

# Steel Structures Residual Life's Determination with the Safety Index

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**Abstract:** The article is dedicated to problem of safety for building steel structures which operations is exceed the design time or very close to the end of design validity. The safety for structure proposes to determine by the reliability index  $\beta$ . The main problem here is the lack of statistical information, which could be obtained only on the results of technical inspections. The methodology of calculation for structures' technical parameters as a basis for determination of index reliability  $\beta$  has done. For each element the reliability index should be calculated first, and then for all structure it should be defined with accordance of elements connection principles. The values of  $\beta$  forms the technical states for structure: satisfactory, need to repair and unfit to use. The residual life of structure also proposed to use as a safety parameter. The approach of calculation for residual life proposed on the results each of theoretical determination of reliability index and also on the results of technical inspections. The residual life could be determined as the time until the approximating curve will not cross the line of relevant  $\beta$  value reflected the related technical state of structure (normally "the unfit to use").

**Keywords:** safety index, residual life, steel structures.

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## I. Introduction

During designing of steel structures, its safety and operation availability achieving by the considering of general conditions that limit states will not become. Under the safety demands, the designer understand an adherence all actions and loads with respective value of reliability until of structure's service life.

An indicator of safety operation is a technical state of structure, which determine on the basis of results of technical inspection. The other characteristic, which also should be determined – the residual life of structure – the guarantee beyond design basis time. The determination of this parameter is one of the main tasks of structural safety, especially for structures close and after design service life.

## II. Literature Review and Related Works

All scientific researches of safety characteristics for structures under operation, based on assumptions that a big number of statistic information exists as a basis for its values' calculations. The reality is different: for building structures usually available a small statistical data that could be processed, because of all information about actual technical state obtained from the results of inspections and has limited number.

From the perspective of probabilistic changing of time-depended parameters, the main equations for limit states calculations should be modified for general safety characteristic specification. In C.A. Cornell's works [2, 3], the safety of building structures proposed to determine by the general reliability parameter – so-called "safety index" or "reliability index" -  $\beta$ .

Actual construction standards – the Eurocodes – recommend the target reliability levels for  $\beta$  that strongly correlate with the possible probability of structure's failure. In ISO 2394 [7] and ISO 13822 [8], the target reliability level of  $\beta$  for determination of reference period intended remaining working life have provided.

Safety philosophy of Eurocodes examined in the paper of J.-A. Calgaro [1]. Numerical values of  $\beta$  determine a structural reliability and safety for concepts of risk and hazards scenario for Eurocodes.

In investigations of R.D.J.M. Steenbergen and A.C.W.M. Vrouwenvelder [10] were determined the required  $\beta$  values for minimum reference period as a function of consequence classes of structures and working life. Two situations were considering for existing structures' safety targets – economic only and human safety arguments. On the basics of these investigations were proposed  $\beta$  values for new, repair and unfit for use structures.

Critical review of risk acceptance approaches to societal, economic and environmental risks based on  $\beta$  values was also summarized in papers of M. Holicky, D. Diamantidis, M. Sykora [4, 9].

But, in all these papers, do not have the methodology how the reliability index  $\beta$  must be determined for existing structures with the real technical parameters' analysis and how it can relate to residual life. So, the

aim of the article is to provide the principles for evaluation of technical state of the structure based on result of structural inspections and  $\beta$  calculation with future residual life's determination.

### III. Research Methodology

The safety demands for steel structure may be realized only if all loads and actions during lifetime period will not exceed design conditions of limit states:

- for ultimate limit state

$$F < R \tag{1}$$

$$R = \Omega \cdot f_y \cdot \gamma_c / \gamma_m \tag{2}$$

$$F = \Sigma F_i \cdot \gamma_{fm} \cdot \psi \cdot C \cdot \gamma_{fi} \tag{3}$$

- for serviceability limit state

$$\delta < [f] \tag{4}$$

$$\delta = \Sigma F_i \cdot C \cdot \gamma_{fi} / E \cdot \Omega \tag{5}$$

where:  $F$  – the loads effect (design actions);  $R$  – the ultimate strength (design strength) in element;  $\Omega$  – the geometrical properties;  $f_y$  – the yield strength of steel;  $\gamma_c$  – the partial factor for work conditions;  $\gamma_m$  – the partial factor for material;  $\Sigma F_i$  – the characteristic value of loads;  $\gamma_{fm}$  – the partial factor for loads;  $\psi$  – the coefficient of loads combinations;  $C$  – the special coefficient;  $\gamma_{fi}$  – the partial factor for certain load;  $\delta$  – the deflection of structure on characteristic loads (design deflections);  $[f]$  – the ultimate structure deflection;  $E$  – the modulus of elasticity.

These main limit states design equations mean the likelihood of changing for loads, steel strength and geometrical properties by the introduction of different partial factors and coefficients.

Abovementioned formulae could be present as general safety characteristic, taking in consideration the probability of its parameters' variation respond to time consideration. In modern building Eurocodes [5, 6] this characteristic named as “reliability index”  $\beta$ :

$$\beta = \frac{\bar{S}}{\sigma(S)} = \frac{\bar{R} - \bar{F}}{\sqrt{\sigma^2(R) + \sigma^2(F)}} \tag{6}$$

and

$$\bar{S} = \bar{R} - \bar{F} \tag{7}$$

$$\sigma(S) = \sqrt{\sigma^2(R) + \sigma^2(F)} \tag{8}$$

where:  $\bar{R}$  – the mean value of load bearing capacity – generalize element strength (structure);  $\bar{F}$  – the mean value of loads affecting on structure;  $\bar{S}$  – the reserve of load bearing capacity for all distribution laws for  $R$  and  $F$ ;  $\sigma^2(R)$  and  $\sigma^2(F)$  – the variance of load bearing capacity and loads affecting on structure respectively.

Reliability index also may be indicated as:

$$\beta = \frac{1}{V(S)} \tag{9}$$

where  $V(S)$  – the coefficient of variation for random variable of load bearing capacity reserve.

Index  $\beta$  defines with relation of either designed or calculated failure probability of structure (element of structure)  $P_f$ :

$$P_f = \Phi(-\beta) \tag{10}$$

where  $\Phi$  – the normal distribution function (Gaussian function).

Probability of failure also could be defined through S function. Structure considered as reliable if  $S > 0$  and failure if  $S \leq 0$ :

$$P_f = Prob(S \leq 0). \tag{11}$$

For another distribution of Sfunction (non-Gaussian), the reliability index  $\beta$  consider as standard measure function of reliability:

$$P_s = (1 - P_f) \tag{12}$$

Evaluation of technical state for structures depending on  $\beta$ , which calculated by equations 6 and 7 should be done with accordance of special procedure, taking in consideration that for ultimate limit state the main characteristic is the stress in the element.

The following procedure for definition of  $\beta$  is proposed.

1. Based on project documentation analysis, the discrete calculation of verified structure must be fulfilled. All loads and geometric properties for cross-section must be taken from normative documents which were in force on the moment of designing. Also, structure should be calculated with accordance of designing rules actual on the moment of investigation(s). Real conditions, related with changes in loads, cross-sections, static schematic must be considered. All stresses in elements define with general statistic parameters: mean value, variation coefficient, distribution law. On this stage all technical parameters may be supposed as independent with normal distribution function. For some variable parameters its theoretical data are given in the table 1.

**Table 1.** Theoretical data of steel structure technical parameters

| Parameter*                     | The unit of measurement                                 | Mean  | Variation coefficient, (%)   | Distribution law    |
|--------------------------------|---|---|--|---------------------|
| yield strength of steel, $f_y$ | N/mm <sup>2</sup> (MPa)                                 | Table 3.1ENV 1993-1-1 [6]                     | 0,066...0,11 (6,6...11,02)   | Gaussian, lognormal |
| modulus of elasticity, $E$     | N/mm <sup>2</sup>                                       | 2,1·10 <sup>5</sup>                           | 0,0476 (4,76)  | Gaussian            |
| Cross-section geometry         | mm; mm <sup>2</sup> ; mm <sup>3</sup> ; mm <sup>4</sup> | Hot rolled section properties, national codes | area – 0,02 (2,0); section modulus – 0,023 (2,3); cross-section dimensions – 0,025...0,09 (2,25...9,1)                         | Gaussian            |
| Loads and actions              | Pa, N/m <sup>2</sup> ; kg/m <sup>2</sup>                | Eurocode, national standards                  | dead load – 0,033 (3,3); snow – 0,2...0,485 (20%...48,5%); wind – 0,237...0,684 (23,7...68,4%); ice on structures – 0,66 (66%) | Gaussian, Weibull   |

\* readings are based on [8].

2. Probabilistic calculations of structure should be done. Mean value of  $\bar{R}$  and variation coefficient  $VR$  calculates on the results of real characteristics for each steel which exists in structure. For yield strength of design steel, the results of inspection may be taken up.

On this stage the following recommendations for technical structure's parameters as random variables may be proposed.

- cross-section geometrical dimensions determine by the measurements. Design values, decreasing values because of corrosion also as increasing values because of possible reinforcement should be determined. For each life stages related parameters must be done, as an example for area -  $A_{min}, \bar{A}, A_{max}$ ;
- for actions and loads, their values should be taken from inspection data. If these data are absent, it must be taken from normative document with design values for each date when it was available. Based on these loads, the design values of forces -  $N_d$  have to be calculated.
- actions and loads with only characteristic values should be determined. All loads' partial factors set equal to 1. The result –  $N_i$ ;
- actions and loads with all demanded design values should be determined. All loads' partial factors and combination factors set with accordance of actual design rules. The result –  $\bar{N}$ ;
- for  $\bar{N}$  and  $N_n$ , stresses  $\sigma$  should be determined with both  $N$  and  $A$ :  $\sigma_1=f(N_d;A_{min}); \sigma_2=f(N_d;\bar{A}); \sigma_3=f(N_d;A_{max}); \sigma_4=f(\bar{N};A_{min}); \sigma_5=f(\bar{N};\bar{A}); \sigma_6=f(\bar{N};A_{max}); \sigma_7=f(N_n;A_{min}); \sigma_8=f(N_n;\bar{A}); \sigma_9=f(N_n;A_{max})$ . Based on these calculation  $\bar{\sigma}$  and  $V\sigma$  have to be determined.

3. For easy calculation the following equations are proposed instead of formulae 6:

$$\beta = \frac{\xi - 1}{\sqrt{\xi^2 + \xi^2 \cdot VR}}, \tag{13}$$

$$\xi = \frac{\bar{R}}{\bar{\sigma}}. \tag{14}$$

3. For determination of general technical state, the corresponding connection of elements in the structure should be chosen. Some sketches are given in the figure 1.
4. For adopted connection of elements in verified structure, reliability index  $\beta$  should be determined.
  - a. for series connection, the probability of safety – absence of failure, may be present as:

$$P_s(q) = \prod_{i=1}^n [1 - P_{fi}(q)], \tag{15}$$

where  $P_{fi}(q)$  – the probability of failure for  $i$  element.

This connection may be recommended for statically determined systems. Here, the probability of failure for all structure will be higher than for each single element.

- b. For parallel connection, the probability of failure  $i$  numbers of elements for the structure with  $n$  elements from the load  $q$ , define as:

$$P_{f,i(n)}(q) = C_n^i \cdot P_{n-i}(q) \cdot P_{i(n)}(q), \tag{16}$$

where  $C_n^i = n!/i!(n - i)!$  – the number of events, which are the favorable result – the probability of failure of  $i$  number of elements.

This connection coincides for statically undetermined systems. In this case, the probability of failure for all system will be less, than for single element.

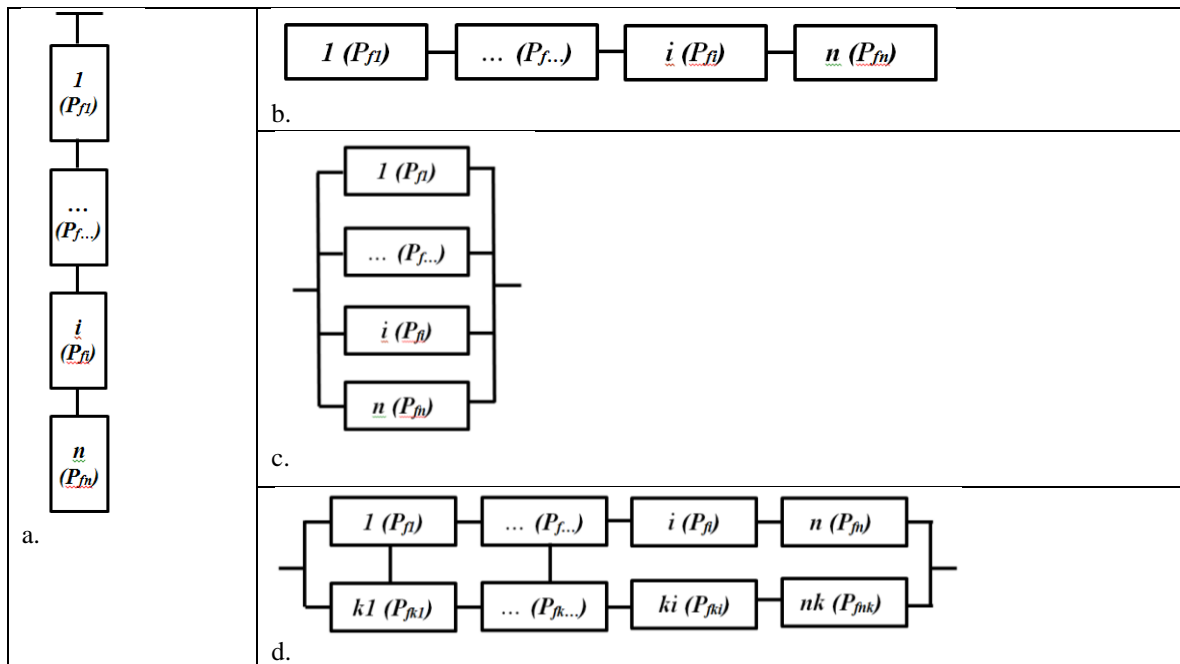


Figure 1. Elements connection principles: a. and b. - series connection; c. – parallel connections; d. – mixed connection

5. Quantity-related ranges for each structure's technical state proposed to determine by the reliability index  $\beta$  [10]. The table 1 contains these ranges related to consequence classes of structure.

Table 1. Technical state of structure related with  $\beta$  and CC.

| Consequence class | $\beta$ | $\beta$             | $\beta$                | $\beta$                              |
|-------------------|---------|---------------------|------------------------|--------------------------------------|
|                   | design  | 1 –satisfactory     | 2 –need for repair     | 3 – unfit for use (state of failure) |
| CC1               | 3.3     | $3.3 < \beta < 2.8$ | $2.8 \leq \beta < 1.8$ | $\beta \leq 1.8$                     |
| CC2               | 3.8     | $3.8 < \beta < 3.3$ | $3.3 \leq \beta < 2.3$ | $\beta \leq 2.3$                     |
| CC3               | 4.3     | $4.3 < \beta < 3.8$ | $3.8 \leq \beta < 2.8$ | $\beta \leq 2.8$                     |

Taking in consideration, that the  $\beta$  ranges may be assumed as a border between acceptable level of structure's operation service and state of possible failure, the time to transition from state 2 to state 3 may be considered as residual life. The residual life for long-term operated structure may be calculated by the methodology defined below.

1. The design reliability index  $\beta_0$  should be received as for the new structure. Calculations fulfilled for design loads on the date of construction project preparation. Design model is ideal – without any imperfections and defects. For probability calculation and determination of  $N_d, N_n, A_{max}, A_{min}, \bar{V}A$  – see as previously given.

If before setting to work, structure have been inspected and some imperfections were found, they must be considered as initial, when  $\beta_0$  calculated. Here, structure is not an ideal, but anyhow exists as a basis for future assessment.

2. Based on results of structural inspections, the reliability indexes  $\beta_1 \dots \beta_n$  for each inspection should be calculated. Calculation should be fulfilled by actual designing standards for the date of structural inspection with related characteristic and designing values for each parameter. It is preferably to apply statistical data for atmospheric loads if they exist.
3. All calculations for determination of  $\sigma_i \dots \sigma_k$  with corresponding values of  $N$  and  $A$  must be accomplished.
4. For total values of  $\sigma_i \dots \sigma_k$  its statistical parameters must be defined: mean value  $\sigma_m = \bar{F}$ , dispersion, root-mean-square deviation, coefficient of variation  $V\bar{F}$ .
5. For each loading relevant values of  $\beta_i$  should be calculated by the (13) and (14); for each value of  $\beta_i$  the probability of failure  $P_f$ , also as probability of safety  $P_s$  should be determined.
6. Taking in consideration of static determination of the structure, the connection of elements should be defined. After that, for each loading (in this case, we suggest that the one and the same loading corresponds to the one and the same structural inspection with its number), the probability of safety defines with the (15) or (16).
7. Plot of  $\beta_i$  against years under operation  $t_i$  prepared for future analysis (see figure 2).

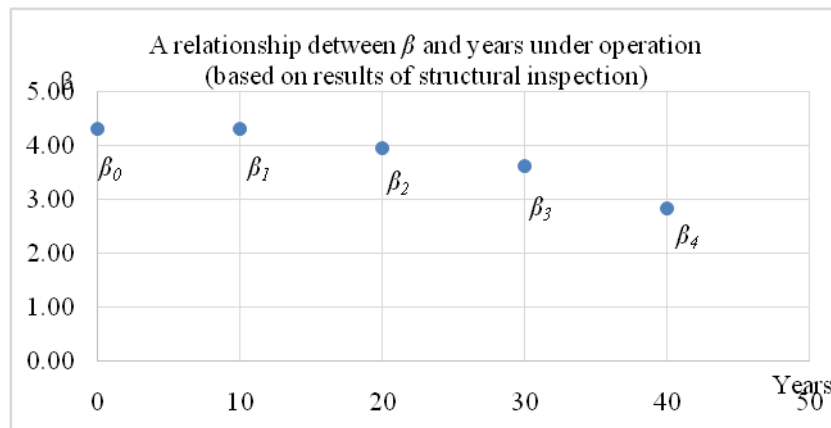


Figure 2. Plot of  $\beta_i$  against years under operation  $t_i$

8. For this plot, the approximating curve (the trend line) defined – see figure 3.

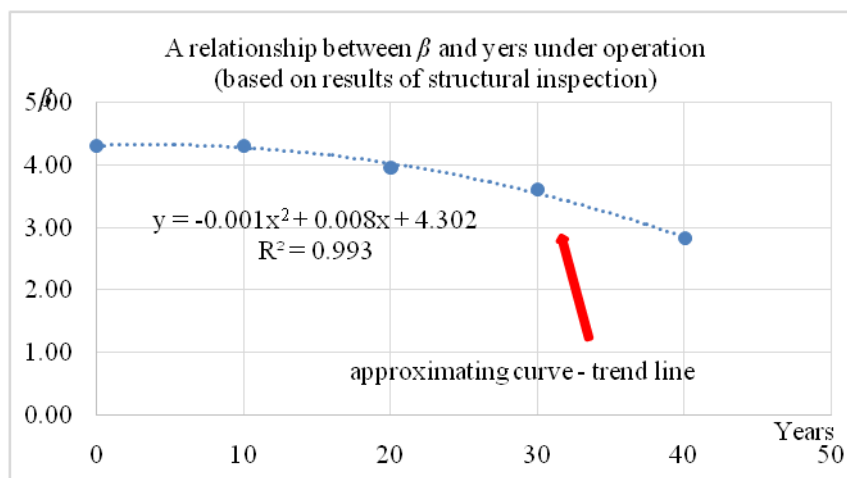


Figure 3. Determination of approximating curve.

9. Based on received equation for approximating curve, the residual life could be determined as the time until this curve will not cross the line of relevant  $\beta$  value (in general - for "unfit for use") for accepted CC of structure, given in the table 1.

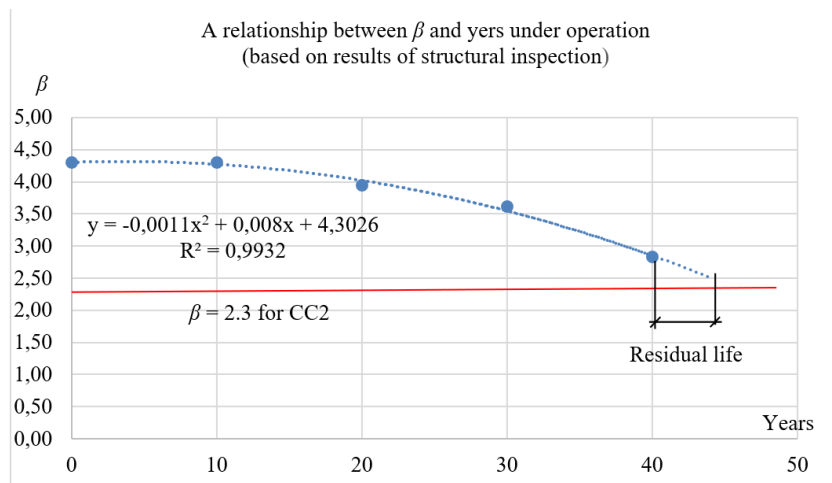


Figure 4. Determination of residual life.

On this stage, the assurance of the residual life's period may be suggested as the equal to the assurance value of  $R^2$  for adopted approximation curve.

10. If residual life was previously calculated by the discrete methods – for example with the regressive relations of corrosion progression, the defined probabilistic result may be compared with discrete ones for more precise solution for future repairing works planning.
11. The steel structures supposed as renovating systems. After repairing works, the  $\beta$  value usually increase. In this case, the residual life must be re-calculated with a new  $\beta$  value (see Figure 5). For calculation a new approximating curve, the old one should be used.

The correction of  $\beta$  new curve proposes to plot after detailed technical inspection of structure. The time to the inspection may be calculated either on the beforehand values of  $\beta$  or on the basis of estimated remaining life of structure as non-restorable system. For correct calculation of a new approximate curve proposed no less than 4 points: design value of  $\beta_0$  (point 1); value of  $\beta$  which calculated after repairing works (point 3); value of  $\beta$  which calculated after first inspection (point 4); Point 2 may be calculate on anytime of operating between time of calculated points 1 and 2 when value of  $\beta$  close to  $\beta_0$ , usually it is the first plan inspection after 10-15 years of operation.

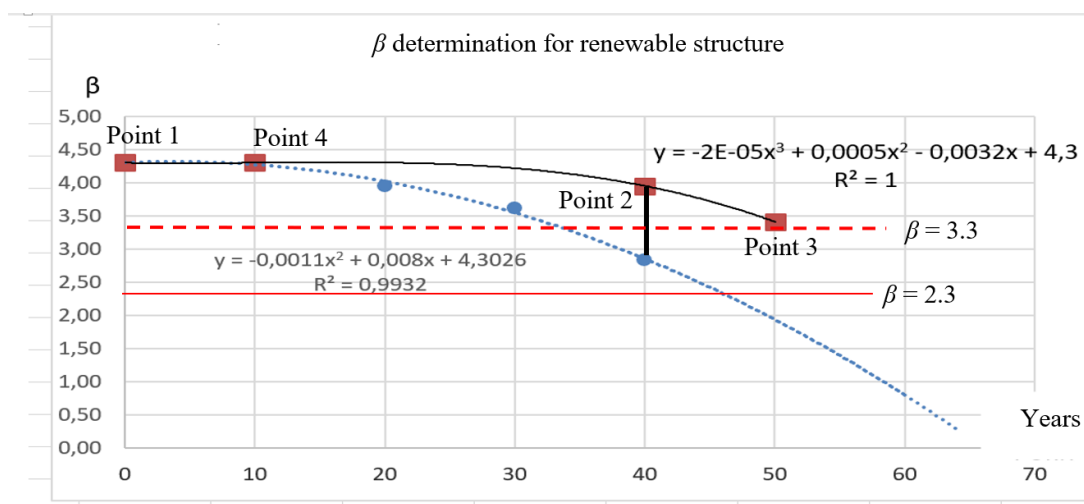


Figure 5. Determination of residual life after repairing works.

#### **IV. Conclusions**

In the paper a methodology for determination of stresses for statistics values which needed for safety assessment on designing and technical inspection results have proposed. It helps to define a technical state of structure not on the subjective imaginations, but strongly on the basis of reliability index calculation.

Also it make possible to plot an approximating curve for structure reliability index  $\beta$  and to determine the residual life as the time until this curve will not cross the line of relevant  $\beta$  value for accepted CC of structure.

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