Numerical Investigations of Narrow Sandwich Beams

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Abstract

This research paper describes a numerical study conducted to evaluate the stresses and deflections in sandwich beams under transverse mechanical loading. The study includes simulations of simply supported three-layer $(0^{0}/core/0^{0})$ and $(90^{0}/core/90^{0})$, five-layer $(0^{0}/90^{0}/core/90^{0}/0^{0})$, and seven-layer $(0^{0}/90^{0}/0^{0}/core/0^{0}/90^{0})$ sandwich beams using ANSYS software. The results are compared with those reported in literature. The study presents the normalized transverse displacement, in-plane normal stresses, and transverse shear stress for different aspect ratios. The simulations use an eight node SHELL 281 element, which is well-suited for analysing layered structures. The accuracy of the numerical results is confirmed by comparing them to existing literature results, which demonstrate good agreement. Overall, this research contributes to a better understanding of the behaviour of sandwich beams under transverse mechanical loading.

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Introduction

Sandwich beams are a class of composite beams that offer high stiffness and strength while maintaining a low weight. They consist of a thick core material that is weak and lightweight, sandwiched between two thin layers called face sheets made of strong material. The primary objective is to increase structural strength without adding extra weight, resulting in an improved strength-to-weight ratio. The selection of face sheet and core materials is critical and depends heavily on their expected performance in the intended environment.

I.

The modelling and analysis of sandwich beams have attracted significant attention due to their broad range of applications. Researchers have utilized various methods to analyse sandwich beams.

Sharma and Rao [1] conducted a detailed investigation of the static deflection and stress analysis of three-layered sandwich cantilever beams that were subjected to both uniform and concentrated loads. Kant and Manjunath [2] developed advanced displacement models for symmetric and asymmetric laminated composite and sandwich beams, based on C0 finite element discretization. The four-node cubic discrete element used in these models had kinematic models with three, four, and five degrees of freedom per node, and a method for computing interlaminar (transverse) stresses was presented. Kant and Manjunath [3] also presented a new set of higher order theories for the analysis of composite and sandwich beams. Barbero et al. [4] presented a formulation of a one-dimensional beam finite element using the layer-wise constant shear (BLCS) approach based on the generalized laminate plate theory. Huang [5] described a study on stress-strain modelling of adhesively bonded sandwich beams to obtain analytical solutions for the displacements, stress, and strain distributions of layers in sandwich beams. Kapuria et al. [6] proposed a new efficient higher-order zigzag theory for the analysis of thermal stresses in laminated beams subjected to thermal loads. Matsunaga [7] conducted an analysis of the displacement and stress distributions of simply supported cross-ply laminated composite and sandwich circular arches subjected to thermal and mechanical loadings, utilizing a global higher order arch theory and three-dimensional equations of equilibrium. Steeves and Fleck [8] presented a systematic approach for comparing the performance of sandwich beams under three-point bending, considering different combinations of materials. Bambole and Desai [9] introduced a new finite element formulation that utilizes a hybrid-interface approach and the minimum energy principle to analyze thick/thin laminated composite beams. Kant et al. [10] proposed a semi-analytical model for static analysis of homogeneous, narrow layered beams under a plane-stress condition based on solving a two-point boundary value problem governed by a set of linear first-order ordinary differential equations. Bardella [11] conducted a study on the accuracy of First-Order Shear Deformation models in computing the deflection of sandwich beams within the linear elastic range. Sankar and Venkataraman [12] investigated sandwich beams with functionally graded cores under sinusoidal transverse loading. Chakrabarti and colleagues [13] developed a finite element (FE) model using the higher-order zigzag theory (HOZT) to analyze the static behavior of laminated sandwich beams with a soft core. Ghugal and Sharma [14] proposed a hyperbolic shear deformation theory for obtaining bending solutions of thick homogeneous,

isotropic, statically beams. Bardella and Tonelli [15] discussed analytical solutions for the precise computation of shear stresses in sandwich beams under flexure. Carrera et al. [16] used various displacement fields-based one-dimensional finite elements to analyze static behavior of laminated beams. Frostig [17] introduced classical and high-order computational models for sandwich panels that contain compressible and incompressible cores. Taranu et al. [18] performed experimental testing on a sandwich panel with a bitumen-impregnated glass mat as the upper facing, a polyurethane foam core, and a bottom facing made of a galvanized cold-formed steel sheet. Kim and Cho [19] proposed an enhanced First-Order Shear Deformation Theory (EFSDT) for laminated and sandwich plates and compared its deflection and stresses with those of the original FSDT and 3D exact solutions. Researchers have studied the behavior of sandwich beams under various loads such as bending, buckling, and vibration, and investigated the effects of material properties, core type, and geometry. Chen and Zhen [20] reviewed recent progress in displacement-based theories and finite element models for analyzing laminated composite plates.

In a concise overview, Carrera and Brischetto [21] summarized recent advances and research papers related to modelling sandwich structures. Various plate theories were described and evaluated for assessing the bending and vibration behavior of sandwich structures. Davidovic et al. [22] conducted a comparison between analytical solutions of the first-order shear deformation plate theory of Mindlin and the classical theory. Gara et al. [23] carried out an experimental study on a construction system that uses completed in-situ sandwich panels with non-shear connectors, focusing on the use of vertical panels as structural walls. Vel et al. [24] proposed an analytical approach to model tapered sandwich members with isotropic facings and a honeycomb core, considering the elastic couplings between bending-transverse shear and extension transverse shear due to the facings' involvement in resisting transverse shear loads.

This study presents numerical investigations conducted using ANSYS Students Version [26] to explore the behaviour of multi-layered sandwich beams under mechanical transverse loads with simple supports. The obtained results were compared with the available literature, demonstrating good agreement.

Finite Element Modellingusing ANSYS

In this study, the authors utilized the eight-node SHELL281 element, which has six degrees of freedom at each node, making it a suitable tool for analysing plate and shell structures. Additionally, this element can be utilized for layered applications in order to model laminated composite shells and sandwich constructions. ANSYS software was employed for the numerical investigations, and it uses the First Order Shear Deformation Theory, commonly known as the Mindlin-Reissner shell theory, for modelling SHELL281 as shown in Fig.1. It should be noted that ANSYS student version was used for this purpose.



Figure 1 SHELL 281 input geometry [26]

II. Results and Discussions

Example 1: 3 layered $(0^{0}/core/0^{0})$ *sandwich beams.*

This section presents numerical investigations of narrow, multi-layered sandwich beams. The first example involves analysing a simply supported laminated sandwich beam with a $0^{0}/\text{core}/0^{0}$ configuration under transverse loading. The core has a thickness of 0.8h, while the two laminated faces are each 0.1h thick, where h = 25mm is the overall thickness of the beam. The beam's span is determined from its aspect ratio and thickness. Transverse load of 1N/mm is applied. The normalized values of transverse displacement (\overline{w}), in-plane normal

stress ($\overline{\sigma}_x$), and maximum transverse shear stress ($\overline{\tau}_{xz}$) are presented in Table 1 for different aspect ratios (l/h) ranging from 4 to 100. The numerical results are compared to existing results from literature and validated using normalization factors for displacements and stresses, as described by Kant et al. [11]. To model the beam, an eight-node SHELL281 element with six degrees of freedom at each node is used, which is suitable for analysing plate and shell structures and can be applied to layered structures such as laminated composite shells and sandwich constructions. The ANSYS software, specifically the ANSYS Student Version, utilizes the First Order Shear Deformation Theory, also known as the Mindlin-Reissner shell theory, for modelling the SHELL281 element.

$$\overline{w} = \frac{100E_2h^3w(\frac{l}{2},0)}{p_0L^4}, \ \overline{\sigma}_x = \frac{h^2\sigma_x(\frac{l}{2},0)}{p_0L^2} \text{ and } \overline{\tau}_{xz} = \frac{h\tau_{xz}(0,z)}{p_0L}$$

To validate the numerical results obtained for the 3-layered sandwich beam example, Kant et al. [11] semi-analytical solutions for narrow sandwich beams under plane stress were used as a benchmark. The percentage error between the presented results and the semi-analytical method was calculated using the formula % error = [Present result - Kant et al. [11]] x 100 / Kant et al. [11]. The Table 1 presents the results obtained for the orthotropic properties of the sandwich beam subjected to transverse load.

Table 1 Comparison of normalized inplane normal stresses ($\overline{\sigma}_x$) transverse shear stresses ($\overline{\tau}_{xz}$) and transverse displacements (\overline{w}) of a simply supported three layered (0⁰/core/0⁰) symmetric sandwich beam in plane stress condition under transverse loading

Stresses/displacement					
S	Source	$(\overline{\sigma}_x)$	$(\overline{\sigma}_x)$	$(\overline{\tau}_{xz})$	(\overline{W})
(1/n)		(l/2, + h/2)	(l/2, -h/2)	(max.)	(1/2,0)
4	Present (ANSYS)	1.7901	-1.7901	0.5524	13.8491
		[-31.23]	[-34.14]	[-3.13]	[0.717]
	Kant et al. [11]	2.6032	-2.7180	0.5703	13.7505
10	Present (ANSYS)	1.5348	-1.5348	0.5524	3.3490
		[-11.23]	[-11.08]	[5.42]	[0.57]
	Kant et al. [11]	1.7290	-1.7260	0.5240	3.3300
20	Present (ANSYS)	1.5446	-1.5446	0.5524	1.7937
		[-2.46]	[-2.44]	[3.56]	[0.011]
	Kant et al. [11]	1.5836	-1.5833	0.5334	1.7935
30	Present (ANSYS)	1.5350	-1.5350	0.5518	1.5088
		[-1.38]	[-1.37]	[3.098]	[0.08]
	Kant et al. [11]	1.5565	-1.5564	0.5358	1.5076
40	Present (ANSYS)	1.5348	-1.5348	0.5524	1.4082
		[-0.78]	[-0.78]	[2.92]	[0.05]
	Kant et al. [11]	1.5469	-1.5469	0.5367	1.4075
50	Present (ANSYS)	1.5344	-1.5344	0.5524	1.3611
		[-0.525]	[-0.525]	[2.848]	[0.000]
	Kant et al. [11]	1.5425	-1.5425	0.5371	1.3611
100	Present (ANSYS)	1.5343	1.5343	0.5525	1.2987

[]% error w.r.t. present ANSYS and Kant et al. [11]

Comparison and validation of the present numerical results with the semi-analytical solutions of Kant et al. (2007) have been conducted for the normalized quantities of in-plane normal stresses ($\overline{\sigma}_x$) at the top and bottom of the orthotropic layer, transverse shear stresses ($\overline{\tau}_{xz}$) at mid-plane, and transverse displacement (\overline{w}) at mid-plane, for aspect ratios (S = l/h) of 4, 10, 20, 30, 40, 50, and 100. The percentage error between the present results and the semi-analytical method of Kant et al. (2007) has been calculated using the formula % error = [(Present result - Kant et al. [11]) / Kant et al. [11]] x 100. The results, shown in Table 1, demonstrate good agreement between the present numerical results and the semi-analytical solutions of Kant et al. [11]. The modeling and contour plots of the three-layered (0⁰/core/0⁰) sandwich beams in ANSYS have been presented in Fig. 2. Fig. 2 shows the variation of in-plane normal stress ($\overline{\sigma}_x$) and transverse displacement (\overline{w}) with respect to the aspect ratio (S = 10) of the three-layered ($\overline{0}^0$ /core/0⁰) sandwich beams in ANSYS. Additionally, Fig. 3 provides Variation of inplane normal stress ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S = 10) of three layered ($\overline{0}^0$ /core/0⁰) sandwich beams in ANSYS. Fig. 4 presents comparison of normalized quantities of inplane normal stresses ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S) of three layered (0^0 /core/0⁰) sandwich beams in ANSYS. Fig. 4 presents comparison of normalized quantities of inplane normal stresses ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S) of three layered (0^0 /core/0⁰) sandwich beams in ANSYS. Fig. 4 presents comparison of normalized quantities of inplane normal stresses ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S) of three layered (0^0 /core/0⁰) sandwich beams.



(c) Transverse shear stresses ($\overline{\tau}_{xz}$) (d) Transverse displacements (\overline{w}) Figure 2Modelling and contour plots of three layered (0⁰/core/0⁰) sandwich beams in ANSYS



Figure 3 Variation of inplane normal stress ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S =10) of three layered (0^0 /core/ 0^0) sandwich beams in ANSYS.



Figure 4 Comparison of normalized quantities of inplane normal stresses ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S) of three layered (0^0 /core/ 0^0) sandwich beam.

Example 2: investigates a symmetric sandwich beam $(90^{\circ}/\text{core}/90^{\circ})$ under transverse loading, with a core thickness of 0.8h and laminated faces of 0.1h each, where h=25mm is the overall thickness.

Table 2 presents the non-dimensional stresses (in-plane normal and transverse shear) and displacements (transverse) for a simply supported sandwich beam subjected to transverse loading. The aspect ratio (l/h) is considered as 100, and the full beam is analyzed with mesh divisions of 0.50, 1, 3, 5, 10, 16, 20, 30, 40, 50 and 100. The results show that the displacements converge at a mesh division of 10, while more mesh divisions are required for the convergence of the stresses. Therefore, a mesh division of 100 is taken for subsequent analyses to obtain sufficiently accurate results for both displacements and stresses.

All numerical results are normalized and presented in Table 3, where the normalized quantities of inplane normal stresses (at the top and bottom of the orthotropic layer), transverse shear stresses (at mid-plane), and transverse displacement (at mid-plane) are compared with the presented numerical results.

The beam is analyzed for various aspect ratios (S= L/h) including 4, 10, 20, 30, 40, 50, and 100. Fig. 5 shows the modelling and contour plots of three-layered ($90^{0}/core/90^{0}$) sandwich beams in ANSYS. Fig. 6 presents the variation of in-plane normal stress and transverse displacement with respect to aspect ratio (S=10) of three-layered ($90^{0}/core/90^{0}$) sandwich beams in ANSYS. Fig. 7 presents normalized quantities of in-plane normal stresses, transverse shear stresses, and transverse displacements with respect to aspect ratio (S) of three-layered ($90^{0}/core/90^{0}$) sandwich beams.

Mesh Size	\overline{W}	$\bar{\sigma}_{x}$	$\overline{ au}_{\scriptscriptstyle XZ}$
	(1/2,0)	(//2, + h/2)	(0, 0)
0.5	30.7148	1.4730	0.5555
1	30.7112	1.4730	0.5307
3	30.7111	1.4730	0.5297
5	30.7111	1.4730	0.5592
10	30.7109	1.4730	0.5182
16	30.7198	1.4730	0.5269
20	30.7179	1.4729	0.5260
30	30.7007	1.4728	0.5237
40	30.6938	1.4727	0.5219
50	30.6922	1.4718	0.5297
100	30.5767	1.4707	0.5091

Table 2 Non-dimensional displacements and stresses at different locations o	f a laminated
$(90^{\circ}/\text{core}/90^{\circ})$ sandwich beam $(l/h = 100)$ for convergence study.	

However, more mesh divisions are required for the convergence of the stresses as expected. As such a mesh division of 100 is taken for all subsequent analysis to get sufficiently accurate results corresponding to displacement as well as stresses. All numerical results are normalized in the following manner;

$$\overline{w} = \frac{100E_2h^3w(\frac{l}{2},0)}{p_0L^4}, \ \overline{\sigma}_x = \frac{h^2\sigma_x(\frac{l}{2},0)}{p_0L^2} \text{ and } \overline{\tau}_{xz} = \frac{h\tau_{xz}(0,z)}{p_0L}$$

The normalized in-plane normal stresses ($\overline{\sigma}_x$) at the top and bottom of the orthotropic layer, transverse shear stresses ($\overline{\tau}_{xz}$) at the mid-plane, and transverse displacement (\overline{w}) at the mid-plane have been compared with the numerical results and are presented in Table 3.

Table 3 Comparison of normalized inplane normal stress ($\overline{\sigma}_x$), transverse shear stresses ($\overline{\tau}_{xz}$) and transverse displacements (\overline{w}) of a simply supported three layer (90⁰/core/90⁰) symmetric sandwich beam in plane stress condition under transverse loading

Stresses/displacements					
S (<i>l</i> / <i>h</i>)	Source	$(\overline{\sigma}_x)$	$(\overline{\sigma}_x)$	$(\overline{\tau}_{xz})$	(\overline{w})
		(l/2, + h/2)	(l /2, - h/2)	(max.)	(172,0)
4	Present (ANSYS)	1.4657	-1.4657	0.5038	43.5863
10	Present (ANSYS)	1.4718	-1.4718	0.5197	32.7518
20	Present (ANSYS)	1.4727	-1.4727	0.5282	31.2057
30	Present (ANSYS)	1.4730	-1.4730	0.5262	30.8989
40	Present (ANSYS)	1.4729	-1.4729	0.5276	30.8192
50	Present (ANSYS)	1.4730	-1.4730	0.5282	30.7729
100	Present (ANSYS)	1.4729	-1.4729	0.5292	30.7111

The beam is also analyzed for different aspect ratios (S = l/h) 4, 10, 20, 30, 40, 50 and 100. Modeling and contour plots of three layered (90⁰/core/90⁰) sandwich beams in ANSYS as shown in Fig. 5. Variation of inplane normal stress ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S =10) of three layered (90⁰/core/90⁰) sandwich beams in ANSYS as shown in Fig. 6. Normalized quantities of inplane normal stresses ($\overline{\sigma}_x$), transverse shear stresses ($\overline{\tau}_{xz}$) and transverse displacements (\overline{w}) with respect to aspect ratio (S) of three layered (90⁰/core/90⁰) sandwich beam are shown in Fig. 7.



Figure 6 Variation of inplane normal stress ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S =10) of three layered (90⁰/core/90⁰) sandwich beams in ANSYS.

100

-.2

-.241

-.282

-. 323.

-.364. -.405

20

40

DIST

60

80

90

100

20

30

40

50 DIST 60

80

-12.845.

-14.966

-17.087.

-19.208.

-21.329.

-23.45 0



Figure 7 Normalized quantities of inplane normal stresses ($\overline{\sigma}_x$), transverse shear stresses ($\overline{\tau}_{xz}$) and transverse displacements (\overline{w}) with respect to aspect ratio (S) three layers (90%/core/90%) of sandwich beam

Example 3: 5 layered $(0^{0}/90^{0}/C/90^{0}/0^{0})$ *sandwich beam.*

A simply supported symmetric sandwich beam $(0^0/90^0/\text{core}/90^0/0^0)$ is analyzed in this example under transverse loading to evaluate stresses and deflections. The layup is (1/2/C/2/1) having a distribution of thickness among the layers as (0.05h/0.05h/0.05h/0.05h), where h (25mm.) is the overall thickness of the sandwich

beam. The values of non-dimensional transverse displacement (\overline{w}), the in-plane normal stress ($\overline{\sigma}_x$) and the maximum transverse shear stress ($\overline{\tau}_{xz}$) are calculated. The beam is also analyzed for different aspect ratios (S = l/h) 5, 10, 20, 50 and 100.Numerical results are validated with existing result from literature. The present result found to be in excellent agreement with the presented numerical results. Normalization factors for displacements and the stresses as described by Chakrabarti et al. [14] are used.

$$\bar{W} = \frac{100E_T h^2 w \left(\frac{l}{2}, z\right)}{p_0 l^4}, \quad \bar{\sigma}_x = \frac{\sigma_x \left(\frac{l}{2}, z\right)}{p_0}, \quad \bar{\tau}_{xz} = \frac{\tau_{xz}(0, z)}{p_0}$$

The variations of the transverse displacement (at the mid span), the in-plane normal stress (at the mid span) and the transverse shear stress (at the boundary) across the depth of the sandwich beam obtained by the present FOST model are shown in Table 4 with those obtained by the Chakrabarti et al. [14] and Vo and Thai [27] has been presented a new finite element (FE) model based on higher order zigzag theory (HOZT), higher order beam theory (HOBT) and sinusoidal shear beam theory (SSBT) for the static analysis of laminated sandwich beam with soft core. The variations of the results from these two researchers are found to match quite well in all the cases. The percentage error between present (ANSYS) and HOZT is calculated as, % error = [Present (ANSYS) - HOZT] x 100 / HOZT.

Table 4 Comparison of normalized inplane normal stress ($ar\sigma_x$), transverse shear stresses ($ar au_{xz}$) and
transverse displacements (\overline{w}) of a simply supported five layered (0 ⁰ /90 ⁰ /core/90 ⁰ /0 ⁰) symmetric sandwich
heam under transverse loading

S	Source			(\overline{W})
(<i>l</i> / <i>h</i>)		(O_x)	(ι_{xz})	(1/2)
		(l/2, h/2)	(max.)	(1/2,0)
5	Present (ANSYS)	52.3903	2.04085	9.9920
	$HO7T^2$	63.84	1.80	7 9568
	nozi	[_17 93]	[13 38]	[25 55]
	HOZT ³	69.8196	2 65/1	9.8243
	nozi	[-24 96]	[-23 10]	[1 68]
	HOBT ³	[21.90]	[25.10]	9 4743
	SSBT ³			9 3801
10	Present (ANSYS)	207 941	4 0817	3 7950
10	HOZT ²	226.11	3.62	3 3060
		[-8.03]	[12,75]	[14.79]
	HOZT ³	225.7728	5.6411	3.7909
		[-7.9]	[-27.64]	[0.11]
	HOBT ³			3.7328
	SSBT ³			3.7235
20	Present (ANSYS)	830.144	8.1634	2.2455
	HOZT ²	874.97	7.25	2.1380
		[-5.12]	[12.6]	[5.03]
	HOZT ³	874.1813	11.6171	2.2423
		[-5.04]	[-29.73]	[0.52]
	HOBT ³			2.2338
	SSBT ³			2.2324
50	Present (ANSYS)	5184.59	20.4084	1.8055
	$HO7T^2$	5417 30	18.12	1 8112
	nozi	[-4 28]	[12 63]	[-0.31]
	HOZT ³	5198 7218	29 5560	1 8057
		[-0.27]	[-30.95]	[-0.01]
	HOBT ³			1.8095
	SSBT ³			1.8093
100	Present (ANSYS)	20739.8	40.8168	1.7495
	HOZT ²	21640.22	36.29	1.7640
		[-4.16]	[12.47]	[-0.82]
	HOZT ³	20739.8185	59.5080	1.7432
		[0.00]	[-31.41]	[0.36]
	HOBT ³			1.7487
	SSBT ³			1.7487

[]% error w. r. t.² Chakrabarti et al. [14] and ³Vo and Thai [27] and present ANSYS

Modeling and contour plots of five layered $(0^{0}/90^{0}/\text{core}/90^{0}/0^{0})$ sandwich beams in ANSYS as shown in Fig. 8. Variation of inplane normal stress ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S =10) of five layered $(0^{0}/90^{0}/\text{core}/90^{0}/0^{0})$ sandwich beams in ANSYS as shown in Fig. 9.Comparison of $(\overline{\sigma}_x)$, transverse shear stresses $(\overline{\tau}_{xz})$ and transverse normalized quantities of inplane normal stresses displacements (\overline{w}) with respect to aspect ratio (S) of five layered ($0^{0}/90^{0}/core/90^{0}/0^{0}$) sandwich beam as shown in Fig. 10.





125

112.5

100

87.5

-.203

-.226

0

50

37.5

2

37.5

12.5

62.5 DIST

47.205

-52.39

100

87.5

125

112.5



Figure 10 Comparison of normalized quantities of inplane normal stresses ($\overline{\sigma}_x$), transverse shear stresses ($\overline{\tau}_{xz}$) and transverse displacements (\overline{w}_1) with respect to aspect ratio (S) of five layer ($0^0/90^0$ /core/90 $^0/0^0$) sandwich beam

Example 4: Numerical investigation of seven layered $(0^{0}/90^{0}/0^{0}/core/0^{0}/90^{0}/0^{0})$ sandwich beam.

Here, new results are generated for a simply supported $(0^0/90^0/0^0/\text{core}/0^0/90^0/0^0)$ symmetric sandwich beam to calculate stresses and deflections through the thickness. The beam is analyzed under transverse load. The layup is (1/2/3/C/3/2/1) having a distribution of thickness among the face layers and the core as (0.04h/0.035h/0.025h/0.04h/0.035h/0.025h), where h (25 mm.) is the overall thickness of the beam.

In Table 5 the results for the non-dimensional displacements (transverse) and stresses (the in-plane normal and transverse shear) are presented to study the rate of convergence and validation of the displacements for simply supported sandwich beam which is subjected to transverse load. The aspect ratio (l/h) is considered as 100. The full beam is analyzed taking mesh divisions 0.50, 1, 3, 5, 10, 16, 20, 31, 40, 50 and 100 as shown in Table 5. However, more mesh divisions are required for the convergence of the stresses as expected. As such a mesh division of 100 is taken for all subsequent analysis to get sufficiently accurate results corresponding to displacement as well as stresses.

For the comparison of the present results a computer program is also developed in matlab to generate results based on first order shear deformation theory solution under plane stress condition. By taking a thickness ratio (l/h) of 10 with those obtained by using FE based software package ANSYS. Normalization factors for displacements and the stresses as described by Chakrabarti et al. [14]are used.

$$\bar{W} = \frac{100E_T h^2 w \left(\frac{l}{2}, z\right)}{p_0 l^4}, \quad \bar{\sigma}_x = \frac{\sigma_x \left(\frac{l}{2}, z\right)}{p_0}, \quad \bar{\tau}_{xz} = \frac{\tau_{xz}(0, z)}{p_0}$$

Modeling and contour plots of seven layered $(0^0/90^0/0^0/\text{core}/0^0/90^0/0^0)$ sandwich beams in ANSYS as shown in Fig. 11. Variation of inplane normal stress ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S =10) of seven layered $(0^0/90^0/0^0/\text{core}/0^0/90^0/0^0)$ sandwich beams in ANSYS as shown in Fig. 12.Comparison of normalized quantities of inplane normal stresses ($\overline{\sigma}_x$), transverse shear stresses ($\overline{\tau}_{xz}$) and transverse displacements (\overline{w}) with respect to aspect ratio (S) of seven layered $(0^0/90^0/0^0/\text{core}/0^0/90^0/0^0)$ sandwich beam as shown in Fig. 13.

(0.750.707001670.750.707) sandwich beam $(u/h = 100)$ for convergence study					
Mesh Size	₩ (1/2,0)	$\overline{\sigma}_{x}$ (//2, + h/2)	$\overline{\tau}_{xz}$ (0, 0)		
0.5	33.5288	35957.7	58.8234		
1	33.5286	35957.2	58.6856		
3	33.5286	35957.3	58.6856		
5	33.5286	35957.3	58.6863		
10	33.5286	35957.6	58.686		
16	33.5285	35956.7	58.6856		
20	33.5284	35956.5	58.6859		
31	33.5281	35955.4	58.6858		
40	33.5278	35954.2	58.6858		
50	33.5286	35966.8	58.6856		
100	33.5229	35938	58.6856		

Table 5 Non-dimensional displacements and stresses at different locations of a symmetric $(0^0/90^0/0^0/\text{core}/0^0/90^0/0^0)$ sandwich beam (l/h = 100) for convergence study

The results for above orthotropic properties of sandwich beam subjected to transverse load and tabulated in Table 6. Normalized quantities of inplane normal stresses ($\overline{\sigma}_x$) at the top and bottom of the orthotropic layer, transverse shear stresses ($\overline{\tau}_{xz}$) at mid-plane and transverse displacement

 (\overline{w}) at mid-plane are compared with presented numerical results. The beam is also analyzed for different aspect ratios (S = l/h) 5, 10, 20, 40, 50 and 100. It can be observed in Table 6 that the present results are sufficiently close with those obtained by using FE based software package ANSYS.

Table 6 Comparison of normalized inplane normal stress (σ_x), transverse shear stresses ($ au_{xz}$)
and transverse displacements (\overline{W}) of a simply supported seven layer ($0^0/90^0/0^0/core/0^0/90^0/0^0$) symmetric
sandwich beam in plane stress condition under transverse loading

S (<i>l</i> / <i>h</i>)	Source	$(\overline{\sigma}_x)$ (l/2, h/2)	$(\overline{\tau}_{xz})$ (0,0)	(\overline{W}) (l/2,0)
5	Present (ANSYS)	90.8294	2.9343	29.079
10	Present (ANSYS)	360.509	5.8686	9.8181
20	Present (ANSYS)	1439.23	11.7371	5.0029
40	Present (ANSYS)	5752.67	23.4742	3.8079
50	Present (ANSYS)	8988.82	29.3428	3.6537
100	Present (ANSYS)	35956.7	58.6856	3.4615



(a) Modelling

(b) Inplane normal stress $(\overline{\sigma}_x)$



(c) Transverse shear stresses ($\overline{\tau}_{xz}$) (d) Transverse displacements (\overline{w}) Figure 11Modelling and contour plots of seven layered ($0^0/90^0/0^0/core/0^0/90^0/0^0$) sandwich beams in ANSYS



Figure 12 Variation of inplane normal stress ($\overline{\sigma}_x$) and transverse displacements (\overline{w}) with respect to aspect ratio (S = 5) of seven layered (0⁰/90⁰/0⁰/core/0⁰/90⁰/0⁰) sandwich beams in ANSYS.



Figure 13 Normalized quantities of inplane normal stresses ($\overline{\sigma}_x$), transverse shear stresses ($\overline{\tau}_{xz}$) and transverse displacements (\overline{w}) with respect to aspect ratio (S) of seven layer ($0^0/90^0/0^0/core/0^0/90^0/0^0$) sandwich beam.

Conclusion

- 1. For three $(0^{0}/\text{core}/0^{0})$ layered sandwich beam, it is seen from comparison that the values of transverse displacements (w) match extremely well with the semi-analytical solutions, but normalized quantities of inplane normal stresses ($\overline{\sigma_{x}}$) are near to lower aspect ratio. Also transverse shear stresses (τ_{xz}) are constant throughout the beam thickness for all aspect ratio.
- 2. For five layered $(0^{0}/90^{0}/\text{core}/90^{0}/0^{0})$ sandwich beam, present numerical results are shows good agreement with HOBT, SSBT and HOZT theory.
- 3. The normalized quantities of inplane normal stresses (σ_x) at the top and bottom of the orthotropic layer, transverse shear stresses (τ_{xz}) at mid-plane and transverse displacement (w) at mid-plane for $(0^0/90^0/\text{core}/90^0/0^0)$ layered sandwich beams obtained from ANSYS are slightly higher than HOBT and near to HOBT
- 4. As number of sheet layers increases, the normalized quantities of inplane normal stresses (σ_x) at the top and bottom of the orthotropic layer and transverse shear stresses (τ_{xz}) at mid-plane are increases as aspect ratio from 4 to 100. Also transverse displacements (w) at mid-plane are decreases as aspect ratio from 4 to 100.

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