

Scheduling Repetitive Construction Projects Using Fuzzy Linear Programming

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Abstract: *Scheduling problem for repetitive construction projects involves three conflicting objectives. These objectives are project duration, project total cost, and project total interruption time. This paper presents a multi-objective fuzzy linear programming model (FLP) for resolving this problem. Literature concerned with scheduling problems for repetitive construction projects was reviewed. Multi-objective fuzzy linear programming was then explained. The proposed model formulation was then presented.*

A bridge project from pertinent literature was selected for model validation purpose. An optimization of each individual objective was performed with a linear programming (LP) software (Lindo) that gave the upper and lower bounds for the multi-objective analysis. Fuzzy linear programming was then applied to optimize the solution. Two scenarios were adopted in solution. In the first scenario, the three above objectives were considered simultaneously. Analysis of the results revealed that project duration, and project total cost are deviated only by 7.2%, and 1.8%, respectively as compared to their corresponding ideal values in the crisp LP model. On the other hand, the percentage of total interruption time to project duration in FLP is 5.3% against zero in LP. In the second scenario, each two objectives were considered in a single run. The purpose is to explain how can the model's user generate and evaluate the optimal tradeoff solution between any two objectives that suit his demands.

Keywords: *Fuzzy Linear Programming; Linear Programming; Repetitive Construction Projects; Multi Objective Analysis.*

I. Introduction

Linear repetitive construction projects require large amounts of resources which are used in a sequential manner. Therefore effective resource management is very important in terms of project total cost, duration, and interruption time. Interruption time in repetitive projects is the queuing time between two activities of the same type at two consecutive units (Liu and Wang) [1].

The methodologies such as critical path method and the repetitive scheduling method optimize the schedule with respect to a single factor. This factor may be to achieve minimum project duration, project total cost, or to minimize resource work breaks. However, real life scheduling decisions are more complicated and project managers must make decisions that address the various cost elements in a holistic way (Ipsilandis) [2].

LP is a technique that widely used for optimal decision-making in a rigid environment. In LP model, the objective function and the constraints cannot be represented precisely in linguistic form causing difficulty in representing a real world problem. Usually, decision making in real world takes place in an environment in which the goals, constraints and the consequences of possible actions are not known precisely. Uncertainty has been one of the major factors that influence project performance and determines its ultimate success. To obtain more realistic solution to the problems, a degree of flexibility is required to be introduced into the crisp constraint inequalities. Accordingly, certain degree of flexibility is to be incorporated in the model parameters of the LP solution to optimize more than one objective simultaneously. Fuzziness in the problem stems from the imprecise aspiration levels attained by the decision maker to the objectives. The objective function of the FLP is to maximize the membership value of intersection of the objectives, which forms the fuzzy decision (Faheem et al.) [3].

This paper introduces a multi objective fuzzy linear programming model for scheduling repetitive construction projects, which takes into consideration project duration, project total cost, and total interruption time simultaneously. Chang et al. [4] explained the advantage of fuzzy multi objective optimization over deterministic approach as; 1) fuzzy uncertainties embedded in the model parameters can be directly reflected and communicated into the optimization process; 2) the variation of the decision maker's aspiration level in the model can be incorporated and thereby generate a more confident solution set for decision maker; 3) Regardless of the orientation of decision maker's aspiration level (maximization or minimization), each objective has its own independent membership function and different aspiration level.

The paper is organized as follows. The first section is devoted to review literature concerned with linear scheduling problems. The second section explains fuzzy linear programming as an optimization

technique. The third section presents the proposed model formulation. Following, a bridge project from pertinent literature is selected for model validation purpose. Analysis of the bridge example helps indicate the contributions of the proposed model. Conclusions are drawn in the last section.

II. Literature Review

Numerous techniques for handling linear scheduling problems have been developed in recent decades. Some studies have adopted heuristic algorithm for repetitive activity scheduling process such as Bragadin and Kahkonen [5]. The algorithm is a semi/automatic procedure that can help inexperienced planner in repetitive construction project scheduling. The optimization process carry out resource timing in two phases: resource–space network implementation and schedule optimization. In the first phase a traditional Precedence Diagramming Method network is plotted on a resource – space chart and the as soon as possible project schedule is performed. In the second phase the algorithm search for resource scheduling optimization by minimization of resource idle time in every resource path on repetitive space units. The work continuity constraint is relaxed in order to maintain the Precedence Diagramming Method minimum project duration. Ammar [6] proposed an integrated CPM and LOB model to schedule repetitive projects in an easy non graphical way considering both logic dependency and resource continuity constraints. Overlapping activities of a single typical unit are used to model duration and logical relationships of repetitive activities.

Another studies used dynamic programming such as Selinger [7] to minimize project duration for linear construction projects. Russell and Caselton [8] extended the work of Selinger in developing a two state variable, N-stage dynamic programming formulation that minimizes the duration of linear construction projects. Handa and Barcia [9] presented an integer dynamic programming model to optimize the project duration. Additional research has utilized dynamic programming in minimizing total cost or project duration by integrating cost, time, or heuristic rules (El-Rayes and Moselhi; Moselhi and El-Rayes; Senouci and El-din; Moselhi and Hassanien)[10-13].

Another group of studies have adopted linear programming and integer programming. Perera [14] proposed a linear programming model to maximize the construction rate of the activities in a repetitive project. Reda [15] utilized a linear programming formulation to minimize the project direct cost for a given project duration. Huang and Halpin [16] proposed a graphical based approach called POLO system to assist in the linear programming modeling of linear scheduling problems. Mattila and Abraham [17] presented an integer programming model for leveling the resources of activities in a linear construction project.

Given the rapid development of computer based techniques, researchers have used artificial intelligence techniques, such as knowledge based systems, neural networks and genetic algorithm to solve the increasing complexity of construction projects. For example, Shaked and Warszawskiz [18] developed a knowledge based system for construction planning of high rise buildings. Adeli and Kariem [19] developed a neural dynamic model to schedule and optimize repetitive projects. They applied the model for highway construction scheduling. Hegazy and Ayed [20] used a neural network approach to manage construction cost data and developed a parametric cost estimating model for highway projects. Leu and Hwang [21] addressed a GA-based resource constrained linear scheduling model. Hyari and El-Rayes [22] constructed a multi objective optimization model that includes genetic algorithm for planning and scheduling repetitive construction projects. This model helps planners in evaluating optimum plans by minimizing project duration and maximizing work continuity simultaneously. Agrama [23] presented a practical and efficient model for time and cost optimizations of horizontal and vertical repetitive works on a spread sheet interface. The model employed the genetic algorithm as an optimization technique. Elbeltagi et al. [24] presented a model for planning and scheduling of repetitive projects. The objective was to optimize total construction cost with a genetic algorithm procedure to search for the optimum schedule. Liu and Wang [1] presented a flexible model depending on constraint programming for linear scheduling problems that accommodates different optimization objectives such as minimizing project total cost or duration. Ipsilandis [2] presented a multi objective linear programming model for scheduling linear repetitive projects, which takes into consideration cost elements regarding the project's duration, the idle time of resources, and the delivery time of the project's units.

Fuzzy linear programming was recently applied as a new technique for handling optimization of multi objective problems. Raju and Kumar [25] developed a FLP model for the evaluation of management strategies of irrigation for a case study of Sri Ram Sagar project, Andhra Pradesh, India. Three conflicting objectives; net profits, crop production and labour employment were considered in the irrigation planning scenario. Kumar et al. [26] applied fuzzy linear programming in construction projects. They illustrated the practicability of applying fuzzy linear programming to civil engineering problem and the potential advantages of the resultant information. Trakiris and Spiliotis [27] applied FLP for problems of water allocation under uncertainty. In their work, a fuzzy set representation of the unit revenue of each use together with a fuzzy representation of each set of constraints, were used to expand the capabilities of the linear programming formulations. Eshwar and Kumar [28] used FLP to identify the optimum number of pieces of equipment required to complete the construction project in the

targeted period with fuzzy data. Mohan and Jothi [29] used FLP for optimal crop planning for irrigation system dealing with the uncertainty and randomness for the various factors affecting the model. Cross and Cabello [30] applied fuzzy set theory to optimization problems, where multiple goals exist. They solved a multi-objective LP problem with fuzzy parameters for borrowing/lending problem. Faheem et al. [3] demonstrated the applicability of fuzzy linear programming for project least-cost scheduling. They presented a practical application of fuzzy linear programming in a real-life project network problem with two objectives. These objectives were minimum completion time and crashing costs required to be optimized simultaneously. Regulwar and Gurav [31] developed a multi objective fuzzy linear programming approach for crop planning in command area of Jayakwadi project stage I, Maharashtra State, India. Four objectives were optimized (maximized) simultaneously. These objectives were the Net Benefits (NB), Crop/Yield Production (YP), Employment Generation (EG) and Manure Utilization (MU). However, literature review demonstrated that FLP has not been adopted for linear scheduling problems of repetitive construction projects for optimization purposes. This paper presents a multi-objective fuzzy linear programming for repetitive construction projects by incorporating three objectives simultaneously; minimization project duration, minimization project total cost, and minimization of total interruption time.

III. Multi Objective Fuzzy Linear Programming

Raju and Kumar [25] explained that fuzzy linear programming problem associates fuzzy input data by fuzzy membership functions. They added that FLP model assumes that objectives and constraints in an imprecise and uncertain situation can be represented by fuzzy sets. The fuzzy objective function can be maximized or minimized. In FLP the fuzziness of available resources is characterized by the membership function over the tolerance range (Raju and Kumar)[25]. However, in conventional LP, the problem is defined as follows (Zimmerman)[32]:

$$\text{Maximize } Z = CX \tag{1}$$

$$\text{Subject to } AX \leq B \tag{2}$$

$$\text{And } X \geq 0 \tag{3}$$

In the fuzzy linear programming the problem can be restated as

Find X such that

$$CX \leq Z \tag{4}$$

$$AX \leq B \tag{5}$$

$$\text{and } X \geq 0 \tag{6}$$

The membership function of the fuzzy set "decision model" [$\mu_D(X)$] is given by Eq.7

$$\mu_D(X) = \min_i \{ \mu_i(X) \}; i=1,2, n \tag{7}$$

$\mu_i(X)$ can be interpreted as the degree to which X fulfils the fuzzy inequality $CX \leq Z$ and n is the number of objective functions. In the planning scenario, decision maker is not interested in a fuzzy set but in crisp optimum solution, maximizing Eq.7 gives Eq. 8.

$$\text{Max}_{X \geq 0} \mu_D(X) = \text{Max}_{X \geq 0} \min_i \{ \mu_i(X) \} \tag{8}$$

Membership function $\mu_i(X)$ is represented as

$$\begin{aligned} \mu_i(X) &= 0 && \text{for } Z \leq Z_L \\ &= \frac{Z - Z_L}{Z_U - Z_L} && \text{for } Z_L < Z < Z_U \\ &= 1 && \text{for } Z \geq Z_U \end{aligned} \tag{9}$$

Z_U = Aspired level of objective

Z_L = Lowest acceptable level of objective

$\mu_i(X)$ reflects the degree of achievement. Value of $\mu_i(X)$ will be 1 for perfect achievement and 0 for no achievement (worst achievement) of a given strategy and some intermediate values otherwise. The model can be transformed as follows:

$$\text{Max}_{X \geq 0} \min_i \frac{Z - Z_L}{Z_U - Z_L} \tag{10}$$

Subject to

$$AX \leq B \tag{11}$$

$$X \geq 0 \tag{12}$$

Introducing a new variable λ , the FLP problem can be formulated as equivalent LP model.

Max λ

Subjected to

$$\frac{Z - Z_L}{Z_U - Z_L} \geq \lambda \tag{13}$$

For each objective

$$AX \leq B \tag{14}$$

$$0 \leq \lambda \leq 1 \tag{15}$$

$$X \geq 0 \tag{16}$$

and all the existing constraints

Briefly the FLP algorithm is divided into six steps:

1. Solve the problem as a linear programming problem by taking only one of the objectives at a time.
2. From the results of step 1, determine the corresponding values of every objective at each solution derived.
3. From step 2, best (Z_U) and worst (Z_L) values can be calculated.
4. Formulate the linear membership function.
5. Formulate the equivalent linear programming model for the fuzzy multi objective.
6. Determine the compromise solution along with degree of truth (λ).

IV. Model Formulation

The aim of this study is to identify the optimum solution that minimizes project duration, project total cost, and project total interruption simultaneously under a set of constraints. These constraints should fit the characteristics of scheduling repetitive construction projects which will be described in the following subsections:

4.1 Activities Logical Relationships

Four typical scheduling relationships and job continuity logic of repetitive activities are shown as follows:

Finish to Start (FS)

$$S_j^i \geq F_j^{i-1} \tag{17a}$$

Start to Start (SS)

$$S_j^i \geq S_j^{i-1} \tag{17b}$$

Finish to Finish (FF)

$$F_j^i \geq F_j^{i-1} \tag{17c}$$

Start to Finish (SF)

$$F_j^i \geq S_j^{i-1} \tag{17d}$$

Where:

S_j^i : Start date of repetitive activity type i in section j .

F_j^i : Finish date of repetitive activity type i in section j .

Also, for each activity, the following precedence logic is used.

$$F_j^i = S_j^i + D_j^i \tag{18}$$

Where:

D_j^i : Duration of repetitive activity type i in section j .

4.2 Duration Constraints

For activity linear time/direct cost relationship, Eq.19 is used

$$D_{Lj}^i \leq D_j^i \leq D_{Uj}^i \tag{19}$$

Where:

D_{Lj}^i Lower limit of duration of repetitive activity type i in section j .

D_{Uj}^i Upper limit of duration of repetitive activity type i in section j .

For activity discrete time/direct cost relationship, Eq(s). 20 and 21 are used

$$D_j^i = \sum_{k=1}^K D_{jk}^i B_{jk}^i \tag{20}$$

$$\sum_{k=1}^K B_{jk}^i = 1 \tag{21}$$

Where:

D_{jk}^i Duration of repetitive activity i in section j for crew formation k .

B_{jk}^i Integer (zero/one) variable for activity i in section j for crew formation k

K Number of crew formations for activity i .

4.3 Interruption Constraint

For each crew formation, queuing time between two activities of the same type at two consecutive units is defined as an interruption. Eq.22 represents the interruption constraint.

$$S_j^i = F_{j-1}^i + Inter_{j-1}^i, \quad j \geq 2 \tag{22}$$

Where:

F_{j-1}^i Finish date of repetitive activity type i in section $j-1$

$Inter_{j-1}^i$ Interruption time of repetitive activity type i between section $j-1$ and j

4.4 Objective Functions

Objective 1: Minimization of project duration (PD)

Minimization of project duration (PD) can be expressed as

$$\text{Min. PD} = F_{Lj}^{Li} \tag{23}$$

Where:

F_{Lj}^{Li} Finish date of last repetitive activity Li in last section Lj

Objective 2: Minimization of project total cost (TC)

Minimization of project total cost (TC) can be expressed as

$$\text{Min. TC} \tag{24}$$

In this research, project total cost (TC) equals to the sum of direct cost (DC), indirect cost (IC), the total penalty interruption cost (TPIC) for the whole project and contract penalty/bonus cost (CPC) as illustrated in Eq.25. Direct cost comprises material, equipment, and labor costs (see Eq. 26). Indirect cost calculated on a daily basis is defined as given in the expression of Eq. (27). In addition to direct and indirect cost a penalty interruption cost which is the penalty applied when work continuity is violated (i.e. when an interruption to a crew occurred) should be estimated. Penalty interruption cost is applied to the total interruption time for each activity; Eq(s) 28-31 illustrate this issue.

$$TC = DC + IC + TPIC + CPC \tag{25}$$

$$DC = MC + EC + LC \tag{26}$$

$$MC = \sum_1^I \sum_1^J Q_j^i * MC^i$$

$$(26a)$$

$$LC = \sum_1^I \sum_1^J D_j^i * LC_k^i \tag{26b}$$

$$EC = \sum_1^I \sum_1^J D_j^i * EC_k^i \tag{26c}$$

$$IC=ICP*PD \tag{27}$$

$$TPIC = \sum_1^I TPI C^i \tag{28}$$

$$TPI C^i = Interr^i * PIC_k^i \tag{29}$$

$$Interr^i = \sum_{j=2}^J Interr_{(j-1)}^i \tag{30}$$

$$PIC_k^i = LC_k^i + EC_k^i \tag{31}$$

$$CPC=P (PD-D_{target}) \tag{32}$$

Where:

MC Total material cost

LC Total labor cost

EC Total equipment cost

I Number of activities

J Number of sections

MC^i Unit material cost per cubic meter for repetitive activity type i .

LC_k^i Unit labor cost per day with crew formation k for repetitive activity type i .

EC_k^i Equipment cost per day with crew formation k for repetitive activity type i

ICP Indirect cost per day

$TPI C^i$ Total penalty interruption cost for activity i .

PIC_k^i Penalty interruption cost per time unit for activity i with crew formation k

$Interr^i$ Total interruption of activity i

P Daily penalty/bonus cost

D_{target} Targeted project duration

For activity linear time/direct cost relationship, in addition to total material cost, a linear equation is extracted, between labor & equipment costs and duration for each repetitive activity at each section. The penalty interruption cost per time unit for activity i will be the direct cost of labor and equipment in normal condition. Thus, TPIC is calculated using Eq.(s) 28-31. For discrete activity time/direct cost relationship, in addition to total material cost, Eq.(s) 33 and 34 are used for calculating direct cost of labor and equipment for each repetitive activity i at each section j (C_j^i). Eq.(s) 28, and 35-38 are used to estimate TPIC for discrete activity time/ direct cost relationships.

$$C_j^i = \sum_{k=1}^K (LC_{jk}^i + EC_{jk}^i) D_{jk}^i B_{jk}^i \tag{33}$$

$$\sum_{k=1}^K B_{jk}^i = 1 \tag{34}$$

$$CI_{j-1}^i = \sum_{k=1}^K (PIC_{(j-1)k}^i) * Interr_{(j-1)k}^i * L_{(j-1)k}^i, \quad j \geq 2 \tag{35}$$

$$Interr_{(j-1)}^i = \sum_{k=1}^K Interr_{(j-1)k}^i * L_{(j-1)k}^i \quad j \geq 2 \tag{36}$$

$$\sum_{k=1}^k L_{(j-1)k}^i = 1 \tag{37}$$

$$TPI C^i = \sum_{j=2}^J CI_{j-1}^i \tag{38}$$

Where:

LC_{jk}^i Labor cost of repetitive activity i in section j for crew formation k .

EC_{jk}^i Equipment cost of repetitive activity i in section j for crew formation k .

CI_{j-1}^i Penalty interruption cost of repetitive activity i between sec. $j-1$ and j .

$PIC_{(j-1)k}^i$ Penalty interruption cost per time unit of repetitive activity i between sec. $j-1$ and j for crew formation k .

$Interr_{(j-1)k}^i$ Interruption time of repetitive activity i between section $j-1$ and j for crew formation k .

$L_{(j-1)k}^i$ Integer (zero/one) variable for repetitive activity i in section j for crew

Formation k concerned with interruption and penalty interruption cost

Objective 3: Minimization of total interruption time (TIT)

Minimization of total interruption time (TIT) for the whole project can be expressed as

$$\text{Min TIT} = \sum_{i=1}^I \sum_{j=2}^J Interr_{(j-1)k}^i \tag{39}$$

V. Model Validation

The bridge example originally introduced by El-Rayes and Moselhi [10] is adopted to validate the proposed model. Table 1 shows the data of bridge example. The indirect cost is \$1000 per day. In this contract, there is no penalty or bonus cost. An individual optimization for each objective will be performed and a comparison of solutions will then be presented through the following subsections. Two scenarios will be adopted in applying Multi objective FLP. The first scenario, considers optimization of the three objectives simultaneously. The second scenario, considers optimization of each two objectives in a single run.

5.1 Individual Optimization

An optimization of each individual objective: project duration, project total cost, and total interruption time is performed with linear programming software (Lindo). Discrete direct cost/ time relationships were developed for activities foundation, columns, beams, slabs for calculating cost of labor, equipment and penalty interruption cost. Comparison of solutions for the above objectives is given in Table 2. It can be seen that the objectives are conflict with one another. Thus, there is a need to strike a balance and develop a tradeoff relationship between minimizing project duration, minimizing project total cost, and minimizing total interruption time (maintaining work continuity). The goal is to select a compromise alternative to meet the chosen levels of satisfaction as would be demanded in the decision making process. The upper and lower bounds for the multi-objective analysis was obtained and presented in Table 3. Ideal and worst values are denoted with an asterisk and plus, respectively.

5.2 Multi Objective Fuzzy Linear Programming

Since the objective is to minimize project duration, project total cost, and project total interruption time simultaneously, best values (Z_U) will be the minimum values obtained in individual optimization process for each objective. This implies that worst values (Z_L) will be the maximum values obtained in individual optimization process. Also, Eq.13 will become as presented in Eq.39. Best values (Z_U) and worst values (Z_L) are substituted in Eq.40 for each objective. Eq.(s)41-44, and all the existing constraints constitute the complete formulation for the example problem.

$$\frac{Z - Z_L}{Z_U - Z_L} \leq \lambda \tag{40}$$

Table 1: Example Data for Model Validation (El-Rayes and Moselhi [10])

Repetitive activity	Quantity (m ³)				Crew formations' data					Mater. cost (\$/m ³)
	sec.1	sec.2	sec.3	sec.4	Crew formation No.	Crew size	Output (m ³ /day)	Labor cost (\$/day)	Equip. cost (\$/day)	
Excavation	1147	1434	994	1529	1	6	91.75	340	566	0
Found.(s)	1032	1077	943	898	1	10	89.77	3804	874	92
					2	8	71.81	2853	655	92
					3	6	53.86	1902	436	92
Columns	104	86	129	100	1	10	5.73	1875	285	479
					2	12	6.88	2435	371	479
					3	14	8.03	3000	456	479
Beams	85	92	104	80	1	7	9.90	3934	315	195
					2	6	8.49	3238	259	195
					3	5	7.07	2544	204	195
Slabs	0	138	114	145	4	4	5.66	1850	148	195
					1	9	8.73	2230	177	186
					2	8	7.76	1878	149	186

Max λ subjected to (41)

$$\frac{PD - 128.3}{107 - 128.3} \leq \lambda \tag{42}$$

$$\frac{TC - 1549724.9}{1477141.8 - 1549724.9} \leq \lambda \tag{43}$$

$$\frac{TIT - 16.9}{zero - 16.9} \leq \lambda \tag{44}$$

Results for the optimum values for the three objectives when applying FLP (scenario 1) are presented in Table 3. It is observed that project duration, and total project cost, are deviated by 7.2% and 1.8%, respectively with degree of truth (λ) 0.638 as compared to ideal values in the crisp linear programming (LP) model. On the other hand, the total interruption days are only 6.1 days in FLP against zero in LP. The results of applying FLP for each two objectives in a single run (scenario 2) are also given in Table 3. Scenario 2 enables the model's user to choose the optimal tradeoff solution that suits his ordering of preferences and priorities. Details of the schedule for scenario 1 are given in Table 4.

Table 2: Comparison of Solutions for Duration, Total Cost, and Interruption Minimization

Section	Activity									
	Excavation		Foundation		Columns		Beams		Slabs	
	S*	F**	S	F	S	F	S	F	S	F
Project duration minimization	0.0	12.5	12.5	26.9	26.9	39.8	39.8	51.8		
Sec. 1	12.5	28.1	28.1	40.1	40.1	50.8	51.8	61.1	61.1	76.9
Sec. 2	28.1	38.9	40.3	50.8	50.8	66.9	66.9	77.4	77.4	90.4
Sec. 3	38.9	55.5	55.5	68	68	80.5	80.5	88.5	90.4	107
Sec. 4	0		6.1		1.4		8.9		0.5	
Interruption										
Project duration	107 days									
Project total cost	\$1549724.9									
Total interruption	16.9 days									

Project total cost minimization	0	12.5	12.5	31.7	31.7	46.8	46.8	61.8		
Sec. 1	12.5	28.1	31.7	46.7	47.3	59.8	62.3	78.6	78.6	96.4
Sec. 2	28.1	38.9	46.7	59.8	59.8	78.6	78.6	97	97	111.7
Sec. 3	38.9	55.5	59.8	76.5	78.6	93.1	97	111.1	111.7	128.3
Sec. 4	0		0		0.5		0.5			0.6
Interruption										
Project duration	128.3 days									
Project total cost	\$1477141.8									
Total interruption	1.6 days									
Total interruption minimization	0	12.5	12.5	31.7	31.7	46.8	46.8	58.8		
Sec. 1	12.5	28.1	31.7	46.7	46.8	57.5	58.8	75.1	75.1	90.9
Sec. 2	28.1	38.9	46.7	57.2	57.5	73.6	75.1	89.8	90.9	103.9
Sec. 3	38.9	55.5	57.2	69.7	73.6	86.1	89.8	103.9	103.9	122.6
Sec. 4	0		0		0		0			0
Interruption										
Project duration	122.6 days									
Project total cost	\$1491469									
Total interruption	0 days									
* Start time of activity	** Finish time of activity									

Table 3: Ideal Values for Individual Optimization, Three and Two Objectives FLP

Objective	Individual optimization			Three objectives FLP
	Min. PD	Min. TC	Min. TI	
Project duration (days)	107 ⁺	128.3 ⁺	122.6	114.7
Total project cost (\$)	1549724.9 ⁺	1477141.8 [*]	1491469	1503380.8
Total interruption (days)	16.9 ⁺	1.6	0 [*]	6.1
Associated (λ)				0.638
Objective	Two objectives FLP			---
	Min (PD+TC)	Min. (PD+TI)	Min. (TC+TI)	
Project duration (days)	114.7	113.8	125.6	
Total project cost (\$)	1503380.8	1518988.9	1479351.6	
Total interruption (days)	5.7	5.4	0	
Associated (λ)	0.638	0.680	0.969	

Table 4: Optimum schedule for three objectives fuzzy linear programming

Section	Activity									
	Excavation		Foundation		Columns		Beams		Slabs	
	S	F	S	F	S	F	S	F	St	F
Fuzzy linear programming										
Sec. 1	0	12.5	12.5	26.9	26.9	42	42	57		
Sec. 2	12.5	28.1	28.1	40.1	42	52.7	57	67.8	67.8	84
Sec. 3	28.1	38.9	40.1	53.2	53.2	69.3	69.3	84	84	97.7
Sec. 4	38.9	55.5	55.5	68	69.5	84	84	98.1	98.1	114.7
Interruption	0		3.5		0.7		1.5		0.4	
Project duration	114.7 days									
Project total cost	\$1503380.8									
Total interruption	6.1 days									

VI. Summary and Conclusions

This study presents a flexible model for handling the optimization problems for linear construction projects. The proposed model depends on multi-objective fuzzy linear programming for optimizing project duration, project total cost, and total interruption time simultaneously. A case study from pertinent literature was used for model validation purposes.

An individual optimization for each objective was performed separately with linear programming software (Lindo) that gave the upper and lower bounds for the multi-objective analysis. Two scenarios for multi-objective solutions were adopted. The first scenario considers the three objectives simultaneously, whereas the second scenario considers each two objectives in a single run. Examining the results of the case study revealed that; (1) fuzzy linear programming is simple and suitable tool for multi-objective problems; (2) the model can be extended to any number of objectives by incorporating only one additional constraint in the constraint set for each additional objective function; (3) in scenario1; project duration, project total cost in fuzzy linear programming were deviated by 7.2% and 1.8%, respectively as compared to ideal values in the crisp linear programming. On the other hand, the percentage of total interruption time to project duration in FLP is 5.3% against zero in LP; and (4) scenario 2 explains that the model enables construction planners to generate and evaluate all optimal tradeoff solutions between any two objectives: project duration and crew work continuity; project duration, and project total cost; or project total cost and crew work continuity, that suit their ordering of preferences and demands.

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