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Analysis of Buckling in Thin Laminated Composite Plates using FE Method

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Abstract: *In order to investigate the phenomenon of buckling in thin laminated composite plates, the first order shear deformable plate theory (FSDT) is used. The finite element technique, also known as FEM, is applied in order to get a numerical solution for the differential equations that control the system. An investigation on the buckling behavior of laminated plates having a rectangular cross-section is carried out for a number of different combinations of end circumstances and aspect ratios. Buckling loads are examined and verified with other work that is accessible in the literature in order to verify the correctness of the approach that is currently being used. The trustworthiness of the finite element approach that was utilized is shown by the fact that it is in excellent agreement with other data that is accessible. With the purpose of generating new numerical results for uniaxial and biaxial compression loads on symmetrically laminated composite plates, new results have been developed. For both uniaxial and biaxial compression loading, it was discovered that the influence of boundary conditions on buckling load rises as the aspect ratio increases. This was the case for both types of loading. Additionally, it was shown that the change in buckling load with aspect ratio becomes virtually constant for larger values of elastic modulus ratio. This was another intriguing discovery.*
Key words: *FE Method, Buckling, Thin plates, Laminated composites.*

I. INTRODUCTION

Composite materials find extensive application across a diverse range of modern engineering fields, spanning from conventional areas like automobiles, robotics, and everyday appliances to advanced sectors such as the aerospace industry. This can be attributed to their outstanding high strength-to-weight ratio, modulus-to-weight ratio, and the ability to control structural properties through variations in fiber orientation, stacking scheme, and the number of laminates. The mechanical behavior of rectangular laminated plates, a key aspect of the structural performance of composite material structures, has garnered significant attention. Specifically, analyzing the buckling phenomena in these plates is crucial for ensuring an efficient and reliable design, as well as for the safe application of the structural element. The analysis of composite laminated plates is typically more complex than that of homogeneous isotropic materials, owing to their anisotropic and coupled material behavior. The components and configurations made from laminated composite materials tend to be quite thin, making them more susceptible to buckling. The buckling phenomenon poses significant risks to structural components, as the buckling of composite plates typically happens at lower applied stress levels and results in substantial deformations. This resulted in an emphasis on examining the buckling behavior of composite materials. Comprehensive overviews of the buckling behavior of elastic structures and laminated plates are available in sources such as Refs. {[1] – [14]}. Nonetheless, the data currently accessible are limited to idealized loading scenarios, specifically uniaxial or biaxial uniform compression.

Given the significance of buckling considerations, there exists a vast array of studies addressing related stability issues. These investigations employ a diverse range of analytical approaches, which can be categorized as either closed-form analytical methods or fall under the semi-analytical or purely numerical analysis techniques. Exact closed-form solutions for the buckling problem of rectangular composite plates exist solely for a restricted set of boundary conditions and lamination schemes. The focus is on cross-ply symmetric and angle-ply anti-symmetric rectangular laminates that feature at least two opposite edges simply supported. Additionally, it encompasses similar plates with two opposite edges clamped yet free to deflect (referred to as guided clamps) or configurations where one edge is simply supported while the opposite edge is equipped with a guided clamp. The majority of the precise solutions examined in the works of Whitney, who formulated an exact solution for the critical buckling of solid rectangular orthotropic plates with all edges simply supported, as well as those by Reddy and Leissa and Kang, along with references [7] and [21]. Bao et al. [22] formulated an exact solution for configurations with two edges simply supported and two edges clamped, while Robinson [23] provided an exact solution for the critical buckling stress of an orthotropic sandwich plate with all edges simply supported. For all other configurations, where only approximated results exist, various semi-analytical and numerical techniques have been established. The Rayleigh – Ritz method, the finite strip method (FSM), the element free Galerkin method (EFG), the differential quadrature technique, the moving least square differential quadrature method, and the widely utilized finite element method (FEM) are among the most prevalent approaches. A variety of authors have employed the finite element method to accurately predict in-plane stress distribution, which is subsequently utilized to address the buckling problem. Zienkiewicz [30] and Cook [31] have effectively outlined a method for determining the buckling strength of plates.

This involves initially addressing the linear elastic problem for a reference load, followed by solving the eigenvalue problem to identify the smallest eigenvalue. The product of this eigenvalue and the reference load yields the critical buckling load of the structure. A comprehensive examination of the evolution of plate finite elements over the last 35 years was provided by Yang et al. [32]. A significant number of studies on the buckling analysis of composite plates found in the literature are typically conducted alongside vibration analyses. These studies are grounded in two-dimensional plate theories, which can be categorized into classical and shear deformable types. Classical plate theories (CPT) fail to consider shear deformation effects, leading to an overestimation of the critical buckling loads for thicker composite plates, as well as for thinner ones exhibiting higher anisotropy. The majority of shear deformable plate theories typically rely on an assumption regarding the displacement field, which involves five unknown displacement components. Since three of these components align with those in CPT, the extra components are multiplied by a specific function of the thickness coordinate and incorporated into the displacement field of CPT to account for shear deformation effects. Considering these functions as linear and cubic forms results in the development of the uniform or Mindlin shear deformable plate theory (USDPT) [33], as well as parabolic shear deformable plate theories (PSDPT) [34]. Various forms were also utilized, including hyperbolic shear deformable plate theory (HSDPT) [35] and trigonometric or sine functions shear deformable plate theory (TSDPT) [36]. Due to the inability of these shear deformation theories to meet the continuity conditions across multiple layers of composite structures, the zig-zag or corrugated plate theories proposed by Di Sciuva [37] and Cho and Parmeter [38] were introduced to address interlaminar stress continuities. Recently, Karama et al. [39] introduced a novel exponential function, specifically the exponential shear deformable plate theory (ESDPT), to describe the displacement field in composite laminated structures. This approach aims to accurately represent the shear stress distribution throughout the thickness of these composite structures. The authors also conducted a comparison of their findings for both static and dynamic problems of composite beams against the sine model. The theory applied in this study falls within the category of displacement-based theories. Extensions of these theories that incorporate the linear terms in z in u and v, along with only the constant term in w, to address higher-order variations and laminated plates, are detailed in the research conducted by Yang, Norris and Stavsky [40], Whitney and Pagano [41], and Phan and Reddy [42]. This theory, known as first-order shear deformation theory (FSDT), posits that the transverse planes, initially normal and straight to the mid-plane of the plate, are assumed to remain straight but may not necessarily remain normal after deformation. As a result, a shear correction factor is utilized in this theory to adjust the transverse shear stress, which remains constant throughout the thickness.This study assumes that the composite media are devoid of imperfections, including initial geometrical distortions of the structure, as well as material and constructional flaws such as broken fibers, delaminated areas, cracks in the matrix, foreign inclusions, and small voids resulting from suboptimal selection of fibers or matrix materials and manufacturing defects. Consequently, it is assumed that the fibers and matrix exhibit perfect bonding.

II. BOUNDARY CONDITIONS

Consider a thin plate of length a, breadth b, and thickness h as shown in Figure 1a, subjected to in – plane loads and as shown in Figure 1b.

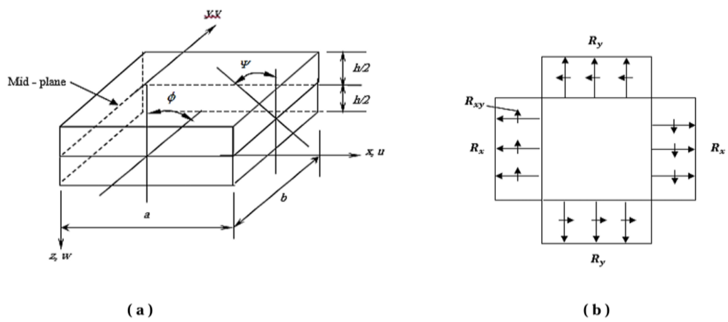

Figure 1

The analyses presented in this paper have been conducted with the assumption that the plate is subjected to either identical or varying support conditions along its four edges. The five sets of edge conditions utilized in this study are categorized as clamped – clamped (CC), simply – simply supported (SS), clamped – simply supported (CS), clamped – free (CF), and simply supported – free (SF), as presented in table 1 below.

Table 1 Boundary conditions

Boundary Conditions	Plate dimensions in y – coordinate $x = 0, x = a$	Plate dimensions in x – coordinate $y = 0, y = b$
CC	$w = \phi = \psi = 0$	$w = \phi = \psi = 0$
SS	$w = \psi = 0$	$w = \phi = 0$
CS	$w = \phi = \psi = 0$	$w = \phi = 0$
CF	$w = \phi = \psi = 0$	–
SF	$w = \psi = 0$	–

III. VALIDATION OF FE PROGRAM

Upon examination of table 4.2, it becomes apparent that the forecast of the buckling loads made by the current research is more comparable to the one made by Reddy J. N. [43]. It should be brought to your attention that the current analysis does not take into account the connection that exists between bending and extensions. A much significant coupling effect may be seen in antisymmetric angle-ply laminates that have a limited number of layers. When there are a considerable number of layers, the coupling effect becomes insignificant, as is the case with the eight-layer laminate that is being compared in the table.

Table 2 Buckling load for simply supported (SS) plate for different moduli and aspect ratios

Aspect Ratio a/b	Modular Ratio	Uniaxial Compression		Biaxial Compression	
	E_1/E_2	10	25	10	25
0.5	Present	24.348	55.790	19.480	44.630
	Ref. [43]	23.746	53.888	18.999	43.110
1.0	Present	18.124	42.690	9.062	21.345
	Ref. [43]	17.637	41.166	8.813	20.578
1.5	Present	18.977	44.476	6.170	14.383
	Ref. [43]	18.565	43.091	6.001	13.877

IV. NUMERICAL RESULTS

It was chosen to conduct a study case and provide findings of buckling loads for cross-ply symmetrically laminated composite plates with a thickness of 0/90/90/0 and 0/90/0. These results will serve as benchmarks for other researchers to employ in their own investigations.

A number of parameters, including the aspect ratio, the boundary conditions, and the modulus ratio, have a significant impact on the buckling stresses that are imposed on the plates. Due to the restricted area that is supplied by this publication, it is not possible to provide the large quantity of data that has been created which cannot be presented. You may find the findings in the tables 3, 4, 5, and 6 that are located below.

Table 3 Buckling load for 0/90/90/0 plate with different boundary conditions and aspect ratios

$$\left(\bar{P} = Pa^2/E_1h^3\right).\ E_1/E_2 = 40, \quad G_{12} = 0.5E_2 \ \text{ and } \ \nu_{12} = 0.25$$

(a) Uniaxial loading

a/b	CC	SS	CS	CF	SF
0.5	2.8999	0.7355	2.8116	2.8816	0.7354
1.0	3.3568	0.8823	2.9888	2.9860	0.8777
1.5	5.1730	1.4268	3.3877	3.3576	1.3822

(b) Biaxial loading

a/b	CC	SS	CS	CF	SF
0.5	1.0827	0.4213	1.0022	0.9852	0.4207
1.0	1.3795	0.4411	1.0741	1.0372	0.4354
1.5	1.6367	0.4391	1.2466	1.1473	0.4372

Table 4 Buckling load for 0/90/90/0 plate with different boundary conditions and aspect ratios $\left(\bar{P} = Pa^2/E_1h^3\right)$. $E_1/E_2 = 5$, $G_{12} = 0.5E_2$ and $\nu_{12} = 0.25$

(a) Uniaxial loading

a/b	CC	SS	CS	CF	SF
0.5	3.1453	0.8598	3.0821	3.0789	0.8556
1.0	4.3829	1.3969	3.5498	3.4952	1.3294
1.5	8.3429	2.9125	4.7780	4.4925	2.5354

(b) Biaxial loading

a/b	CC	SS	CS	CF	SF
0.5	1.8172	0.6877	1.6838	1.6578	0.6874
1.0	2.2064	0.6985	1.8328	1.8125	0.5990
1.5	2.8059	0.8962	1.7618	1.6983	0.8953

Table 5 Buckling load for 0/90/0 plate with different boundary conditions and aspect ratios (¯ Pa^2/E_1h^3). $E_1/E_2 = 5$, $G_{12} = 0.5E_2$ and $\nu_{12} = 0.25$

(a) Uniaxial loading

a/b	CC	SS	CS	CF	SF
0.5	3.3624	0.9142	3.3112	4.2781	0.9105
1.0	4.3977	1.3969	3.7376	3.6940	1.3439
1.5	7.7135	2.6763	4.7942	4.5828	2.4048

(b) Biaxial loading

a/b	CC	SS	CS	CF	SF
0.5	1.7380	0.6871	1.6337	1.5690	0.6872
1.0	2.1744	0.6984	1.7113	1.6820	0.6986
1.5	2.5075	0.8235	1.7622	1.6814	0.8239

Table 6 Buckling load for 0/90/0 plate with different boundary conditions $\left(\bar{P} = Pa^2/E_1h^3\right)$. $E_1/E_2 = 40$, $G_{12} = 0.5E_2$ and $\nu_{12} = 0.25$

(a) Uniaxial loading

a/b	CC	SS	CS	CF	SF
0.5	2.7304	0.8011	2.6555	2.6435	0.8010
1.0	3.3700	0.8823	3.2149	3.2142	0.8809
1.5	4.1817	1.1421	3.4017	3.3947	1.1313

(b) Biaxial loading

a/b	CC	SS	CS	CF	SF
0.5	0.7529	0.3325	0.7201	0.7143	0.3319
1.0	0.9511	0.3489	0.7932	0.7803	0.3478
1.5	1.1763	0.3514	0.8099	0.7940	0.3472

V. CONCLUSIONS

The development of a finite element model has been done with the intention of accomplishing the objective of calculating the buckling loads of laminated plates that have a rectangular cross-section. This was something that had to be done in order to ensure that there was consistency. The buckling loads are explored and validated by the process of comparison, which involves comparing them to prior research that has been published in the relevant literature in the past. The reason that this is done is to guarantee that the approach that is currently being used produces correct results, which is the reason why this is done. There were other comparisons that were carried out, and the results that were supplied by the ANSYS software as well as the findings from the tests were compared with all of the other findings. The comparisons were carried out in more detail. With a high degree of concordance with the data that is not only available but also readily accessible, the finite element technique that was utilized demonstrates that it is dependable. This is shown by the fact that it has a high degree of alignment with the data.

REFERENCES

[1]. Timoshenko S.P., Gere J.M.,' Theory of elastic stability', second edition, New York: McGraw Hill book company, Inc.; (1961).
[2]. Chai G.B., Hoon K.H.,' buckling of generally laminated composite plates', Composite science technology; (1992), 45:PP. (125 –
[3]. Liew K.M., Wang C.M.,' Pb – 2 Rayleigh – Ritz method for general plate analyses', Engineering structures; (1993), 15:PP. (55 –
[4]. Chai G.B., Khong P.W.,' The effect of varying support conditions on the buckling of laminated composite plates', Composite structures; (1993), No.24(2): PP.(99 –
[5]. Narita Y. Fukushi K.,' Analysis program for generally laminated rectangular plates based on classical plate theory', Trans. J. SME, (A); (1996), PP. (2621 –
[6]. Ashton J.E., Whitney J.M,' theory of laminated plates', Stamford: Technomic publishing co., Inc.; (1970).
[7]. Jones R.M.,' Mechanics of composite materials', Washington: Scripta Book Co.; (1975).
[8]. Turvey G.J., Marshall I.H.,' Buckling and post buckling of composite plates', London: Chapman and Hall; (1995).
[9]. Singer J., Arbocz J., Weller T.,' Buckling experiments: Experimental methods in buckling of thin walled strictures', vol.1, New York: John Willey and sons ;(1998).
[10]. Singer J., Arbocz J., Wetter T.,' buckling experiments: Experimental methods in buckling of thin walled structures: shells, Built up structures, composites and additional topics', vol.2. , New York: John Willey and sons;
[11]. Reddy J.N., second edition,' Mechanics of laminated composite plates and shells', Boca Raton CRC press; (2004).
[12]. Leissa A.W.,' Buckling of laminated composite plates and shells panels', Air force Wright – Patterson aeronautical laboratories, final report ;(1985).
[13]. Herakovich C.T., Tarnopolskii Y.M.,' Hand book of composites structures and design', vol.2, Amsterdam: North Holland; (1989).
[14]. Berthelot J.M.,' Composite materials mechanical behavior and structural analyses', New York: Springer; (1999).
[15]. Whitney J.M.,' Structural analysis of laminated anisotropic plates', Lancaster, PA: Technomic Publishing; (1987).
[16]. Reddy J.N.,' Mechanics of laminated composite plates: Theory and Analyses', Boca Raton: CRC press; (1997).
[17]. Reddy J.N.,' Energy and variational methods in applied mechanics', New York: Wiley; (1984).
[18]. Reddy J.N., Phan N.S.,' Stability and natural vibration of isotropic, orthotropic and laminated plates according to a higher order shear deformable theory', Journal of sound and vibration; (1985), 9 (82).
[19]. Reddy J.N., Khdeir A.A.,' Buckling and vibration of laminated composite plates using various plate theories', AIAA; (1989), 27 (12).
[20]. Leissa A.W., Kang J.,' Exact solutions for vibration and buckling of an SS – C – SS – C rectangular plate loaded by linearly varying in – plane stresses', International Journal of mechanical sciences;(2002),44.
[21]. Iyengar N.G.,' Structural stability of columns and plates', Chichester, England: Ellis Harwood limited publishers;
[22]. Bao G., Jiang W., Roberts J.C.,' Analytic and finite element solutions for bending and buckling of orthotropic rectangular plates', International Journal of solids and structures ; (1997),34(14).
[23]. Robinson J.R.,' The buckling and bending of orthotropic sandwich panels with all edges- simply supported', Aero Q; (1955), 6(2):PP. (125).
[24]. Baharlou B., Leissa A.W.,' Vibration and buckling of generally laminated composite plates with arbitrary edge conditions', International Journal of mechanical sciences;(1987),29(8):
[25]. Dawe D.J., Wang S.,' Spline finite strip analysis of the buckling and vibration of rectangular composite laminated plates', International Journal of mechanical sciences; (1995),37(6):
[26]. Liu G.R., Chen X.L., Reddy J.N.,' Buckling of symmetrically laminated composite plates using the element free Galerkin method', International Journal of structural stability dynamics; (2002), 2(3):
[27]. Bert C.W., Malik M.,' Differential quadrature: A powerful new technique for analysis of composite structures', composite structures; (1997), 39(3– 4):
[28]. Huang Y.Q., Li Q.S.,' Bending and buckling analysis of anti – symmetric laminates using the least square differential quadrature method', Computer methods in applied mechanics and engineering, 193; (2004):
[29]. Kim Y.S., Hoa S.V.,' biaxial buckling behavior of composite rectangular plates', composite structures; (1995), 31(4):
[30]. Zienkiewicz O.C.,' The finite element method', England: McGraw Hill; (1977).
[31]. Cook R.D.,' Concepts and applications of finite element analyses', New York: Wiley; (1981).
[32]. Yang H.T.Y., Saigal S., Masud A., Kapania R.K.,' A survey of recent shell element', International Journal for numerical methods in engineering,47 (1–3); (2000):
[33]. Mindlin R.D.,' Influence of rotatory inertia and shear on flexural motions of isotropic elastic plates', Trans. ASME Journal of applied mechanics; (1951), 18:
[34]. Reddy J.N.,' A refined non – linear theory of laminated plates with transverse shear deformation', International Journal of solids and structures; (1984), 51:
[35]. Soladatos K.P.,' a transverse shear deformation theory for homogeneous monoclinic plates', Acta Mech.; (1992), 94:
[36]. Touratier M.,' An efficient standard plate theory', International Journal of engineering sciences; (1991), 29(8):
[37]. Di Sciuva M.,' An improved shear deformation theory for moderately thick multi – layered anisotropic shells and plates', Journal of applied mechanics; (1987), 54:

[38]. Cho M., Parmeter R.R.,' Efficient higher – order composite plate theory for laminated composites', AIAA Journal; (1993), 31 – 7:
[39]. Karama M., Afaqk S., Mistou S.,' Mechanical behavior of laminated composite beam by the new multi – layered laminated composite structures model with transverse shear stress continuity', International Journal of solids and structures;(2003),40:
[40]. Constance P. Yang, Charles H.Norris and Yehuda Stavsky, ' Elastic Wave Propagation in Heterogeneous Plates', International Journal of Solids and Structures, Vol.2, (1966).
[41]. Whitney J.M. and Pagano N.J.,' Shear Deformation in Heterogeneous Anisotropic Plates ', Journal of Applied Mechanics, Vol.4, (1970).
[42]. Phan N.D. and Reddy J.N., 'Analysis of Laminated Composite Plate using Higher-Order Shear Deformation Theory', International Journal of Numerical Method in Engineering, Vol. 21, (1985).
[43]. Reddy J.N. "Mechanics of laminated composite plates and shells" 2nd edition, CRC Press, (2004).