IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 18, Issue 6 Ser. II (Nov. – Dec. 2021), PP 16-25
www.iosrjournals.org

Application of Ritz Analysis for the Building with Rigid Underground Stories: A Comparative Study to Common Approaches

Toshif Patel[1], Ravi Karkar[2], Abdullah Lala[3], Dr. Jigar Sevalia[4]
[1,2,3,4] (Department of Civil Engineering, *Sarvajanik College of Engineering and Technology, Surat, 395001, Gujarat, India)*

Abstract: *To analyze the structures with rigid underground or bottom stories, two approaches are common among the practicing engineers. In the first approach, the entire structure including the basement walls is analyzed for the first few modes by mode superposition method using Eigen vectors while the similar second approach neglects the rigidity of the basement walls. There exists another approach in which two-stage analysis is performed but due to implementation issues, it is not performed for large structures. The present study provides a comparison of the two common approaches and also deals with the effect of mode truncation for building with rigid underground stories. Further, it is suggested that complete seismic response of the buildings with rigid underground stories should be captured and for that a computationally efficient method based on load-dependent Ritz vector is recommended.*

Key Word*: Underground stories; Mode superposition analysis; Ritz vector; Eigen vector; Missing mass*

Date of Submission: 01-12-2021 Date of Acceptance: 15-12-2021

I. Introduction

In the modern era, most of tall buildings are generally provided with numbers of underground stories. In such cases, the underground structure becomes much more rigid than the above-ground structure and the seismic mass associated with it does not excite. To analyse such structures structural engineers do not have the consensus and follow three different approaches.

In the first approach entire structure including the basement walls is included in the analysis model while in the second approach basement walls are not included in the analysis model instead the weight of the basement wall is applied on the frame element. However, in both the approach, analysis is performed by mode superposition method using Eigen vectors.

The frequency and mode shapes obtained from Eigen vector analysis are used for seismic analysis by response spectrum method. In order to capture the complete response of the structure large number of modes are required which may consume significant computational effort and may lead to practical difficulties. To overcome this practical difficulty higher frequency modes are truncated and the properties of the first few lower modes are used in the analysis.

Many building codes require truncation of modes to satisfy certain percentage of mass participation. Similarly, IS 1893:2016 [1] requires that number of modes should be selected such that at least 90% mass participation is obtained in the direction considered for analysis.

As the Eigen vector analysis is based on free vibration and the seismic mass associated with the rigid underground stories does not excite resulting in many modes with higher frequencies for which the inertial effects are negligible [2]**.** The study conducted by M. Dhileep, P. Arumairaj, and G. Hemalatha [3] concluded that truncation of higher frequency mode generally ignores some mass participation and can cause significant error especially in the calculation of the response of stiff structural systems.

In order to capture higher frequency modes, it is essential to consider large number of modes and also require missing mass correction procedure. M. Dhileep and P. Bose [4] studied the methods to consider the effect of missing mass like mode acceleration method, residual method, and static correction method.

In order to avoid difficulty in capturing the missing mass and to save the computational effort, many building codes like ASCE 7-16 [5] suggested a two-stage equivalent lateral force analysis procedure provided that substructure is 10 times stiffer than superstructure. Here the flexible super structure is analysed as a separate structure by either equivalent static method or response spectrum method and the rigid substructure is analysed as a separate structure by equivalent static method.

In the two-stage analysis procedure, the reactions from individual columns of flexible structure is to be input manually and require amplification by the ratio of Response reduction factor of the upper and lower portion. For these two separate teams are required to carry out analysis and design processes especially for large structures.

Owing to the implementation challenges mentioned above, two stage analysis procedure is not common amongst the practicing engineers and *hence the present study focuses on*

- *Comparison of two commonly used approaches*
- *Effect of mode truncation on building with rigid underground stories.*
- *Further this paper emphasizes on capturing the complete seismic response. However, capturing complete seismic responses consumes significant time and effort. Therefore, this paper also suggests a computationally efficient load-dependent Ritz vector analysis by comparing its results for both approaches.*

To brief about the methods used in the study, this paper begins by introducing the Eigen vector analysis, static correction method and load dependent Ritz vector method.

Eigen Vector

The Eigen vectors are based on free vibration and require numbers of numerical operations to evaluate natural frequencies and mode shapes. Frequency values indicate possible resonant conditions and mode shapes associated with higher time period indicates the regions of flexibility [6].

The equation of motion for multi-degree of freedom system without damping is given as [7],

$$M\ddot{U} + KU = P(t) \qquad \text{(}\textbf{\textit{Error! Bookmark not defined.}}\text{)}$$

By putting $p(t) = 0$ gives the governing equation of motion for free vibration system

$$M\ddot{U} + KU = 0 \qquad (2)$$

Above equation represent total N numbers of differential equation which are coupled with the mass and stiffness matrices and the solution of above equation given as

$$U(t) = u(t)_i \emptyset_i \qquad (1)$$

Substituting Equation no. (3) in to Equation no. (2) gives,

$$[-\omega^2 M\emptyset_i + K\emptyset_i]u(t) = 0 \qquad (2)$$

Above equation can be solved in two ways. Either substituting $u(t)$=0 which indicate that structure has no motion or by evaluating the values of natural frequency and mod e shapes which satisfy the following algebraic equation.

$$[K\emptyset_i] = \omega^2 M\emptyset_i \qquad (3)$$

Above equation is known as matrix eigenvalue problem. Here ω is natural frequency; $\emptyset_i$ is Eigen vector; K is the stiffness matrices; M is the mass matrices;

Static correction method:

In this method, response of the higher frequency mode shape is determined by the static analysis. For each of N-n high-frequency modes the response Ui at any time 't' is calculated by ordinary static analysis [4],

$$U = \sum_{i=1}^{n} \emptyset_i X_i + [K^{-1} U_b \ddot{u}_g - \sum_{i=1}^{n} F_i U_{bi} \ddot{u}_g] M \qquad (4)$$

where, U is the displacement vector; U_b is the static displacement vector when the base of the structure undergoes a unit deflection in the direction of earthquake; $\ddot{u}_g$ is the ground acceleration; K is the stiffness matrices; M is the mass matrices; $\emptyset_i$ is the Eigen vector determined from Eigen value problem; X_i is the modal coordinate and $F_i = \frac{\emptyset_i \emptyset_i^T}{\emptyset_i^T K \emptyset_i}$.

For the structure having rigid underground stories or stiff supports Static correction method is used in addition to modal analysis for getting accurate response of the structure. However, the major drawback of this method is that it cannot capture dynamic modes and hence the obtained Eigen vectors are not true Eigen vectors.

Ritz vector analysis:

E. L. Wilson, M. W. Yuan and J. M. Dickens have been demonstrated [6] that dynamic analyses based on a special set of load-dependent Ritz vectors yield more accurate results than the use of the same number of natural mode shapes.

In this method, load-dependent Ritz vectors are generated by considering the spatial distribution of the dynamic loads. The power of this method is that it will capture those mode firsts that have got a large portion of participation. However, the first Ritz vector is a static displacement vector that serves as the starting load vector and it is determined from the following relationship [7],

$$\mathrm{K}\mathrm{Y}_1 = S \tag{5}$$

Where, K = Stiffness matrices; S = spatial distribution of dynamic loading, and Y_1 is static displacement vector. The Y_1 vector in equation (5) is orthogonalized and normalized to obtain first Ritz vector given below,

$$\Psi_1 = \frac{Y_1}{\sqrt{Y_1^T M Y_1}} \tag{6}$$

The remaining vectors are generated from above determined Ritz vector and are used as the load vector for the next static solution which follows the relationship given below [7],

$$KY_n = M\Psi_{n-1} \tag{7}$$

Further, Standard Eigen solution techniques are used to orthogonalize the set of generated Ritz vectors which then provides the final set of Ritz vector modes and are given by,

$$\Psi' = \phi\Psi \tag{8}$$

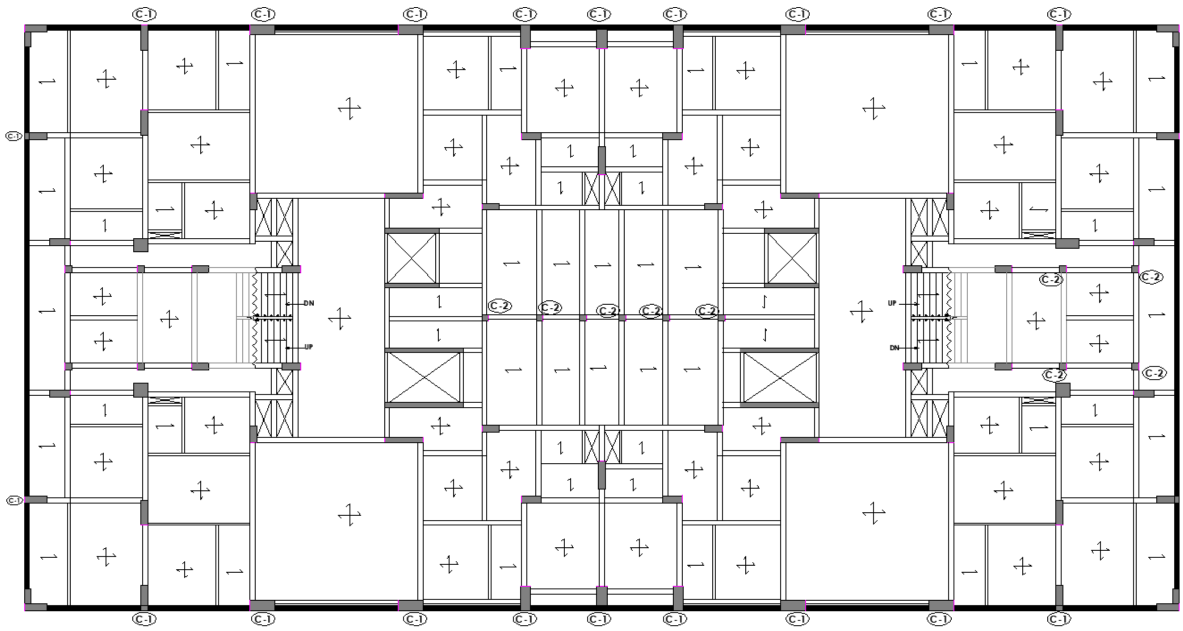
Where, Ψ_1 is the first Ritz vector; Ψ' is final set of Ritz vector; n is number of modes to be considered.

II. Modeling And Analysis

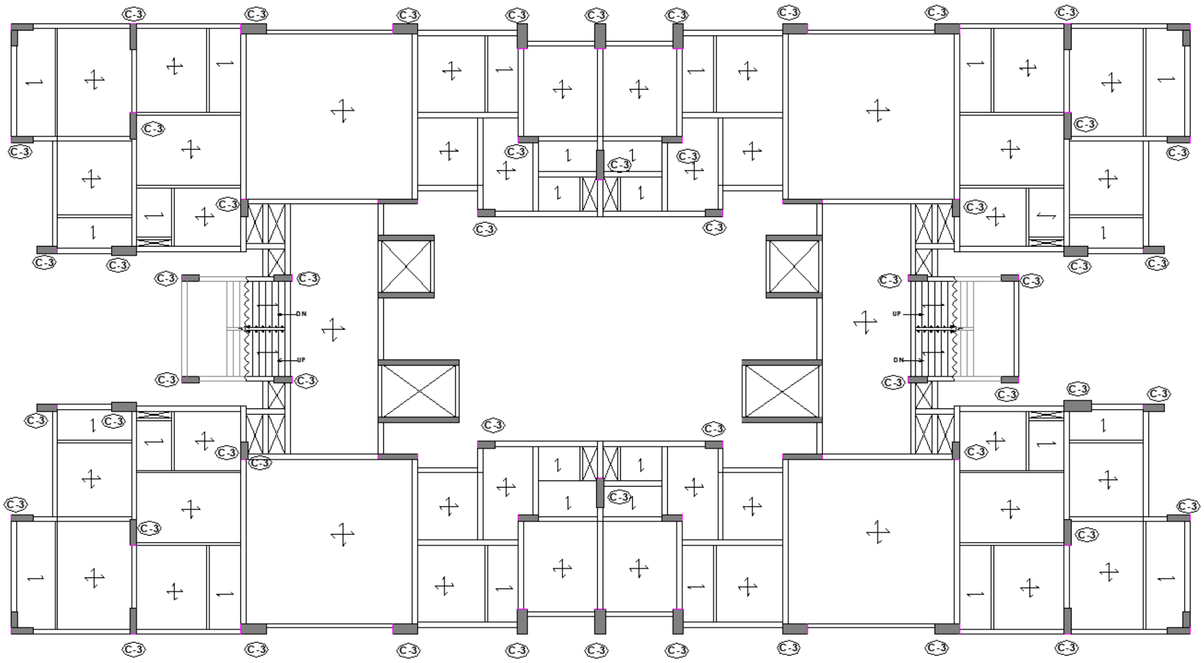
To demonstrate the research, a 17-story building having a story height of 3 meters with the structural plan shown in figure 1 is considered and the number of below-grade levels is varied from 1 to 5 stories. It is assumed that the below-grade stories are of the same height as that of the above-grade stories.

Structural Plan

The structural plan used for the present study is shown in figure 1



(a)



(b)

Fig. 1 (a) Underground stories plan view (b) Typical stories plan view

Modeling parameters

Modelling of the building is done by using special-purpose finite element software ETABS. The modelling parameters like grade of materials, section properties, stiffness modifiers, loading is shown from table 1 to table 4.

Table 1 Grade of materials

Characteristic compressive strength of concrete (N/mm^2)				
Columns	Beams	Slabs	Shear walls	Basement walls
30	25	20	30	30
Yield strength of rebar (N/mm^2)				
415				

Table 2 Section properties

Columns (mm)		**Beams (mm)**	**Slab (mm)**	**Shear wall (mm)**		**Basement wall thickness (mm)**
Perimeter column only for underground stories (C1)	600 x 1200	300 x 600	125	Core walls	200	600
all square columns (C2)	500 x 500			Remaining walls	300	
Remaining columns (C3)	400 x 1200					
Note: Slab is modelled for membrane behavior while shear walls and basement walls are modelled for shell thin behaviour.						

Table 3 Stiffness modifiers

Columns	**Beams**	**Slabs**	**Shear wall**	**Basement wall**
0.70Ig	0.35Ig	No modifier	0.70Ig	No modifier
Here Ig is the gross moment of inertia				

Table 4 Loading

Floor finish (KN/m^2)	**Live load (KN/m^2)**	**Wall load (on all beams) (KN/m)**
1.5	2	13.8

Seismic parameters

The data used to perform response spectrum analysis as per IS 1893:2016 [1] is shown in table 5

Table 5 Seismic parameters

Data type	**Value**
Seismic zone	Ⅲ

Soil type	II
Response reduction factor (R)	5
Importance factor (I)	1

Analysis

Modal analysis is performed by using Eigen vectors and Ritz vectors in finite element software ETABS. To evaluate the base shear response spectrum method is performed as per IS 1893:2016 [1]. In the present study, analysis is performed *to represent the comparison of two approaches adopted by practicing engineers, Effect of mode truncation and Recommendation of load depended Ritz vector analysis for building with rigid underground stories* by comparing four cases as given below,

Case 1: Exact Base Shear including basement wall by Eigen vector analysis with static correction
Case 2: Exact Base Shear including basement wall by Ritz vector analysis
Case 3: Base shear for 90% mass participation by Eigen vector analysis with static correction
Case 4: Base shear for 90% mass participation by Ritz vector analysis

III. Result and Discussion

The modal analysis is performed by Eigen vector and Ritz vector analysis for the two common approaches where in the first approach entire structure including the basement walls is included in the analysis model while in the second approach basement walls are not included in the analysis model instead the weight of basement walls is applied on beam element.

Comparison of two commonly used approaches

Generally, seismic response is obtained in the form of base shear. The complete seismic response is said to be captured when the base shear is obtained for complete mass participation. In the present, the base shear associated with complete mass participation is termed as exact base shear. In Figures 2 and 3, the two approaches are compared. It can be seen that when the basement walls are not included in the analysis model, the results are underestimated by 10 % to 55 % in the X direction and by 8 % to 46 % in the Y direction depending upon the rigidity of the lower portion. Notably, both approach provides identical result up to 2 Numbers of underground stories.

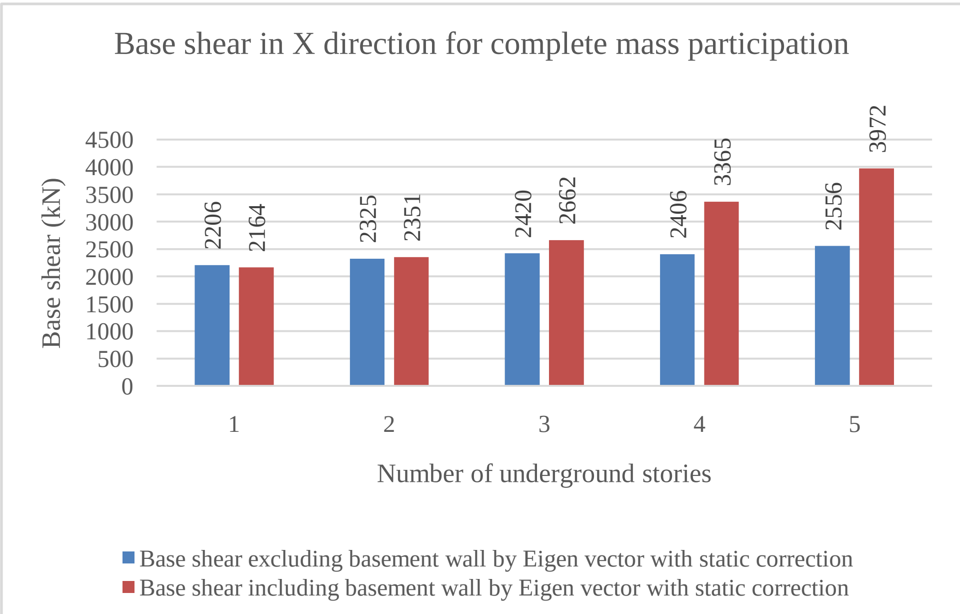


Fig. 2 Base shear in X direction for complete mass participation

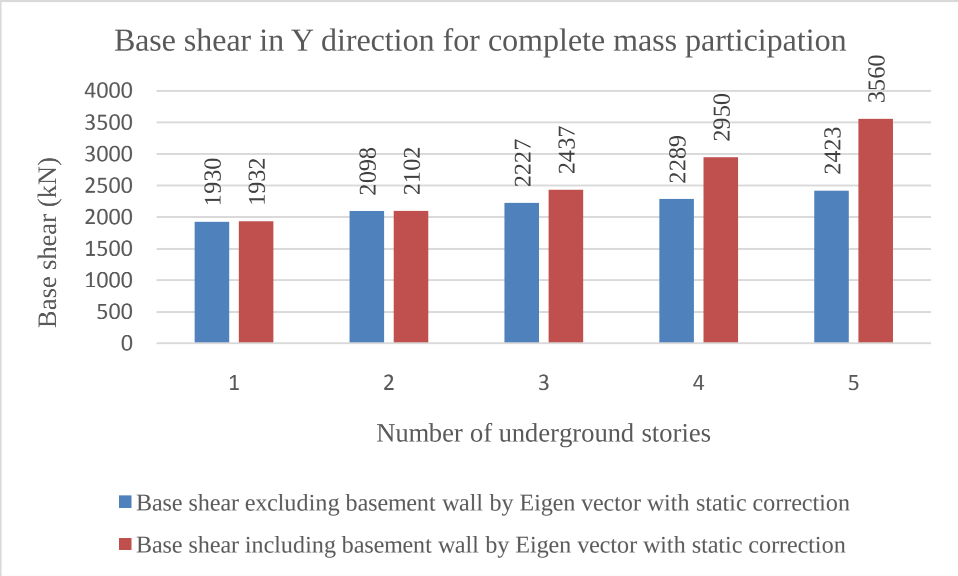


Fig. 3 Base shear in Y direction for complete mass participation

Effect of mode truncation on building with rigid underground stories.

Many building codes recommend that modes should be truncated in such a way that mass participation is at least 90%. It can be seen from Figures 4 and 5 that when modes are truncated, mass participation of rigid lower portion which contains higher frequency modes are ignored and result in higher base shear in the X direction by 21% to 66% in Eigen vector analysis. Similarly, deviation observed in the Y direction varies from 22% to 50% for Eigen vector analysis. It is observed that, case 3 overestimates the base shear and gives an uneconomical design. For this reason, mode truncation should be avoided in Eigen vector analysis when there are rigid underground stories.

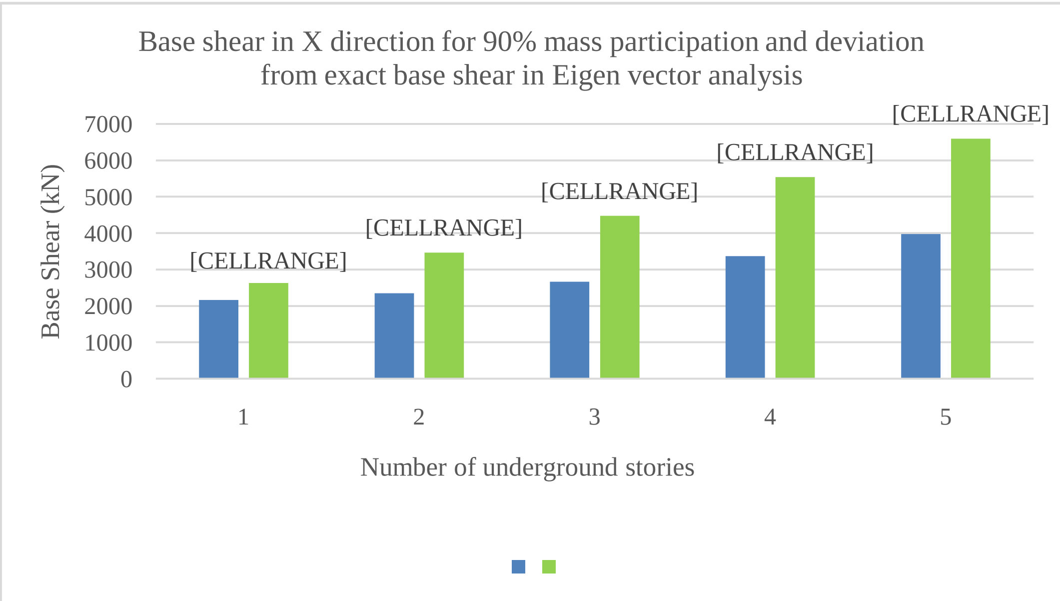


Fig. 4 Base shear in the X direction for 90% mass participation in Eigen vector analysis

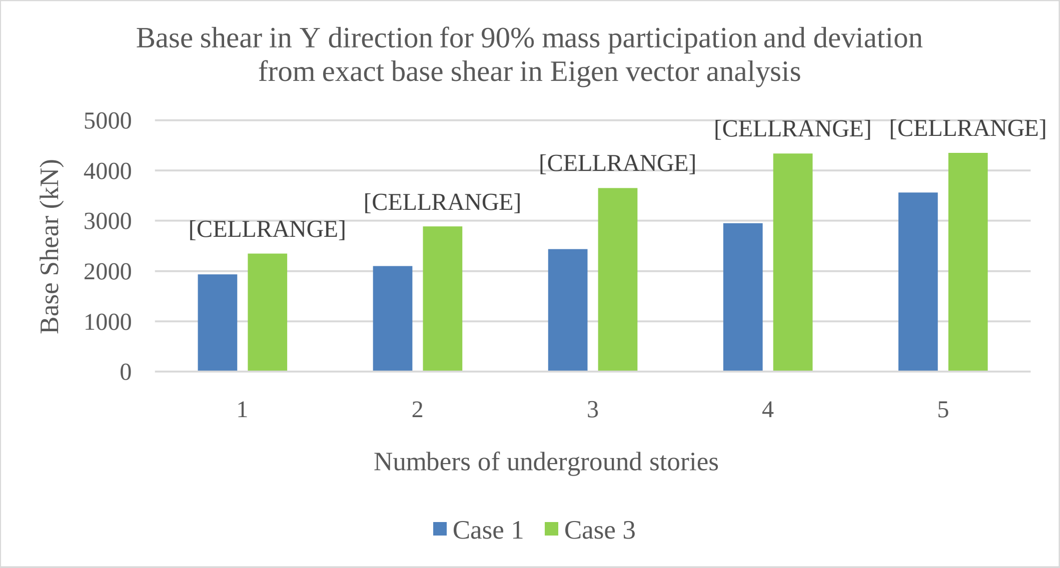


Fig. 5 Base shear in the Y direction for 90% mass participation in Eigenvector analysis

Recommendation of load-dependent Ritz vector analysis for Building with Rigid underground stories

As shown in figure 6, the number of modes required to capture the complete mass participation is excessive in Eigen vector analysis including missing mass correction and hence consume significant computational time and effort. on the other hand, Ritz vector analysis considers spatial distribution of dynamic loading and provides approximately similar base shear to Eigen vector analysis by considering only a few numbers of modes shown in Figures 6,7 and 8. Moreover, missing mass correction is automatically considered in Ritz vector analysis.

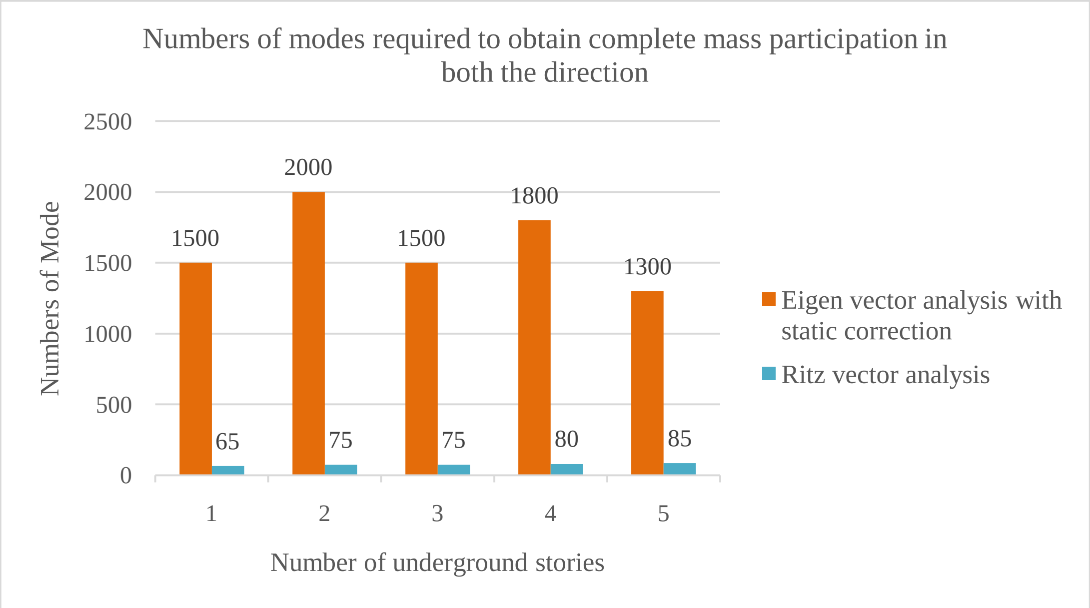


Fig. 6 Numbers of modes required to obtain complete mass participation in both the direction

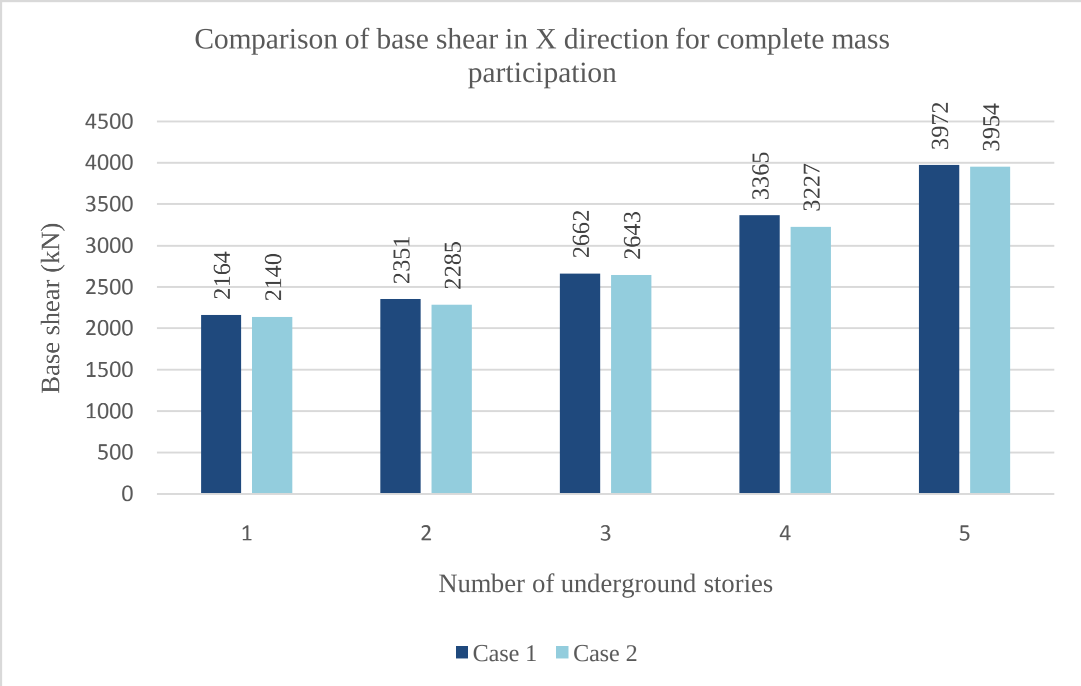


Fig. 7 Base shear in the X direction for complete mass participation

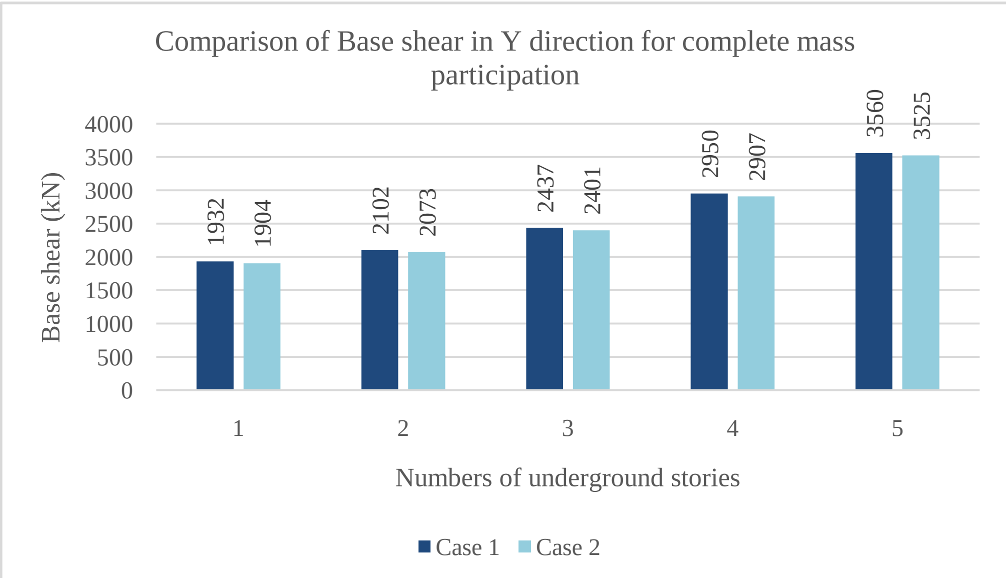


Fig. 8 Base shear in the Y direction for complete mass participation

Now, the question is arising whether mode truncation should be allowed or not in Ritz vector analysis?
It can be seen from Figures 9 and 10 that when modes are truncated, mass participation of rigid lower portion which contains higher frequency modes are ignored and result in higher base shear in the X direction by 55% to 91% in Ritz vector analysis. Similarly, deviation observed in the Y direction varies from 22% to 77% for Ritz vector analysis. Figures 9 and 10 shows that, case 4 overestimates the base shear and gives an uneconomical design. Hence, mode truncation should also avoid in Ritz vector analysis when there are rigid underground stories.

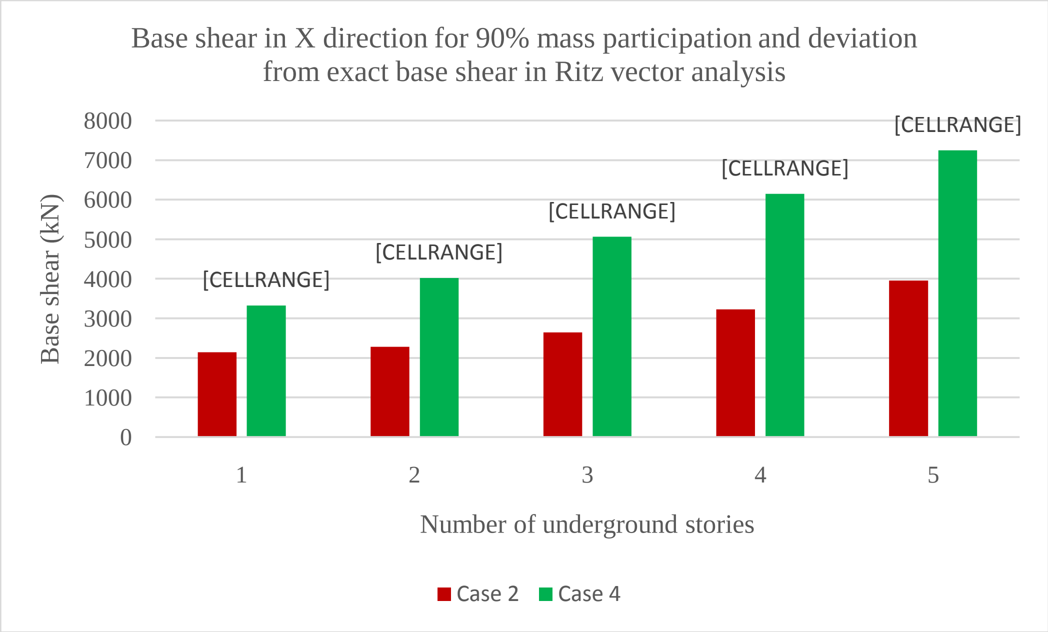


Fig. 9 Base shear in the X direction for 90% mass participation in Ritz vector analysis

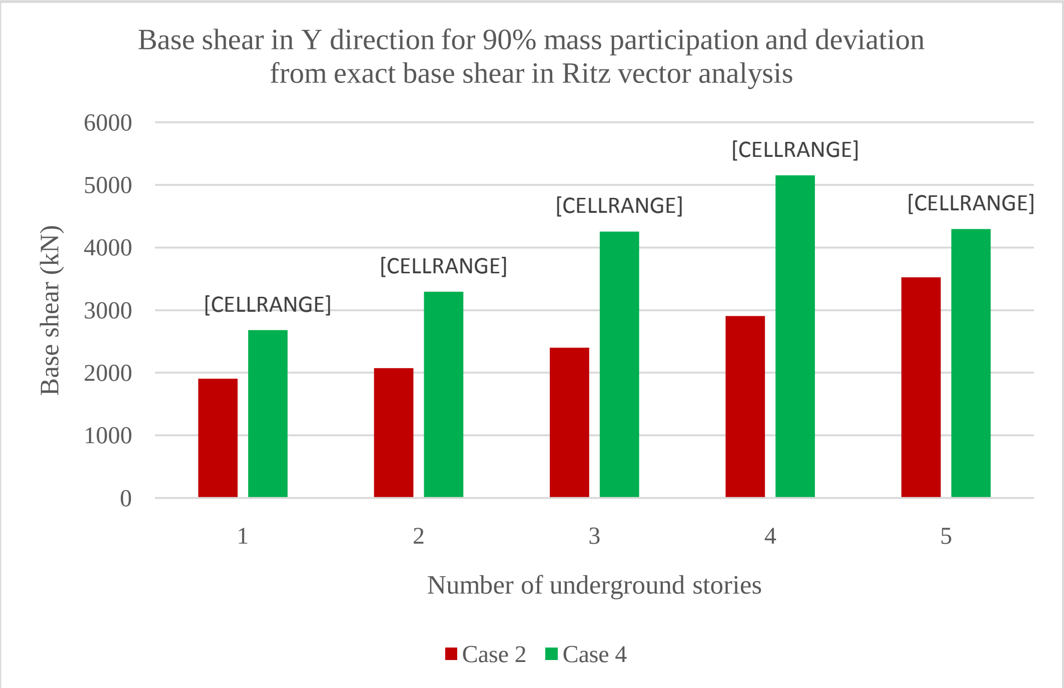


Fig. 10 Base shear in the Y direction for 90% mass participation in Ritz vector analysis

IV. Conclusion and recommendation

Upon studying two approaches followed by practicing engineers for analysing buildings with rigid underground stories, the following major conclusions are obtained:

- It is observed that up to two underground stories by excluding basement walls from the analysis model give approximately similar base shear. However, when the numbers of underground stories exceed two the basement walls should be included in the analysis model.
- In order to provide an economical design solution, truncation of modes should be avoided and it is recommended that complete seismic response should be captured especially for situations involving rigid underground stories.
- To obtain a complete seismic response, Eigenvector analysis requires significant computational time involving a tedious procedure.

- Mode superposition analysis by using Ritz vector automatically includes missing mass and provides results with the same level of accuracy as that of Eigenvector analysis with reduced computational time.

From the author's point of view, it is recommended to perform mode superposition analysis by using Ritz vector for the buildings with rigid underground stories. Further, this study can be extended to compare the accuracy of mode superposition analysis by Ritz vector with a two-stage analysis procedure.

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Toshif Patel. "Application of Ritz Analysis for the Building with Rigid Underground Stories: A Comparative Study to Common Approaches." *IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)*, 18(6), 2021, pp. 16-25.