.59	27.82

THE REGRESSION EQUATION

The solution of equation 19, given the responses in table 3, gives the unknown coefficient of the regression equation as follows:

$\propto_1 = -5685.38$ $\propto_{12} = 3415.259$ $\propto_{24} = 835.9397$
$\propto_2 = -6692.39$ $\propto_{13} = -3595.77$ $\propto_{25} = 553.3372$
$\propto_3 = 21943.39$ $\propto_{14} = -566.255$ $\propto_{34} = -1386.23$
$\propto_4 = 771.4913$ $\propto_{15} = 814.8985$ $\propto_{35} = -2371.04$
$\propto_5 = -18.9654$ $\propto_{23} = -2049.49$ $\propto_{45} = -72.7589$

Hence, from equation 15b, the regression equation is given by:

$$Y = -5685.38Z_1 - 6692.39Z_2 + 21943.39Z_3 + 771.4913Z_4 - 18.9654Z_5 + 3415.259Z_1Z_2 -3595.77Z_1Z_3 -566.255Z_1Z_4 + 814.8985Z_1Z_5 -2049.49Z_2Z_3 + 835.9397Z_2Z_4 + 553.3372 Z_2Z_5 - 1386.23Z_3Z_4 - 2371.04Z_3Z_5 -72.7589Z_4Z_5 \quad (21)$$

Equation (21) is the mathematical model for the optimization of compressive strength of Pulverised Fuel Ash-cement concrete based on Osadebe's second-degree polynomial.

TEST OF ADEQUACY OF THE MODEL

The test for adequacy for Osadebe's Regression Model was done using Fischer test at 95% accuracy level. The compressive strength at the control points
(i.e. $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}$). In this, two hypotheses were set aside:

a) **Null Hypothesis**: At 95% accuracy level, that there is no significant difference between the laboratory concrete cube strength and model predicted strength results.

b) **Alternative Hypothesis:** At 95% accuracy level, there is a significant difference between the laboratory concrete cube strength and model predicted strength results.

The test is carried out as shown in Table 5 .

Table 5: Fischer-statistical test computations for Osadebe's Regression Model

Control points	y_e	y_m	$y_e - \bar{y}_e$	$y_m - \bar{y}_m$	$(y_e - \bar{y}_e)^2$	$(y_m - \bar{y}_m)^2$
C_1	41.44	41.38	10.13	11.34	102.62	128.69
C_2	33.11	30.12	1.8	0.08	3.24	0.006
C_3	23.29	23.39	-8.02	-6.63	64.32	44.22
C_4	33.69	31.07	2.38	1.03	5.66	1.06
C_5	24.40	23.09	-6.91	-6.95	47.75	48.30
C_6	33.14	33.38	1.86	3.34	3.46	11.16
C_7	40.39	43.77	9.08	13.73	82.45	88.51
C_8	28.88	25.77	-2.43	-4.27	5.90	18.23
C_9	37.33	34.39	6.02	4.35	36.24	18.92
C_{10}	29.55	26.16	-1.39	-3.88	1.93	15.05
C_{11}	29.40	26.58	-1.91	-3.46	3.65	11.97
C_{12}	25.51	23.98	-5.8	-6.06	33.64	36.72
C_{13}	32.73	32.85	1.43	2.81	2.04	7.80
C_{14}	28.11	26.80	-3.20	-3.29	10.24	10.50
C_{15}	28.59	27.82	-2.72	-2.22	7.40	4.93
Sum	$\sum ye = 469.6$	$\sum ym = 450.55$			$\sum (ye - \bar{y}e)^2 = 410.54$	$\sum (ym - \bar{y}_m)2 = 546.076$
Mean	$\bar{y}e = 31.31$	$\bar{y}_m = 30.04$				

Note: ye is the experimental compressive strength, while ym is the model compressive strength

$$S_e^2 = \frac{\sum (ye - \bar{y}e)^2}{N-1} = \frac{410.39}{14} = 29.32$$

$S_m^2 = \frac{\sum(ym - \bar{y}m)^2}{N-1} = \frac{546.076}{14} = 39.00$

$F_{calculated} = \frac{S_1^2}{S_2^2}$

Where S_1^2 is the greater of S_e^2 and S_m^2, while S_2^2 is the smaller of the two.
Here $S_1^2 = S_m^2 = 39.00$ and $S_2^2 = S_e^2 = 29.32$

$F_{calculated} = \frac{39.00}{29.32} = 1.330$

The model is acceptable at 95% confidence level if:

$\frac{1}{F_{\propto(V1,V2)}} < \frac{S_1^2}{S_2^2} < F_{\propto(V1,V2)}$

Where
Significant level, $\propto = 1 - 0.95 = 0.05$
Degree of freedom, V = N-1 = 15-1= 14
From standard F-statistic table, $F_{\propto(V1,V2)} = 2.443$ and.

$\frac{1}{F_{\propto(V1,V2)}} = \frac{1}{2.443} = 0.4093$

Hence the condition: $\frac{1}{F_{\propto(V1,V2)}} < \frac{S_1^2}{S_2^2} < F_{\propto(V1,V2)}$ which is 0.4093< 1.330 < 2.443 is satisfied.

Therefore, the null hypothesis that "there is no significant difference between the experimental and the model expected result" is accepted. This implies that Osadebe's Regression Model is adequate.

IV. Conclusion

Using Osadebe's second degree polynomial regression equation, mix design model for a five component Pulverised Fuel Ash- cement concrete cube was developed. This model could predict the compressive strength of PFA-Cement concrete cube when the mix ratios are known and vice versa. The predictions from this model were tested at 95% accuracy level using statistical Fischer test and found to be adequate.

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