IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 21, Issue 3 Ser. 3 (May. – June. 2024), PP 20-26
www.iosrjournals.org

A Comparison Of Several Shell Theories For Free Vibration Analysis Of Circular Cylinders.

G.Tarakrishnamanohar, Dr. M. Chandrasekhar Reddy
Research Scholar, Department Of Mechanical Engineering, Osmania University, Hyderabad, India
Professor, Department Of Mechanical Engineering, Osmania University, Hyderabad, India

Abstract
A detailed discussion of a semi-analytical approach for studying the free vibration of simply supported circular cylindrical shells is presented. This method approximates simply supported boundary conditions using the beam function. A survey of the literature indicates that beam functions are often employed to estimate shell natural frequencies. There is no need to deal with laborious computations because this approach does not need boundary condition equations. Thus verifying the correctness of this approximation method is crucial. So this strategy is used to several distinct shell theories like Donnell-Mushtari, Love-Timoshenko, Amold-Warburton, Hougton-Johns, Flugge-Byme-Iur'ye, Reissner-Naghdi-Berry, Sanders, Vlasov, Kennard-Simplified, and Soedel are the ones that make up the list. The estimated approach performed well when compared to the experimental data. Finally, the effects of length, radius, and thickness were investigated on amplitude ratios.
Keywords*: Beam Function, boundary conditions, Natural Frequency, Free Vibration, Circular Cylindrical Shell,*

Date of Submission: 03-06-2024 Date of Acceptance: 13-06-2024

I. Introduction

Cylindrical shells, like beams and plates, are practical components of a variety of engineering structures, including pipelines and ducts, vehicle bodies, space shuttles, aircraft fuselages, ship hulls, submarines, and construction buildings. However, determining the dynamic characteristics of cylindrical shells is more challenging than those of beams and plates. This is due mostly to the more complicated equations of motion for cylindrical shells when coupled with boundary conditions.

In 1973, Liessa[1] provided a detailed synthesis and discussion of shell theories, including natural frequencies and mode shape information. The Flugge theory [2] is founded on the Kirchhoff-Love hypothesis for thin elastic shells. This theory may be used to calculate strain-displacement relationships and curvature changes on the center surface of a cylindrical shell. The simplified Donnell's theory would be by ignoring a few terms in Flugge equations. Many studies, including Flugge, built on Love's pioneering work [3] and his first approximation theory. Livanov[4] solved the axisymmetrical vibrations of simply supported cylindrical shells issue by using displacement functions and using Love's assumption. More recently, Amabili and Paidoussis [5], Amabili [6], and Kurylov and Amabili [7] offered notable evaluations from a non-linear perspective.

These theories are concerned not only with simply supported end conditions, but also with alternative limits, such as cantilever cylindrical shells [8], fixed free circular cylinder shells [9], clamped-clamped shells [10], and infinite length shells [11]. Rinchart and Wang [12] studied the vibration of simply supported cylindrical shells stiffened by discrete longitudinal stiffeners, employing Donnell's approximation theory, Flugge's more accurate theory, and Love's assumption for longitudinal wave numbers. In addition to the approximation method, additional ways to finding natural frequencies exist, such as the computer-based numerical method [13],[14] to reduce cumbersome processing work, and the wave propagation technique [15]. Most studies, including those described above, employ the beam function as an estimate for the simply supported boundary conditions in order to discover natural frequencies of vibration using the approximate technique. This method may also be used to do finite element analysis of cylindrical shells using the Hermitain polynomial of the beam function type [16]. More recently, Farshidianfar et al. [17] employed acoustic exitation to determine the natural frequency of long cylindrical shells.

Shells are three-dimensional structures that may freely vibrate, unlike beams or plates, which are typically one- or two-dimensional. This has resulted in complex shell movements at resonance frequencies. Therefore, modal identification of cylindrical shells with their amplitudes has always been of tremendous relevance, even apart from their frequency behavior. Applications such as sound radiation, acoustics, and engineering design heavily rely on the amplitude ratios of cylindrical shells. The amplitude ratios of the mode forms have been determined in earlier research [18], although these investigations are not comprehensive.

The current work proposes a semi-analytical method to examine the free vibration of cylindrical shells with simple supports.As previously mentioned, beam transformations with similar boundary conditions are employed in conventional analysis to approximate wave numbers in the axial direction. This approach is regarded as being approximative.The approximation approach is used to get the natural frequencies based on 10 distinct shell theories.(Donnell-Mushtari, Love-Timoshenko, Amold-Warburton, Houghton-Johns, Flugge-Byme-Lur'ye, Reissner-Naghdi-Berry, Sanders, Vlasov, Kennard-Simplified and Soedel) A comparison of the results with the experimental data demonstrates a fair degree of agreement. Lastly, an analysis was done on the behavior of the circular shells at different aspect and thickness ratios. It was noted that the predominate motion of the shell might shift to radial, tangential, or longitudinal by changing these parameters.

II. Theoretical Analysis

The cylindrical shell that is being studied has the following constants: density (□□, Poisson's ratio (□□, axial length (L), mean radius(R), and modulus of elasticity (E). As seen in Figure 1, the corresponding displacements in the axial, circumferential, and radial directions are indicated by the symbols $u(x,\theta,t), v(x,\theta,t), and\ w(x,\theta,t)$

The cylindrical shell under consideration is with constant thickness h, mean radius R, axial length L, Poisson's ratio □, density □and Young's modulus of elasticity E . Here are the respective displacements in the axial, circumferential and radial directions are denoted by $u(x,\theta,t), v(x,\theta,t), and\ w(x,\theta,t)$ as shown in Figure l.

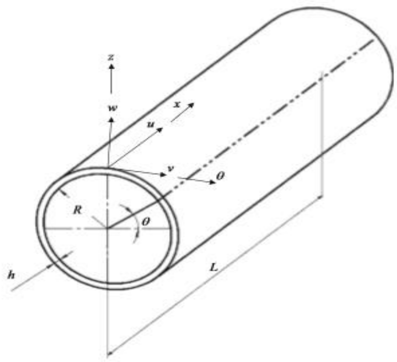

Figure 1. Coordinate system and dimensions of Circular cylindrical shell.

To investigate free vibration of a cylindrical shell, the equations of motion can be stated in matrix form as follows:

$$\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{Bmatrix} u(x,\theta,t) \\ v(x,\theta,t) \\ w(x,\theta,t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (1)$$

where D_{ij} (i,j= l, 2, 3) are differential operators with regard to x, θ, t

Different systems of equations are utilized to simulate the vibration behavior of a circular cylindrical shell. This work analyzed with several shell theories like Donnell-Mushtari, Love-Timoshenko, Amold-Warburton, Hougton-Johns, Flugge-Byme-Iur'ye, Reissner-Naghdi-Berry, Sanders, Vlasov, Kennard-Simplified, and Soedel, to determine natural frequencies for different boundaries.

Assuming a synchronous motion, the first attempt at solving (1) is as follows:

$$\begin{cases} u(x,\theta,t) = U(x,\theta)\, f(t) \\ v(x,\theta,t) = V(x,\theta)\, f(t) \\ w(x,\theta,t) = W(x,\theta)\, f(t) \end{cases} \quad (2)$$

where f(t) is the scalar model coordinate corresponding to the mode shapes $U(x,\theta), V(x,\theta)\ , W(x,\theta)$

The following stage involves separating the spatial dependency of the modal shape between the longitudinally and circumferential directions using the separation of variables approach.

Therefore, the shell's axial, tangential, and radial displacements differ in accordance with:

$$\begin{cases} u(x,\theta,t) = Ae^{\lambda_m x}\sin(n\theta)\cos(\omega t) \\ v(x,\theta,t) = Be^{\lambda_m x}\cos(n\theta)\cos(\omega t) \\ w(x,\theta,t) = Ce^{\lambda_m x}\sin(n\theta)\cos(\omega t) \end{cases} \quad (3)$$

where n and λ_m represents the circumferential wave parameter and the axial wave number, respectively. x, t typically represents a spatial variable, time respectively. ω is circular frequency of natural vibration of shell and A, B, and C are the undetermined constants.

Any of the shell theories may be used to substitute (3) in (1) to create a set of homogeneous equations with the following matrix form.

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (4)$$

where $[C_{i,j}]$ (i,j=1,2,3) are coefficients of matrix with terms of n, λ_m , υ ,k , and Ω^2

The non dimensional frequency parameter(Ω^2) is defined as follows

$$\Omega^2 = \frac{(1-\upsilon^2)\rho\omega^2 R^2}{E} \quad (5)$$

The non dimensional thickness parameter (k) is defined as follows

$$k = \frac{h^2}{12R^2} \quad (6)$$

The coefficient matrix $[C_{i,j}]$ for several shell theories is obtained as follows

Donnell-Mushtari Shell Theory:

$$\begin{bmatrix} \Omega^2 + {\lambda_m}^2 - \frac{(1-\upsilon)}{2}n^2 & -\frac{(1+\upsilon)}{2}n\lambda_m & \upsilon\lambda_m \\ \frac{(1+\upsilon)}{2}n\lambda_m & \Omega^2 + \frac{(1-\upsilon)}{2}{\lambda_m}^2 - n^2 & n \\ -\upsilon\lambda_m & n & \Omega^2 - [1 + k({\lambda_m}^2 - n^2)^2] \end{bmatrix} \quad (7)$$

Love-Timmoshenko Shell Theory:

$$\begin{bmatrix} \Omega^2 + {\lambda_m}^2 - \frac{(1-\upsilon)}{2}n^2 & -\frac{(1+\upsilon)}{2}n\lambda_m & \upsilon\lambda_m \\ \frac{(1+\upsilon)}{2}n\lambda_m & \Omega^2 + (1+2k)\frac{(1-\upsilon)}{2}{\lambda_m}^2 - (1+k)n^2 & n + nk(n^2 - {\lambda_m}^2) \\ -\upsilon\lambda_m & n & \Omega^2 - [1 + k({\lambda_m}^2 - n^2)^2] \end{bmatrix} \quad (8)$$

Amold-Warburton Shell Theory:

$$\begin{bmatrix} \Omega^2 + {\lambda_m}^2 - \frac{(1-\upsilon)}{2}n^2 & -\frac{(1+\upsilon)}{2}n\lambda_m & \upsilon\lambda_m \\ \frac{(1+\upsilon)}{2}n\lambda_m & \Omega^2 + (1+4k)\frac{(1-\upsilon)}{2}{\lambda_m}^2 - (1+k)n^2 & nk[n^2 - (2-\upsilon){\lambda_m}^2] \\ -\upsilon\lambda_m & nk[n^2 - (2-\upsilon){\lambda_m}^2] & \Omega^2 - [1 + k({\lambda_m}^2 - n^2)^2] \end{bmatrix} \quad (9)$$

Houghton-Johns Shell Theory:

$$\begin{bmatrix} \Omega^2 + {\lambda_m}^2 - \frac{(1-\upsilon)}{2}n^2 & -\frac{(1+\upsilon)}{2}n\lambda_m & \upsilon\lambda_m \\ \frac{(1+\upsilon)}{2}n\lambda_m & \Omega^2 + \frac{(1-\upsilon)}{2}{\lambda_m}^2 - n^2 & nk[n^2 - (2-\upsilon){\lambda_m}^2] \\ -\upsilon\lambda_m & nk[n^2 - (2-\upsilon){\lambda_m}^2] & \Omega^2 - [1 + k({\lambda_m}^2 - n^2)^2] \end{bmatrix} \quad (10)$$

Flugge-Byrme-Lur'ye Shell Theory:

$$\begin{bmatrix} \Omega^2 + {\lambda_m}^2 - (1+k)\frac{(1-\upsilon)}{2}n^2 & -\frac{(1+\upsilon)}{2}n\lambda_m & \upsilon\lambda_m - k\lambda_m[{\lambda_m}^2 + (1-\upsilon)n^2] \\ \frac{(1+\upsilon)}{2}n\lambda_m & \Omega^2 - n^2 + (1+3k)\frac{(1-\upsilon)}{2}{\lambda_m}^2 & n\left[1 - \frac{3-\upsilon}{2}k{\lambda_m}^2\right] \\ \upsilon\lambda_m + k\lambda_m[{\lambda_m}^2 + (1-\upsilon)n^2] & n\left[1 - \frac{3-\upsilon}{2}k{\lambda_m}^2\right] & \Omega^2(1+k) - k[({\lambda_m}^2 - n^2)^2 - 2n^2] \end{bmatrix} \quad (11)$$

Reissner-Naghdi-Berry Shell Theory:

$$\begin{bmatrix} \Omega^2+{\lambda_m}^2-\frac{(1-\upsilon)}{2}n^2 & -\frac{(1+\upsilon)}{2}n\lambda_m & \upsilon\lambda_m \\ \frac{(1+\upsilon)}{2}n\lambda_m & \Omega^2+(1+\mathrm{k})\left(\frac{(1-\upsilon)}{2}{\lambda_m}^2-n^2\right) & n\left[1+k(n^2-{\lambda_m}^2)\right] \\ -\upsilon\lambda_m & n\left[1+k(n^2-{\lambda_m}^2)\right] & \Omega^2-\left[1+\mathrm{k}\left({\lambda_m}^2-n^2\right)\right] \end{bmatrix} \quad (12)$$

Sanders Shell Theory:

$$\begin{bmatrix} \Omega^2+{\lambda_m}^2-(1+\frac{k}{4})\frac{(1-\upsilon)}{2}n^2 & -n\lambda_m\left[\frac{1+\upsilon}{2}-\frac{3k(1-\upsilon}{8}\right] & \lambda_m\left(\upsilon-\frac{1-\upsilon}{2}kn^2\right) \\ n\lambda_m\left[\frac{1+\upsilon}{2}-\frac{3k(1-\upsilon}{8}\right] & \Omega^2-(1+\mathrm{k})n^2+(1+\frac{9\mathrm{k}}{4})\frac{(1-\upsilon)}{2}{\lambda_m}^2 & n\left[1+k\left(n^2-\frac{3-\upsilon}{2}{\lambda_m}^2\right)\right] \\ \lambda_m\left(\frac{1-\upsilon}{2}kn^2-\upsilon\right) & n\left[1+k\left(n^2-\frac{3-\upsilon}{2}{\lambda_m}^2\right)\right] & \Omega^2-\left[1+\mathrm{k}({\lambda_m}^2-n^2)^2\right] \end{bmatrix} \quad (13)$$

Vlassov Shell Theory:

$$\begin{bmatrix} \Omega^2+{\lambda_m}^2-\frac{(1-\upsilon)}{2}n^2 & -n\lambda_m\left[\frac{1+\upsilon}{2}\right] & \lambda_m\left[\upsilon-k\left(\frac{1-\upsilon}{2}n^2+{\lambda_m}^2\right)\right] \\ \frac{(1+\upsilon)}{2}n\lambda_m & \Omega^2+\frac{(1-\upsilon)}{2}{\lambda_m}^2-n^2 & n\left(1-\frac{3-\upsilon}{2}k{\lambda_m}^2\right) \\ \lambda_m\left[k\left(\frac{1-\upsilon}{2}n^2+{\lambda_m}^2\right)-\upsilon\right] & n\left(1-\frac{3-\upsilon}{2}k{\lambda_m}^2\right) & \Omega^2-(1+\mathrm{k})-\mathrm{k}\left[({\lambda_m}^2-n^2)^2-2\mathrm{n}^2\right] \end{bmatrix} \quad (14)$$

Kennard-Simplified Shell Theory:

$$\begin{bmatrix} \Omega^2+{\lambda_m}^2-\frac{(1-\upsilon)}{2}n^2 & -\left[\frac{1+\upsilon}{2}\right]n\lambda_m & \upsilon\lambda_m \\ \frac{(1+\upsilon)}{2}n\lambda_m & \Omega^2+\frac{(1-\upsilon)}{2}{\lambda_m}^2-n^2 & n\left[1+\frac{3k\upsilon}{2(1-\upsilon)}(1-n^2)\right] \\ -\upsilon\lambda_m & 0 & \Omega^2-\left(1+\frac{2+\upsilon}{2(1-\upsilon)}\right)-\mathrm{k}\left[({\lambda_m}^2-n^2)^2-\frac{4-\upsilon}{2(1-\upsilon)}\mathrm{n}^2\right] \end{bmatrix} \quad (15)$$

Soedel Shell Theory:

$$\begin{bmatrix} \Omega^2+{\lambda_m}^2-\frac{(1-\upsilon)}{2}n^2 & -\left[\frac{1+\upsilon}{2}\right]n\lambda_m & \upsilon\lambda_m \\ \frac{(1+\upsilon)}{2}n\lambda_m & \Omega^2+(1+\mathrm{k})(\frac{1-\upsilon}{2}{\lambda_m}^2-n^2) & n\left[1+k(n^2-{\lambda_m}^2)\right] \\ -\upsilon\lambda_m & n\left[1+k(n^2-{\lambda_m}^2)\right] & \Omega^2-\left(1+\mathrm{k}\left(-{\lambda_m}^2-n^2\right)^2\right) \end{bmatrix} \quad (16)$$

The coefficient matrix's determinant in equation (4) for a nontrivial solution must be zero

i.e., $det\left|C_{ij}\right| = 0\,; i,j = 1,2,3$ (17)

Expanding (17) yields the two Eigen value problems as follows

(i)There are one or more appropriate values for ω such that (17) vanishes for a given value of λ_m.

(ii)There are one or more appropriate values for $\lambda_m$such that (17 vanishes for a given value of ω.

A cubic equation in terms of the non-dimensional frequency parameter Ω^2 may be obtained by solving equation (17).

The natural frequencies of the cylindrical shell are therefore three positive roots and three negative roots for fixed values of n and λ_m .This allows the shell to be classified as primarily axial, circumferential, and radial. The lowest frequency is typically linked with motion that is largely radial (or flexural).

III. Beam Function Method

In general, it is impossible to solve in closed form the roots of the characteristic equation of (17) for λ_m. Because of this, researchers have a tendency to employ approximation approaches. For closed circular cylindrical shells, estimated displacements and natural frequencies may be obtained using beam functions. With the same boundary conditions, this approach integrates the flexural vibration of a cylindrical shell with a transversely vibrating beam. For a simply supported shell at both ends, the approximation technique defines the nature of the axial mode as

$$\lambda_m = m\pi \frac{R}{L}\sqrt{-1} \qquad (18)$$

By substituting (18) into (17), the only unknown in the characteristic equation is the frequency parameter Ω^2 for a fixed combination of m and n.

IV. Result And Discussion

Because the beam function approach is an estimate for obtaining natural frequencies for thin circular cylindrical shells, it is critical to ensure its correctness. As a result, the natural frequency for simply supported boundary conditions was determined using the beam function in conjunction with above standard cylindrical shell theories.

Table 1 presents a comparison between the approximation technique findings derived from the ten theories and an experiment conducted for a simply supported circular cylindrical shell by Farshidianfar et al. [17]. The aluminum shell under investigation in Table 1 has the following material properties: υ = 0 33, ρ = 2700 Kg/m3, and E = 68.2 GPa. The shell's dimensions considered are h=0.00147 m, R=0.0762 m and L=1.7272 m

m	n	Experiment (Hz)	Donnell-Mushtari (Hz)	Love-Timoshenko (Hz)	Arnold-Warburton (Hz)	Flugge-Byme-Lur' ye (Hz)	Houghton-Johns (Hz)	Reissner-Naghdi-Berry (Hz)	Sanders (Hz)	Vlasov (Hz)	Kennard-Simplified (Hz)	Soedel (Hz)
1	1	138.4	145.78	138.98	138.89	138.81	131.64	138.93	138.89	138.76	139.15	138.41
1	2	190.3	226.92	172.95	172.75	172.73	163.56	172.94	172.75	172.84	192.00	170.20
1	3	502.2	530.20	471.46	471.37	471.39	467.68	471.46	471.37	471.44	492.83	468.92
1	4	884.4	962.42	902.29	902.24	902.28	900.23	902.29	902.24	902.31	924.35	899.83
2	1	464.7	520.01	518.07	517.97	517.91	515.94	518.02	517.97	517.85	518.25	517.47
2	2	310.5	286.06	244.82	244.26	244.20	237.69	244.78	244.27	244.50	259.46	237.00
2	3	477	539.51	481.45	481.10	481.11	477.45	481.44	481.11	481.33	502.96	471.44
3	2	496.6	449.85	449.85	449.85	423.43	449.85	424.15	423.51	423.77	433.61	414.15
3	3	558.9	569.82	514.49	513.75	513.75	510.26	514.47	513.77	514.21	535.51	493.21
4	2	679.8	704.8	688.28	687.48	687.48	684.97	688.23	687.54	687.80	694.77	677.37
4	3	638.3	636.53	586.68	585.53	585.53	582.39	586.64	585.57	586.21	606.20	553.30
5	3	782	748.88	706.10	704.60	704.65	701.9	706.05	704.69	705.4	723.44	662.76

Table 1: Comparison of Approximation Analysis results with Experimental data

The errors of all theories with respect to the experiment are also shown in Table 2.
A relative error is defined as follows:

$$Error_{\omega} = \frac{\omega_{experimental} - \omega_{analytical}}{\omega_{experimental}} \times 100\% \qquad (19)$$

m	n	Donnell-Mushtari (Hz)	Love-Timoshenko (Hz)	Arnold-Warburton (Hz)	Flugge-Byme-Lur' ye (Hz)	Houghton-Johns (Hz)	Reissner-Naghdi-Berry (Hz)	Sanders (Hz)	Vlasov (Hz)	Kennard-Simplified (Hz)	Soedel (Hz)
1	1	-5.33	-0.42	-0.35	-0.30	4.88	-0.38	-0.35	-0.26	-0.54	-0.01
1	2	-19.24	9.12	9.22	9.23	14.05	9.12	9.22	9.17	-0.89	10.56
1	3	-5.58	6.12	6.14	6.14	6.87	6.12	6.14	6.13	1.87	6.63
1	4	-8.82	-2.02	-2.02	-2.02	-1.79	-2.02	-2.02	-2.03	-4.52	-1.74
2	1	-11.90	-11.48	-11.46	-11.45	-11.03	-11.47	-11.46	-11.44	-11.52	-11.36
2	2	7.87	21.15	21.33	21.35	23.45	21.17	21.33	21.26	16.44	23.67
2	3	-13.10	-0.93	-0.86	-0.86	-0.09	-0.93	-0.86	-0.91	-5.44	1.17
3	2	9.41	9.41	9.41	14.73	9.41	14.59	14.72	14.67	12.68	16.60

3	3	-1.95	7.95	8.08	8.08	8.70	7.95	8.07	8.00	4.19	11.75
4	2	-3.68	-1.25	-1.13	-1.13	-0.76	-1.24	-1.14	-1.18	-2.20	0.36
4	3	0.28	8.09	8.27	8.27	8.76	8.09	8.26	8.16	5.03	13.32
5	3	4.24	9.71	9.90	9.89	10.24	9.71	9.89	9.80	7.49	15.25

Table2: Errors between approximation analyses with Experimental data

It is observed that, the beam function method yields close results compared to the experiment as well. It is also concluded that some theories (Love-Timoshenko, Arnold-Warburton, Flugge-Byme-Lur'ye , Reissner-Naghdi-Berry, Sanders, Vlasov and Soedel reveal same results.Kenanrd-Simplified,Reissner-Nigdhi-Berry and Soedell are more accurate than other theories and Donnell_Mushtari and Houghton-Johns theory are not precise compared to other theories

The mode shapes or eigen functions of free vibrations are found by returning to the homogeneous set of equations which yielded the characteristic equation. In case of Donnel-Musthari theory the set is given by eqtn(7) . Any two of the equations are chosen and the third is discarded. The two remaining equations can be solved for the ratios of amplitudes .The most convenient ratios to choose being A/C and B/C. The eqtn(7) can be rewritten as

$$\begin{bmatrix} \Omega^2 + \lambda_m{}^2 - \frac{(1-\upsilon)}{2} n^2 & -\frac{(1+\upsilon)}{2} n\lambda_m \\ \frac{(1+\upsilon)}{2} n\lambda_m & \Omega^2 + \frac{(1-\upsilon)}{2} \lambda_m{}^2 - n^2 \end{bmatrix} \begin{bmatrix} A/C \\ B/C \end{bmatrix} = \begin{bmatrix} \upsilon\lambda_m \\ n \end{bmatrix} \qquad (20)$$

By solving eqtn(20) ,the values of amplitude ratios A/C and B/C can be written as inverting them as follows

$$\frac{A}{C} = \frac{(\upsilon\lambda_m) * \left(\Omega^2 + \frac{(1-\upsilon)}{2}\lambda_m{}^2 - n^2\right) - \left(-\frac{(1+\upsilon)}{2} n\lambda_m\right) * n}{\left(\ \Omega^2 + \lambda_m{}^2 - \frac{(1-\upsilon)}{2} n^2\right) * \left(\Omega^2 + \frac{(1-\upsilon)}{2}\lambda_m{}^2 - n^2\right) - \left(\frac{(1+\upsilon)}{2} n\lambda_m\right) * (-\frac{(1+\upsilon)}{2} n\lambda_m)}$$

$$\frac{B}{C} = \frac{(n) * \left(\Omega^2 + \frac{(1-\upsilon)}{2}\lambda_m{}^2 - n^2\right) - \left(\frac{(1+\upsilon)}{2} n\lambda_m\right) * n}{\left(\ \Omega^2 + \lambda_m{}^2 - \frac{(1-\upsilon)}{2} n^2\right) * \left(\Omega^2 + \frac{(1-\upsilon)}{2}\lambda_m{}^2 - n^2\right) - \left(\frac{(1+\upsilon)}{2} n\lambda_m\right) * (-\frac{(1+\upsilon)}{2} n\lambda_m)}$$

As indicated above, the lowest of three natural frequencies for each λ_m and n combinations usually A/C, B/C ratios less than unity indicating primarily radial motion.

V. Conclusions

Ten distinct thin shell theories have been used to study the free vibration of circular cylindrical shells with simply supported boundary conditions: Donnell-Mushtari, Love-Timoshenko, Amold- Warbunon, Houghton-Johns, Flugge-Byme-Lur'ye, Reissner-Naghdi-Beny, Sanders, Vlasov, Kennard-Simplified, and Soedel. The work focuses on utilizing the beam function as an approximation for boundary conditions to determine the natural frequencies of a shell. The hypotheses were then compared to the results to ensure their correctness, and there was good agreement. Furthermore, the approximation technique based on the Soedel and Kennard-Simplified theories performed better than other theories.

Reference

[1] W. Leissa, "Vibration Of Shells", (Nasa Sp-288), Us Government Printing Office, Washingon Dc, 1973.
[2] W. Flugge, "Stresses In Shells". Sponger. New York. 1973.
[3] A. E. H. Love. "On The Small Free Vibrations And Deformations Of Thin Shells", Philosophical Transactions Of The Roya Society (London) 179a, Pp. 491—546, 1888.
[4] K.K. Livanov, Axisymmetric Vibrations Of Simply Supported Cylindrical Shells", Pmm Vol. 2.6. Pp. 742-745, 1961
[5] M. Amabili , M. P. Paidoussis, "Review Of Studies On Geometrically Nonlinear Vibrations And Dynamics Of Circular Cylindrical Shells And Panels, With And Without Fluid Structure Interaction", Applied Mechanics Review, Vol. 56, Pp. 349-381, 2003
[6] M. Amabili, "Nonlinear Vibrations And Stability Of Shells And Plates", Cambridge University Press, New York, Usa, 2008.
[7] Y. Kurylov, M. Amabili, "Polynomial Versus Trigonometric For Nonlinear Vibrations Of Circular Cylindrical Shells With Different Boundary Conditions", Journal Of Sound And Vibration, Vol.329, Pp. 1435-1449, 2010.
[8] G. B. Warburton, J. Higgs. "Natural Frequencies Of Thin Cantilever Cylindrical Shells", Journal Of Sound And Vibration. Vol 11. Pp. 335-338. 1970.
[9] C. B. Sharma, "Calculation Of Natural Frequencies Of Fixed-Free Circular Cylindrical Shells" , Journal Of Sound And Vibration. Vol. 35. Pp. 55-76. 1974
[10] Callahan. H. Baruh. " A Closed-Form Solution Procedure For Circular Cylindrical Shell Vibrations". International Journal Of Solids And Structures, Vol. 36, Pp. 2973-3013, 1999

[11] F. Moussaouri, R. Benamar, R. G. "The Effects Of Large Vibration Amplitudes On The Mode Shapes And Natural Frequencies Of Thin Elastic Shells, Part I: Coupled Transverse-Circumferential Mode Shapes Of Isotropic Circular Cylindrical Shells Of Infinite Length", Journal Of Sound And Vibration, Vol. 232, Pp. 917-943, 2000

[12] S. A. Rinehart, J. T. S. Wang- "Vibration Of Simply Supported Cylindrical Shells With Longitudinal Stiffeners". Journal Of Sound And Vibration. Vol. 24. Pp. 151-163. 1972

[13] J.W.Tedesco, C. N. Kostem, A. Kalnins, "Free Vibration Analysis Of Circular Cylindrical Shells". Journal Of Computers And Structures. Vol. 25. Pp. 677-685. 1987.

[14] A. B. Sabir. A.,Sfendji,. T G. Hughes, "Strain-Based Finite Element For The Natural Frequencies Of Cylindrical Shells" Journal Of Thin-Walled Structures Vol. 18. Pp. 67-82, 1994.

[15] X. M. Zhang. G, R. Ciu, K. Y. Cam. "Vibration Analysis Of Thin Cylindrical Shels Using Wave Propagation Approach" Journal Of Sound And Vibration, Vol. 239, Pp. 397-403. 2001

[16] S K. Tou, K K, Wong, "High-Precision Finite Element Analysis Of Cylindrical Shells" Journal Of Computers And Structures. Vol 26, Pp. 887-854. 1987.

[17] A. Farshidianfar. M. H. Farshidianfar. M. J. Crocker. W. O. Smith. "The Vibration Analysis Of Long Cylindrical Shells Using Acoustical Excitation", Journal Of Sound And Vibration, Vol. 232, Pp. 917-943, 2010

[18] W. Soedel, "Vibrations Of Shells And Pates", 3rd Ed., Marcel Dekker, Inc., 2004.