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Kane's Method For Suspension Boat Dynamics

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Abstract:
Small boats operating at high speeds are suspected to high vertical accelerations causes large number of crew injuries and boat damages [1]. A concept of suspension boat consists of main hull and four sponsons connected to suspension links spring and shock absorbers patented by Prof. J L Grenestedt [2] in order to reduce the vertical acceleration.
The paper represents a 10DOF dynamic model of the suspension boat using Kane's method.
***Key Word**: Suspension Boat dynamics; Kane's method dynamics; Dynamics; 10 DOF equation of motion.*

U_i	*Generalized speed where i is DOF of the system.*
$\hat{n}_i$	*Base vector of fixed frame N where i=1,2,3*
$\widehat{b_i^0}$	*Base vectors attached to the hull center respect to reverence frame where i=1,2,3*
$\widehat{b_i^j}$	*Base vectors attached to sponsons centers respect to hull, where j=1,2.. , i =1,2,3*
$\overset{N \to A}{V}$	*Velocity of centre of mass of hull respect to reference frame (m/s)*
$\overset{N \to sn}{V}$	*Velocity of nth sponsons respect to reference frame, where n =1,2,3….(m/s)*
$\overset{N \to A}{\omega}$	*Angular velocity of center of hull respect to reference frame (Rad/s)*
$\overset{N \to sn}{\omega}$	*Angular velocity of nth sponsons respect to reference frame. Where n=1,2,3…. (Rad/s)*
$\overset{N \to A}{p}$	*Position of center of mass of hull respect to reference frame (m)*
$\overset{N \to f}{p}$	*position of point on front revolute respect to reference frame (m)*
$\overset{N \to r}{p}$	*position of point on rear revolute respect to reference frame (m)*
$\overset{N \to sn}{p}$	*position of center of mass of sponsons respect to reference frame where n=1,2,.. (m)*
$\overset{N \to wn}{p}$	*Positions of running surface where the water loads reassumed to apply on sponsons. Where n=1,2…(m)*
$\overset{N \to A}{a}$	*Linear acceleration of center of hull respect to reference frame (m/s2)*
$\overset{N \to sn}{a}$	*linear acceleration of nth sponsons respect to reference frame. Where n =1,2….(m/s2)*
$\overset{N \to sn}{\alpha}$	*Angular acceleration of nth sponsons respect to reference frame. Where n =1,2….(Rad/s2)*
$\overset{N \to A}{\alpha}$	*Angular acceleration of center of mass of hull (Rad/s2)*
x	*Horizontal coordinate in Earth-Fixed system aligned with direction of travel (m)*
y	*Horizontal coordinate in Earth-Fixed system, perpendicular with direction of travel (m)*
z	*Vertical coordinate in Earth-Fixed system, position up positive (m)*
θ	*Pitch angle positive bow up (Rad)*

ϕ	*Roll angle positive rolling to the right (Rad)*
ψ	*Yaw angle measured clockwise from North (Rad)*
θ_1	*Sponson1 deflection angle positive when the transom deflect upwards (Rad)*
θ_2	*Sponson2 deflection angle positive when the transom deflect upwards (Rad)*
θ_3	*Sponson3 deflection angle positive when the transom deflect upwards (Rad)*
θ_4	*Sponson4 deflection angle positive when the transom deflect upwards (Rad)*
u_1	*Time derivation of x (m/s)*
u_2	*Time derivation of y (m/s)*
u_3	*Time derivation of z (m/s)*
u_4	*Time derivation of θ (Rad/s)*
u_5	*Time derivation of ϕ (Rad/s)*
u_6	*Time derivation of Ψ (Rad/s)*
u_7	*Time derivation of $\theta 1$ (Rad/s)*
u_8	*Time derivation of $\theta 2$ (Rad/s)*
u_9	*Time derivation of $\theta 3$ (Rad/s)*
u_{10}	*Time derivation of $\theta 4$ (Rad/s)*
M	*Hull mass (Kg)*
m_1	*Sponson1 mass (Kg)*
m_2	*Sponson2 mass (Kg)*
m_3	*Sponson3 mass (Kg)*
m_4	*Sponson4 mass (Kg)*
K_1	*Spring stiffness for first sponson (N/m)*
K_2	*Spring stiffness second sponson (N/m)*
K_3	*Spring stiffness for third sponson (N/m)*
K_4	*Spring stiffness for fourth sponson (N/m)*
C_1	*Damping coefficient first sponson (Ns/m)*
C_2	*Spring stiffness second sponson (Ns/m)*
C_3	*Spring stiffness for third sponson (Ns/m)*
C_4	*Spring stiffness fourth sponson (Ns/m)*

a1	*Distance between transoms of front sponsons (m)*
a2	*Distance between transoms of front sponsons (m)*
a3	*Distance between transoms of rear sponsons (m)*
a4	*Distance between transoms of rear sponsons (m)*
a5	*Distance between transoms of front sponsons (m)*
a6	*Distance between transoms of front sponsons (m)*
a7	*Distance between transoms of rear sponsons (m)*
a8	*Distance between transoms of rear sponsons (m)*
a9	*Distance between transoms of front sponsons (m)*
a10	*Distance between transoms of front sponsons (m)*
a11	*Distance between transoms of rear sponsons (m)*
a12	*Distance between transoms of rear sponsons (m)*
a13	*Distance between transoms of front sponsons (m)*
a14	*Distance between transoms of rear sponsons (m)*
a15	*Distance between transoms of front sponsons (m)*
a16	*Distance between transoms of rear sponsons (m)*
I	*Moment of inertia of center of hull (Ns/m)*
Is	*Moment of inertia of sponsons (Ns/m)*
F_1	*Force applied upward direction on first sponson (N)*
F_2	*Force applied upward direction on second sponson (N)*
F_3	*Force applied upward direction on third sponson (N)*
F_4	*Force applied upward direction on fourth sponson (N)*
g	*Gravitational acceleration (Kg m2)*

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I. Introduction

Small boats operating at high speeds often exposed to high vertical accelerations. As craft speed and wave height increases the higher vertical accelerations causes extreme discomfort and eventually to pain and possible injury for the crew. Hinged flap mechanism suspended to boat hull via shock absorber components was developed and experimentally tested using different design parameters in order to reduce vertical acceleration on high speed boats.

The paper represents a 10DOF dynamic of the suspension boat using Kane's method. The model enables us to extract information about boat dynamic behavior and choose the optimum design parameters.

II. Dynamic Analysis of Suspension Boat with Sponsons

3-dimensional dynamic model describing suspension boat with four sponsons in 10 degrees of freedom using kane' s equation method. The boat with suspension is schematically shown in Figure 1, 2 consists of front and rear sponsons attached via springs and dampers to center-hull.

The generalized coordinates (x , y ,z ,Ψ , θ, ϕ, θ_1 , θ_2 , θ_3 , θ_4) are used where x, y, z are the coordinates of center of mass of center of hull. The orthonormal base vectors $\widehat{n_\iota}, \widehat{n_2}, \widehat{n_3}$ for fixed frame N is used. The base vectors point West ($\widehat{n_\iota}$), North ($\widehat{n_2}$), and down ($\widehat{n_3}$), respectively as shown in Figure 3. θ is pitch angle (positive bow up), ϕ is roll angle (positive rolling to the right), ψ is yaw angle measured clockwise from North, θ_1, θ_2, θ_3, θ_4 are sponson deflection angles measured positive when the transom of the sponsons deflects upwards as shown in figure 4.

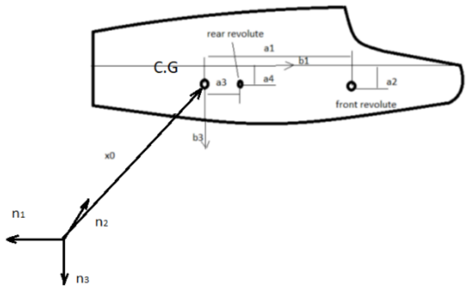

Fig: 1 Boat with Sponsons Side View

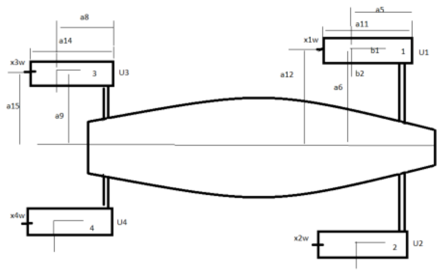

Fig: 2 Boat with Sponsons Top View

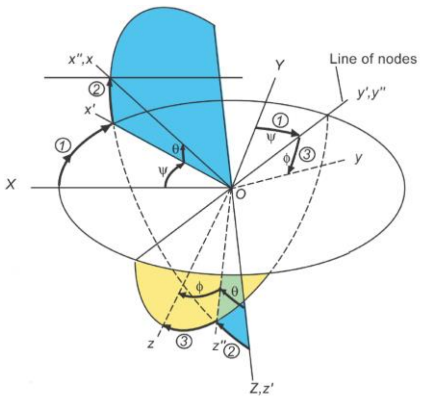

Fig:3 Euler Angles

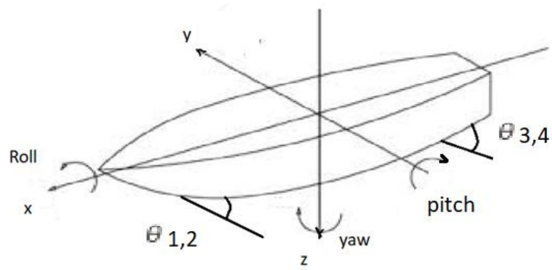

Fig:4 Sponsons angles

III. Rotational Matrices

We start off with the standard definition of the rotations about the three principal axes.
A rotation about the z-axis is defined as Ψ angle.

$$Rz(\psi)=\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A rotation about the y-axis is defined as θ angle.

$$Ry(\theta)=\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

A rotation about the x-axis is defined as ϕ angle.

$$R_x(\phi)=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

Where (ψ, θ, ϕ) are Euler Angles.

$R= R_x(\phi)\ R_y(\theta)\ R_z(\psi)=$

$$\begin{bmatrix} \cos\phi\cos\theta & \sin\phi\cos\theta & -\sin\phi \\ -\sin\theta\cos\psi+\cos\theta\sin\phi\sin\psi & \cos\psi\cos\theta+\sin\theta\sin\phi\sin\psi & +\cos\phi\sin\psi \\ \cos\psi\sin\phi\cos\theta+\sin\psi\sin\theta & \sin\theta\sin\phi\cos\psi-\cos\theta\sin\psi & \cos\phi\cos\psi \end{bmatrix}$$

We would rewrite the same expression $\widehat{b_\iota^0}$ base vectors attached to the hull center respect to reference frame.

$\widehat{b_1^0} = \cos\phi\cos\theta\,\widehat{n_\iota} + \sin\phi\cos\theta\,\widehat{n_2} - \sin\phi\,\widehat{n_3}$

$\widehat{b_2^0} = (-\sin\theta\cos\psi + \cos\theta\sin\phi\sin\psi)\widehat{n_\iota} + (\cos\psi\cos\theta + \sin\theta\sin\phi\sin\psi)\widehat{n_2} + \cos\phi\sin\psi\,\widehat{n_3}$

$\widehat{b_3^0} = (\cos\psi\sin\phi\cos\theta + \sin\psi\sin\theta)\widehat{n_\iota} + (\sin\theta\sin\phi\cos\psi - \cos\theta\sin\psi)\widehat{n_2} + \cos\phi\cos\psi\,\widehat{n_3}$

A sponson in the present design is attached to the center hull via a revolute (1 degree of freedom rotation) near the bow of the sponsons. These revolutes are parallel to the $\widehat{b_2^0}$ axis. $\widehat{b_\iota^J}$ base vectors attached to sponsons centers respect to hull. The front left sponson has the base vectors $\widehat{b_1^1}, \widehat{b_2^1}, \widehat{b_3^1}$

$\widehat{b_1^1} = \cos\theta_1\,\widehat{b_1^0} + \sin\theta_1\,\widehat{b_3^0}$

$\widehat{b_2^1} = \widehat{b_2^0}$

$\widehat{b_3^1} = -\sin\theta_1\,\widehat{b_1^0} + \cos\theta_1\,\widehat{b_3^0}$

Likewise, for the right front.

$\widehat{b_1^2} = \cos\theta_2\,\widehat{b_1^0} + \sin\theta_2\,\widehat{b_3^0}$

$\widehat{b_2^2} = \widehat{b_2^0}$

$\widehat{b_3^2} = -\sin\theta_2\,\widehat{b_1^0} + \cos\theta_2\,\widehat{b_3^0}$

left and right rear sponsons, respectively:

$\widehat{b_1^3} = \cos\theta_3\,\widehat{b_1^0} + \sin\theta_3\,\widehat{b_3^0}$

$\widehat{b_2^3} = \widehat{b_2^0}$

$\widehat{b_3^3} = -\sin\theta_3\,\widehat{b_1^0} + \cos\theta_3\,\widehat{b_3^0}$

$\widehat{b_1^4} = \cos\theta_4\,\widehat{b_1^0} + \sin\theta_4\,\widehat{b_3^0}$

$\widehat{b_2^4} = \widehat{b_2^0}$

$\widehat{b_3^4} = -\sin\theta_4\,\widehat{b_1^0} + \cos\theta_4\,\widehat{b_3^0}$

The Positions of Front and Rear Revolute and Center of Mass of Four Sponsons Respect to Reference Frame:

The revolute of the front and rear sponsons pass through the points $\underset{p}{N \to f}$, $\underset{p}{N \to r}$ respectively can be described as following

$\underset{p}{N \to f} = \underset{p}{N \to A} + a_1\widehat{b_1^0} + a_2\widehat{b_3^0}$

$\underset{p}{N \to r} = \underset{p}{N \to A} + a_3\widehat{b_1^0} + a_4\widehat{b_3^0}$

The positions of the center of mass of the four sponsons respect to reference frame are described as following

$\underset{p}{N \to s1} = \underset{p}{N \to A} + a_1\widehat{b_1^0} + a_2\widehat{b_3^0} - a_5\widehat{b_1^1} - a_6\widehat{b_2^1} + a_7\widehat{b_3^1}$

$\underset{p}{N \to s2} = \underset{p}{N \to A} + a_1\widehat{b_1^0} + a_2\widehat{b_3^0} - a_5\widehat{b_1^2} + a_6\widehat{b_2^2} + a_7\widehat{b_3^2}$

$\underset{p}{N \to s3} = \underset{p}{N \to A} + a_3\widehat{b_1^0} + a_4\widehat{b_3^0} - a_8\widehat{b_1^3} - a_9\widehat{b_2^3} + a_{10}\widehat{b_3^3}$

$\underset{p}{N \to s4} = \underset{p}{N \to A} + a_3\widehat{b_1^0} + a_4\widehat{b_3^0} - a_8\widehat{b_1^4} + a_9\widehat{b_2^4} + a_{10}\widehat{b_3^4}$

The positions of the running surfaces, where the water loads are assumed to apply, of the four sponsons are

$\underset{p}{N \to w1} = \underset{p}{N \to A} + a_1\widehat{b_1^0} + a_2\widehat{b_3^0} - a_{11}\widehat{b_1^1} - a_{12}\widehat{b_2^1} + a_{13}\widehat{b_3^1}$

$\underset{p}{N \to w2} = \underset{p}{N \to A} + a_1\widehat{b_1^0} + a_2\widehat{b_3^0} - a_{11}\widehat{b_1^2} + a_{12}\widehat{b_2^2} + a_{13}\widehat{b_3^2}$

$\underset{p}{N \to w3} = \underset{p}{N \to A} + a_1\widehat{b_1^0} + a_2\widehat{b_3^0} - a_{14}\widehat{b_1^3} - a_{15}\widehat{b_2^3} + a_{16}\widehat{b_3^3}$

$\underset{p}{N \to w4} = \underset{p}{N \to A} + a_1\widehat{b_1^0} + a_2\widehat{b_3^0} - a_{14}\widehat{b_1^4} + a_{15}\widehat{b_2^4} + a_{16}\widehat{b_3^4}$

Angular Velocity of Center of Hull and Four Sponsons Respect to Reference Frame:

$$\underset{\omega}{N \to A} = \begin{bmatrix}\omega_x \\ \omega_y \\ \omega_z\end{bmatrix} = \begin{bmatrix}\dot{\phi} - \dot{\psi}\sin\theta \\ \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\cos\phi \\ -\dot{\theta}\sin\phi + \dot{\psi}\cos\theta\cos\phi\end{bmatrix}$$

Angular velocities of sponsons can be determined by following equations

$$\omega^{N \to s1} = \omega^{N \to A} - \dot{\theta}_1 \widehat{b_2^0}$$
$$\omega^{N \to s2} = \omega^{N \to A} - \dot{\theta}_2 \widehat{b_2^0}$$
$$\omega^{N \to s3} = \omega^{N \to A} - \dot{\theta}_3 \widehat{b_2^0}$$
$$\omega^{N \to s4} = \omega^{N \to A} - \dot{\theta}_4 \widehat{b_2^0}$$

Linear Velocity of Center of Hull and Four Sponsons Respect to Reference Frame:
The linear velocity of the center of mass of the center hull respect to reference frame is described as following

$$V^{N \to A} = p^{N \to A} = \dot{x}\,\widehat{n_1} + \dot{y}\,\widehat{n_2} + \dot{z}\,\widehat{n_3} = u_1\widehat{n_1} + u_2\widehat{n_2} + u_3\widehat{n_3}$$

The linear velocity of the center of mass of four sponsons respect to reference frame are

$$V^{N \to s1} = p^{N \to A} + \omega^{N \to A} \times (a_1\widehat{b_1^0} + a_2\widehat{b_3^0}) + (-\dot{\theta}_1\widehat{b_2^0}) \times (-a_{11}\widehat{b_1^1} - a_{12}\widehat{b_2^1} + a_{13}\widehat{b_3^1})$$
$$V^{N \to s2} = p^{N \to A} + \omega^{N \to A} \times (a_1\widehat{b_1^0} + a_2\widehat{b_3^0}) + (-\dot{\theta}_2\widehat{b_2^0}) \times (-a_5\widehat{b_1^2} + a_6\widehat{b_2^2} + a_7\widehat{b_3^2})$$
$$V^{N \to s3} = p^{N \to A} + \omega^{N \to A} \times (a_3\widehat{b_1^0} + a_4\widehat{b_3^0}) + (-\dot{\theta}_3\widehat{b_2^0}) \times (-a_8\widehat{b_1^3} - a_9\widehat{b_2^3} + a_{10}\widehat{b_3^3})$$
$$V^{N \to s4} = p^{N \to A} + \omega^{N \to A} \times (a_3\widehat{b_1^0} + a_4\widehat{b_3^0}) + (-\dot{\theta}_4\widehat{b_2^0}) \times (-a_8\widehat{b_1^4} + a_9\widehat{b_2^4} + a_{10}\widehat{b_3^4})$$

Angular Acceleration of Center of Hull and Four Sponsons Respect to Reference Frame:
The angular acceleration of center of mass of hull respect to reference frame

$$\alpha^{N \to sn} = \frac{d}{dt}\,\omega^{N \to A}$$

The angular acceleration of center of mass of sponsons respect to reference frame are

$$\alpha^{N \to s1} = \alpha^{N \to A} + \alpha^{A \to s1} + \omega^{N \to A} \times (-\dot{\theta}_1\widehat{b_2^0})$$
$$\alpha^{N \to s2} = \alpha^{N \to A} + \alpha^{A \to s2} + \omega^{N \to A} \times (-\dot{\theta}_2\widehat{b_2^0})$$
$$\alpha^{N \to s3} = \alpha^{N \to A} + \alpha^{A \to s3} + \omega^{N \to A} \times (-\dot{\theta}_3\widehat{b_2^0})$$
$$\alpha^{N \to s4} = \alpha^{N \to A} + \alpha^{A \to s4} + \omega^{N \to A} \times (-\dot{\theta}_4\widehat{b_2^0})$$

Linear Acceleration of Center of Hull and Sponsons Respect to reference Frame:

The linear accelerations of the center of mass of center of hull is

$$a^{N \to A} = \dot{u}_1\widehat{n_1} + \dot{u}_2\widehat{n_2} + \dot{u}_3\widehat{n_3}$$

Linear acceleration of sponsons respect to reference frame

$$a^{N \to s1} = a^{N \to A} + a^{N \to s1} \times p^{N \to s1} + \omega^{N \to s1} \times (\omega^{N \to s1} \times p^{N \to s1})$$
$$a^{N \to s2} = a^{N \to A} + a^{N \to s2} \times p^{N \to s2} + \omega^{N \to s2} \times (\omega^{N \to s2} \times p^{N \to s2})$$
$$a^{N \to s3} = a^{N \to A} + a^{N \to s3} \times p^{N \to s3} + \omega^{N \to s3} \times (\omega^{N \to s3} \times p^{N \to s3})$$
$$a^{N \to s4} = a^{N \to A} + a^{N \to s4} \times p^{N \to s4} + \omega^{N \to s4} \times (\omega^{N \to s4} \times p^{N \to s4})$$

Constructing of Partial Velocities Table
Where u_i is generalized speed
We can use the following expression

$\underset{V_i}{N \to A}$ partial derivative of linear velocity, i =1,2 ,..10

$\underset{\omega_i}{N \to si}$ partial derivative of angular velocity, i= 1, 2,.10

$$\underset{V_1}{N \to A} = \frac{\partial \underset{V}{N\to A}}{\partial u_1}, \underset{V_1}{N \to s1} = \frac{\partial \underset{V_1}{N\to s1}}{\partial u_1}, \underset{V_1}{N \to s2} = \frac{\partial \underset{V_1}{N\to s2}}{\partial u_1},$$

$$\underset{V_1}{N \to s3} = \frac{\partial \underset{V_1}{N\to s3}}{\partial u_1}, \underset{V_1}{N \to s4} = \frac{\partial \underset{V_1}{N\to s4}}{\partial u_1}, \underset{\omega_1}{N \to A} = \frac{\partial \underset{\omega_i}{N\to A}}{\partial u_1}, \underset{\omega_1}{N \to s1} = \frac{\partial \underset{\omega_i}{N\to s1}}{\partial u_1}, \underset{\omega_1}{N \to s2} = \frac{\partial \underset{\omega_i}{N\to s2}}{\partial u_1}, \underset{\omega_1}{N \to s3} = \frac{\partial \underset{\omega_i}{N\to s3}}{\partial u_1}, \underset{\omega_1}{N \to s4} = \frac{\partial \underset{\omega_i}{N\to s4}}{\partial u_1}$$

Generalied speeds u_i	$\underset{V_i}{N \to A}$	$\underset{V_i}{N \to s1}$	$\underset{V_i}{N \to s2}$	$\underset{V_i}{N \to s3}$	$\underset{V_i}{N \to s4}$	$\underset{\omega_i}{N \to A}$	$\underset{\omega_i}{N \to s1}$	$\underset{\omega_i}{N \to s2}$	$\underset{\omega_i}{N \to s3}$	$\underset{\omega_i}{N \to s4}$
u_1	$\underset{V_1}{N \to A}$	$\underset{V_1}{N \to s1}$	$\underset{V_1}{N \to s2}$	$\underset{V_1}{N \to s3}$	$\underset{V_1}{N \to s4}$	$\underset{\omega_1}{N \to A}$	$\underset{\omega_1}{N \to s1}$	$\underset{\omega_1}{N \to s2}$	$\underset{\omega_1}{N \to s3}$	$\underset{\omega_1}{N \to s4}$
u_2	$\underset{V_2}{N \to A}$	$\underset{V_2}{N \to s1}$	$\underset{V_2}{N \to s2}$	$\underset{V_2}{N \to s3}$	$\underset{V_2}{N \to s4}$	$\underset{\omega_2}{N \to A}$	$\underset{\omega_2}{N \to s1}$	$\underset{\omega_2}{N \to s2}$	$\underset{\omega_2}{N \to s3}$	$\underset{\omega_2}{N \to s4}$
u_3	$\underset{V_3}{N \to A}$	$\underset{V_3}{N \to s1}$	$\underset{V_3}{N \to s2}$	$\underset{V_3}{N \to s3}$	$\underset{V_3}{N \to s4}$	$\underset{\omega_3}{N \to A}$	$\underset{\omega_3}{N \to s1}$	$\underset{\omega_3}{N \to s2}$	$\underset{\omega_3}{N \to s3}$	$\underset{\omega_3}{N \to s4}$
u_4	$\underset{V_4}{N \to A}$	$\underset{V_4}{N \to s1}$	$\underset{V_4}{N \to s2}$	$\underset{V_4}{N \to s3}$	$\underset{V_4}{N \to s4}$	$\underset{\omega_4}{N \to A}$	$\underset{\omega_4}{N \to s1}$	$\underset{\omega_4}{N \to s2}$	$\underset{\omega_4}{N \to s3}$	$\underset{\omega_4}{N \to s4}$
u_5	$\underset{V_5}{N \to A}$	$\underset{V_5}{N \to s1}$	$\underset{V_5}{N \to s2}$	$\underset{V_5}{N \to s3}$	$\underset{V_5}{N \to s4}$	$\underset{\omega_5}{N \to A}$	$\underset{\omega_5}{N \to s1}$	$\underset{\omega_5}{N \to s2}$	$\underset{\omega_5}{N \to s3}$	$\underset{\omega_5}{N \to s4}$
u_6	$\underset{V_6}{N \to A}$	$\underset{V_6}{N \to s1}$	$\underset{V_6}{N \to s2}$	$\underset{V_6}{N \to s3}$	$\underset{V_6}{N \to s4}$	$\underset{\omega_6}{N \to A}$	$\underset{\omega_6}{N \to s1}$	$\underset{\omega_6}{N \to s2}$	$\underset{\omega_6}{N \to s3}$	$\underset{\omega_6}{N \to s4}$
u_7	$\underset{V_7}{N \to A}$	$\underset{V_7}{N \to s1}$	$\underset{V_7}{N \to s2}$	$\underset{V_7}{N \to s3}$	$\underset{V_7}{N \to s4}$	$\underset{\omega_7}{N \to A}$	$\underset{\omega_7}{N \to s1}$	$\underset{\omega_7}{N \to s2}$	$\underset{\omega_7}{N \to s3}$	$\underset{\omega_7}{N \to s4}$
u_8	$\underset{V_8}{N \to A}$	$\underset{V_8}{N \to s1}$	$\underset{V_8}{N \to s2}$	$\underset{V_8}{N \to s3}$	$\underset{V_8}{N \to s4}$	$\underset{\omega_8}{N \to A}$	$\underset{\omega_8}{N \to s1}$	$\underset{\omega_8}{N \to s2}$	$\underset{\omega_8}{N \to s3}$	$\underset{\omega_8}{N \to s4}$
u_9	$\underset{V_9}{N \to A}$	$\underset{V_9}{N \to s1}$	$\underset{V_9}{N \to s2}$	$\underset{V_9}{N \to s3}$	$\underset{V_9}{N \to s4}$	$\underset{\omega_9}{N \to A}$	$\underset{\omega_9}{N \to s1}$	$\underset{\omega_9}{N \to s2}$	$\underset{\omega_9}{N \to s3}$	$\underset{\omega_9}{N \to s4}$
u_{10}	$\underset{V_{10}}{N \to A}$	$\underset{V_{10}}{N \to s1}$	$\underset{V_{10}}{N \to s2}$	$\underset{V_{10}}{N \to s3}$	$\underset{V_{10}}{N \to s4}$	$\underset{\omega_{10}}{N \to A}$	$\underset{\omega_{10}}{N \to s1}$	$\underset{\omega_{10}}{N \to s2}$	$\underset{\omega_{10}}{N \to s3}$	$\underset{\omega_{10}}{N \to s4}$

IV. Generalized Forces

Calculate the Generalized Active Forces

where the generalized active force, Fi, is defined as

$$F_i = \sum_i \left(\vec{F}_i \cdot \underset{V_i}{N \to A} + \vec{T}_i \cdot \underset{\omega_i}{N \to A} + \vec{F}_i \cdot \underset{V_i}{N \to si} + \vec{T}_i \cdot \underset{\omega_i}{N \to si} \right)$$

Where F_i (i=1,….10) is the generalized active forces applied upon sponsons in z direction due to water loads $F_i\widehat{n_3}$
The Applied tourque on sponsons due to external forces

$\vec{T}_1 = F_1\widehat{n_3} \times (\underset{p}{N \to w1} - \underset{p}{N \to f})$

$\vec{T}_2 = F_2\widehat{n_3} \times (\underset{p}{N \to w2} - \underset{p}{N \to f})$

$\vec{T}_3 = F_3\widehat{n_3} \times (\underset{p}{N \to w3} - \underset{p}{N \to r})$

$\vec{T}_4 = F_4\widehat{n_3} \times (\underset{p}{N \to w4} - \underset{p}{N \to r})$

Spring moments upon sponsons suspension springs

$\vec{k}_{s1} = k_1\theta_1 \widehat{b_2^1}$

$\vec{k}_{s2} = k_2\theta_2 \widehat{b_2^2}$

$\vec{k}_{s3} = k_3\theta_3 \widehat{b_2^3}$

$\vec{k}_{s4} = k_4\theta_4 \widehat{b_2^4}$

Where k_i (i=1,2,.4) is spring stiffness

Spring moments upon sponsons suspension springs

$\vec{D}_1 = C_1\theta_1\widehat{b_2^1}$
$\vec{D}_2 = C_2\theta_1\widehat{b_2^2}$
$\vec{D}_3 = C_3\theta_1\widehat{b_2^3}$
$\vec{D}_4 = C_4\theta_1\widehat{b_2^4}$

Where C_i where (i=1,2…4) is damping coefficient of dampers
The generalized active forces due to external forces can be calculated as following

$$F_1 = ((-\mathrm{M}g\widehat{n_3}) + (\mathrm{F}_1\widehat{\mathrm{n}_3}) + (\mathrm{F}_2\widehat{\mathrm{n}_3}) + (\mathrm{F}_3\widehat{\mathrm{n}_3}) + (\mathrm{F}_4\widehat{\mathrm{n}_3})).\underset{V_i}{N \to A} + ((-m_1 g\widehat{\mathrm{n}_3}).\underset{V_1}{N \to s1})) + (\vec{T}_1.\underset{\omega_1}{N \to s1}) + (\vec{k}_{s1}.\underset{\omega_1}{N \to s1}) + (\vec{D}_1.\underset{\omega_1}{N \to s1}) + (-m_2 g\widehat{\mathrm{n}_3}).\underset{V_1}{N \to s2} + (\vec{T}_2.\underset{\omega_1}{N \to s2}) + (\vec{k}_{s2}.\underset{\omega_1}{N \to s2}) + (\vec{D}_2.\underset{\omega_1}{N \to s2}) + ((-m_3 g\widehat{\mathrm{n}_3}).\underset{V_1}{N \to s3}) + (\vec{T}_3.\underset{\omega_1}{N \to s3}) + (\vec{k}_{s3}.\underset{\omega_1}{N \to s3}) + (\vec{D}_3.\underset{\omega_1}{N \to s3}) + ((-m_4 g\widehat{\mathrm{n}_3}).\underset{V_1}{N \to s4}) + (\vec{T}_4.\underset{\omega_1}{N \to s4}) + (\vec{k}_{s4}.\underset{\omega_1}{N \to s4}) + (\vec{D}_4.\underset{\omega_1}{N \to s4})$$

Similary, F_2 *to* F_{10} can be calculated where M is hull mass, m_i where (i=1,..4) Is sponsons mass

Calculate The Generalized Inertia Forces

The generalized inertia force, F_i^* is defined as

$$F_i^* = \sum_i(-\mathrm{M}.\underset{a}{N \to A}.\underset{V_i}{N \to A}) - (\underset{a}{N \to A}.\vec{\mathrm{I}} + \underset{\omega}{N \to A} \times \vec{\mathrm{I}}.\underset{\omega}{N \to A}).\underset{\omega_i}{N \to A} - (m_i.\underset{a}{N \to si}.\underset{V_i}{N \to si}) - ((\underset{\alpha}{N \to si}.\vec{\mathrm{I}}_{si} + \underset{\omega}{N \to si} \times \vec{\mathrm{I}}_{si}.\underset{\omega}{N \to si}).\underset{\omega_i}{N \to si})$$

Substituting in the equation the generalized inertial forces can be determined as following

$$F_1^* = (\mathrm{M}.\underset{a}{N \to A}.\underset{V_1}{N \to A}) - (\underset{a}{N \to A}.\vec{\mathrm{I}} + \underset{\omega}{N \to A} \times \vec{\mathrm{I}}.\underset{\omega}{N \to A}).\underset{\omega_1}{N \to A} - (m_1.\underset{a}{N \to s1}.\underset{V_1}{N \to s1}) - (m_2.\underset{a}{N \to s2}.\underset{V_1}{N \to s2}) - (m_3.\underset{a}{N \to s3}.\underset{V_1}{N \to s3}) - (m_4.\underset{a}{N \to s4}.\underset{V_1}{N \to s4}) - ((\underset{\alpha}{N \to s1}.\vec{\mathrm{I}}_{s1} + \underset{\omega}{N \to s1} \times \vec{\mathrm{I}}_{s1} - \underset{\omega}{N \to s1}).\underset{\omega_1}{N \to s1}) - ((\underset{\alpha}{N \to s2}.\vec{\mathrm{I}}_{s2} + \underset{\omega}{N \to s2} \times \vec{\mathrm{I}}_{s2} - \underset{\omega}{N \to s2}).\underset{\omega_1}{N \to s2}) - ((\underset{\alpha}{N \to s3}.\vec{\mathrm{I}}_{s3} + \underset{\omega}{N \to s3} \times \vec{\mathrm{I}}_{s3} - \underset{\omega}{N \to s3}).\underset{\omega_1}{N \to s3}) - ((\underset{\alpha}{N \to s4}.\vec{\mathrm{I}}_{s4} + \underset{\omega}{N \to s4} \times \vec{\mathrm{I}}_{s4} - \underset{\omega}{N \to s4}).\underset{\omega_1}{N \to s4})$$

Likewise, $F_2^*$to F_{10}^* where

$\vec{\mathrm{I}} = \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix}$ Is moment of inertia of Hull

$\vec{\mathrm{I}}_{s1} = \begin{bmatrix} Is_{11} & 0 & 0 \\ 0 & Is_{22} & 0 \\ 0 & 0 & Is_{33} \end{bmatrix}$ Moment of Inertia of sponson

The generalized active forces and the generalized inertial
forces represented by the equations are summarized as follows
$\mathrm{F}_1 + F_1^* = 0$
$\mathrm{F}_2 + F_2^* = 0$
$\mathrm{F}_3 + F_3^* = 0$
$\mathrm{F}_4 + F_4^* = 0$
$\mathrm{F}_5 + F_5^* = 0$
$\mathrm{F}_6 + F_6^* = 0$
$\mathrm{F}_7 + F_7^* = 0$
$\mathrm{F}_8 + F_8^* = 0$
$\mathrm{F}_9 + F_9^* = 0$
$\mathrm{F}_{10} + F_{10}^* = 0$
These dynamic equations can be represented in matrices form

$$\begin{bmatrix} & \cdots & \\ \vdots & LHS\ COFFICENT & \vdots \\ & \cdots & \end{bmatrix} \begin{bmatrix} \dot{u_1} \\ \dot{u_2} \\ \dot{u_3} \\ \dot{u_4} \\ \dot{u_5} \\ \dot{u_6} \\ \dot{u_7} \\ \dot{u_8} \\ \dot{u_9} \\ \dot{u_{10}} \end{bmatrix} = \begin{bmatrix} & \cdots & \\ \vdots & RHS\ COFFICENT & \vdots \\ & \cdots & \end{bmatrix}$$

V. Conclusion

The paper represents procedures of 10 DOF dynamic model of suspension boat with four sponsons patented by Prof. J Grenestent, using Kane's method. The model enables us to extract information about boat dynamic behavior under different particular conditions, and facilitate choose the optimum design parameters.

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