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Prediction of Maximum Stress of an Axially Compressed Simply Supported Isotropic Rectangular Plate with a Circular Opening

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Abstract: *In this paper, an analytical model is developed to predict the maximum stress of an axially compressed simply supported isotropic rectangular plate with a circular opening located at the centre of the plate based on the maximum principal stress theory of elastic failure. An isotropic plate of size 600 x 500 x 12 mm with varying hole to plate width ratios of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 was studied. A MATLAB script was developed based on the analytical model to find the maximum stress occurring on the plate corresponding to each plate opening. An illustrative example was given to demonstrate the applicability of the present models and the results obtained were validated using the results obtained based on finite element model with ANSYS, and were found to coincide at every point of the plate opening. It was also found that under different ratios of opening, the maximum stress and stress reduction factor increased with increase in hole diameter and occurred at the points of discontinuities for all diameters of circular holes.*

Keywords: *Maximum stress, rectangular plate, maximum principal stress theory, circular opening, elastic failure*

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I. Introduction

Plates are structural elements which are which are used to support structural loads. They are majorly used as building slabs and bridge decks and have very large lateral dimensions compared to their thickness. A plate may also, be thought of as a wide flexural member. Plates generally carry loads normal to their plane.

Rectangular plates with circular opening(s) have found wide practical applications in various fields of engineering such as marine, mechanical and structural engineering (Troyani et al., 2002). For plates to fit into such applications, it is practically impossible to avoid creating openings which may lead to the reduction of mechanical and structural strength of the plate leading to the structural failure under service loads. It is therefore of paramount importance to explore into the state of stress in the vicinity of the openings for proper analysis of the load bearing capacity and overall safety of the plate. High stress that emanates from the point of plate discontinuity or abrupt change in geometry of the plate is known as stress concentration and they are mostly established at the edges of holes (Ko, 1985; Dheeraj and Singh, 2013; Stanley and Day, 1993; Pandit et al., 2013). Most of the strength analyses involving stress concentration factors (SCFs) are based on the conditions of infinite-width/diameter of plate because closed form stress distributions are available. For the design of plate with openings, in-depth knowledge of the stresses and stress concentration factors (SCFs) at the edge of hole under an in-plane loading is required. The points of stress of stress concentration in plates are usually prone to failures in the form of fatigue cracking and plastic deformation which weaken the load carrying capacity of plates.

Over the years, the complex nature of plate elements used in civil and structural engineering works led to the formulation of various failure prediction models which may not have reliable applicability (Konish and Whitney, 1975). Also, some existing failure prediction models require a large computational effort which makes them less suitable for quick prediction of maximum stresses at the vicinity of the hole. There is therefore, a need to develop a quick, easy and straightforward approach to prediction of stress and failure in rectangular plates with varying sizes of opening.

In this paper, analytical and numerical (FE) approaches have been used to estimate the maximum stress in a simply supported rectangular isotropic plate with circular opening of varying diameters uniform axial compression. A MATLAB code was developed based on the formulated model to predict the maximum compressive stress. The results of the analytical model were validated using ANSYS.

2.1 Maximum Stress model development

Consider a simply supported rectangular plate of width B and a central circular hole of diameter d subjected to a uniformly distributed load in compression as shown in Fig. 1.

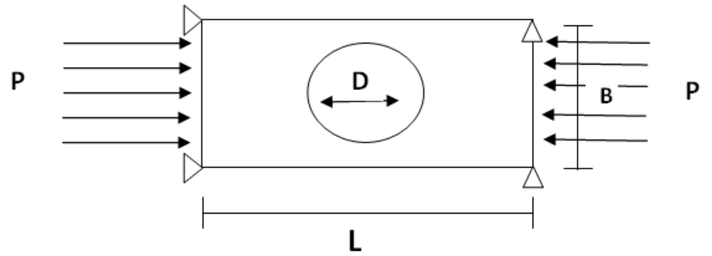

Fig. 1- A rectangular plate with a circular opening subjected to a compressive force

Using the maximum principal stress theory, the plate failure occurs when the maximum principal stress in compression reaches the elastic limit stress for the material in compression. The elastic limit stress for the material in compression is given by:

$$\sigma_{elas} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4*\tau^2}}{2} \quad (1)$$

The elastic limit stress for the material in compression is assumed to be the maximum principal stress for the material in compression and it represents the maximum principal stress at the edge of the hole. Equation (1) now transforms to:

$$\sigma_{Max} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4*\tau^2}}{2} \quad (2)$$

Where: σ_x, σ_y, τ = normal stress in x direction, normal stress in y direction and tangential stress respectively

$$\sigma_x = \sigma_{nom} \quad (3)$$

$$\sigma_y = \tau = 0 \quad (4)$$

Equation (2) now becomes:

$$\sigma_{Max} = \frac{\sigma_x}{2} + \frac{\sqrt{(\sigma_x)^2}}{2} \quad (5)$$

The net area of the plate across the circular hole is given by:

$$A_{net} = (B - d)*t \quad (6)$$

Where: $B, d, t =$ plate width, hole diameter and plate thickness respectively

The nominal stress in the plate at the net cross-sectional area is given by:

$$\sigma_x = \sigma_{nom} = \frac{P}{(B - d)*t} \quad (7)$$

Where: P = axial compressive force on plate

Let:

$$\sigma_{Max} = \frac{\sigma_x}{2} + \frac{\sqrt{(\sigma_x)^2}}{2} = \sigma \quad (8)$$

The maximum stress is assumed to increase with increase in area of the hole. This implies that:

$$\sigma_{Max} = \sigma + \lambda_i\sigma = \sigma(1 + \lambda_i) \quad (9)$$

Where: $\lambda_i =$ area fraction of a particular hole diameter

Using Equation (9), equation (3.5) now becomes:

$$\sigma_{Max} = \frac{\sigma_x}{2} + \frac{\sqrt{(\sigma_x)^2}}{2} * (1 + \lambda) \quad (10)$$

The area of an ith circular hole is given by:

$$A_{Hi} = 2\pi * r * t \quad (11)$$

$$r = \frac{d_i}{2} \quad (12)$$

Where: r = radius of hole
Substituting for r in equation (11) yields:

$$A_{Hi} = \pi * d_i * t \quad (13)$$

The area fraction corresponding to an ith hole diameter is given by:

$$\lambda_i = \frac{A_{Hi}}{A_P} = \frac{(\pi * d_i * t)_i}{B * t} \quad (14)$$

Where: A_{Hi} = area fraction of an ith hole

$A_P = B * t$ = area of plate without hole

2.2 Finite element model

From strength of materials, the stress-strain component relation for elastic materials is given by:

$$\{\sigma\} = [D] * \{\varepsilon\} \quad (15)$$

Where: D = elastic constants matrix
The elastic constants matrix is given by:

$$D = \frac{E(1-\mu)}{(1+\mu)(1-2\mu)} \begin{pmatrix} 1 & \frac{\mu}{1-\mu} & 0 \\ \frac{\mu}{1-\mu} & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2(1-\mu)} \end{pmatrix} \quad (16)$$

From equation (15), the strain equation is given by:

$$\varepsilon = \frac{\sigma}{[D]} = \quad (17)$$

The equilibrium equation which connects the nodal forces and the corresponding nodal displacements of the finite elements is given by:

$$\{F\} = [K^e] * \{d\} \quad (18)$$

Where: K^e = element stiffness matrix
The structural stiffness matrix is given by:

$$K = \sum_{i=1}^{n} K^e \quad (19)$$

The equilibrium equation connecting the nodal forces and the corresponding nodal displacements is given by equation (20).

$$\{F\} = [K] * \{d\} \quad (20)$$

$\{F\}$ = nodal force vector, $[K]$ = global stiffness matrix and $\{d\}$ = nodal displacement vector

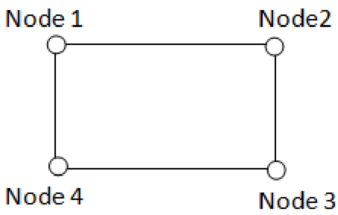

Fig. 2- A four node finite element used in this study

2.3 An illustrative example

A plate of finite width shown in Fig.3 is used to demonstrate the applicability of the present analytical model. The geometric parameters and values of elastic constants are shown in table 1.

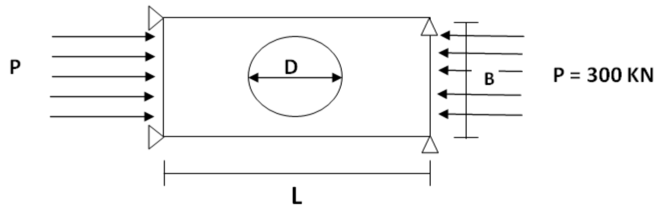

Fig. 3- A plate of finite width for numerical study

Table 1- Geometric characteristics of the plate

Material type	Steel
Dimension (mm)	600 * 500
Thickness (mm)	12
Young's modulus (N/mm^2)	2 * 10^5
Poisson's Ratio	0.30
Force (KN)	300 KN
Diameter of circle opening (mm)	0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90% and 100%
Ratio of diameter of opening (d) to width of plate (B)	0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 0.7, 0.8 and 0.9

According to Pilkey and Peterson (1997), the nominal stress on a plate of finite with a central hole subjected to an axial load in compression is given by:

$$\sigma_{nom} = \frac{P}{t[B-d]} \quad (21)$$

Where: P = axial load, t = plate thickness, d = hole diameter and B = plate width

The stress reduction factor, α corresponding to an ith opening ratio, β is defined as the ratio of maximum stress to the maximum stress at zero opening ratio.

II. Results

The results of maximum stress obtained for both the present analytical and finite element method are presented in Table 1.

Table 2: Comparison of results of analytical and FE method

Opening Ratio (β)	Maximum Stress (N/mm^2) (Present Method)	Maximum (Numerical) N/mm^2	Stress Reduction Factor, α (Present Method)	Stress Reduction Factor (α) (Numerical)
0	120.695	71.921	1.00	1.00
0.1	134.157	145.30	1.112	2.02
0.2	150.652	155.55	1.2482	2.16
0.3	172.184	168.79	1.4266	2.35
0.4	201.416	187.79	1.6688	2.61
0.5	240.931	223.82	1.9962	3.11
0.6	301.182	288.40	2.4956	4.01
0.7	403.298	392.66	3.3546	5.45
0.8	604.371	627.99	5.0074	8.73

0.9	1219.56	1235.8	10.105	17.18

The relationship between analytical and numerical values of maximum compressive stresses and stress reduction factors and the various percentages of openings are shown in Figs. 4 and 5 respectively.

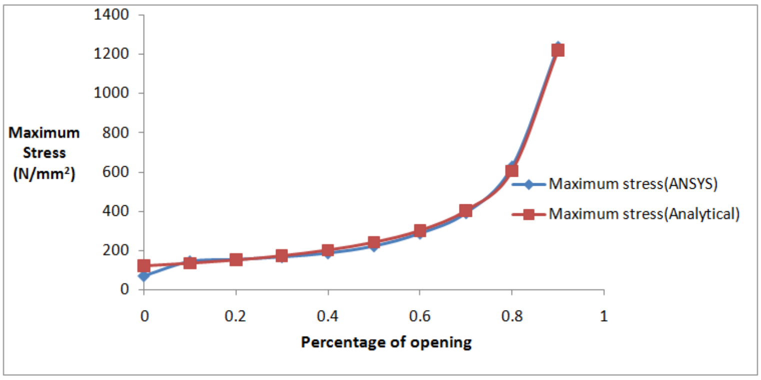

Fig. 4- Comparison of analytical and numerical values of maximum stress for different ratio of openings

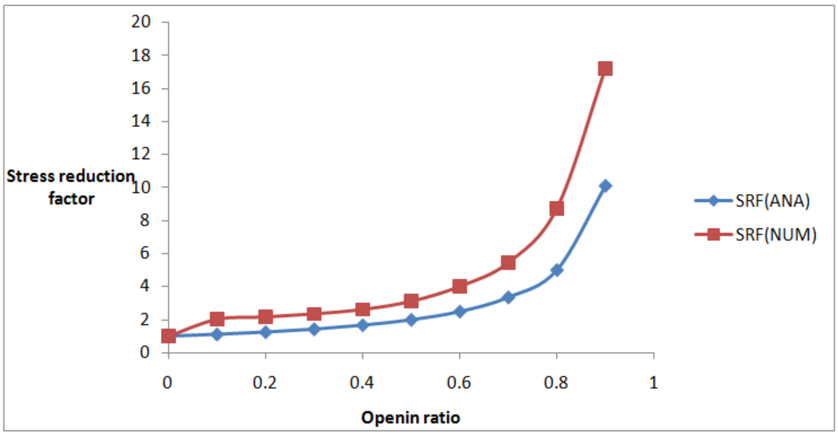

Fig. 5- Comparison of analytical and numerical values of stress reduction factors for different ratio of openings

Fig 4 shows the relationship between analytical values of maximum stress and the various opening ratios. It can be observed from Fig.4 that the maximum stress increases with increase in opening ratio (β) for both the analytical and numerical method for the various opening ratios. This is in agreement with the results obtained by Babulal and Tewari (2015). It can also be observed from Figure 4 that the results obtained from both the analytical and numerical method are almost identical at different opening ratios. The disparity between the analytical and predicted results of maximum stress may be due to model errors.

Based on the derived curve (Fig. 4), the equations that relate the maximum stress and opening ratio (β) for both the analytical and numerical method are shown in equations (22) and (23) respectively. An average value of coefficient of determination (R^2) of 0.893 was obtained from both curves.

$$\sigma_{Maxanalitical} = 2187 * \beta^2 - 1067 * \beta + 211.7 \qquad (22)$$

$$\sigma_{Maxnumerical} = 2207 * \beta^2 - 1051 * \beta + 193.7 \qquad (23)$$

Also, Fig. 5 shows that the stress reduction factor increases with increase in opening ratio of the plate for both methods. The equations that relate the stress reduction factor (α) and opening ratio (β) for both the analytical and numerical method are shown in equations (24) and (25) respectively. The little disparity between the analytical and predicted results of stress reduction factors may be due to errors involved in the model derivations. An average value of coefficient of determination (R^2) of 0.893 was from both plots.

$$\alpha = 30.68 * \beta^2 - 14.61 * \beta + 2.693 \qquad (24)$$

$$\alpha = 18.12 * \beta^2 - 8.83 * \beta + 1.753 \qquad (25)$$

The rectangular mesh ready for finite element analysis with ANSYSIS is shown in Fig. 6.

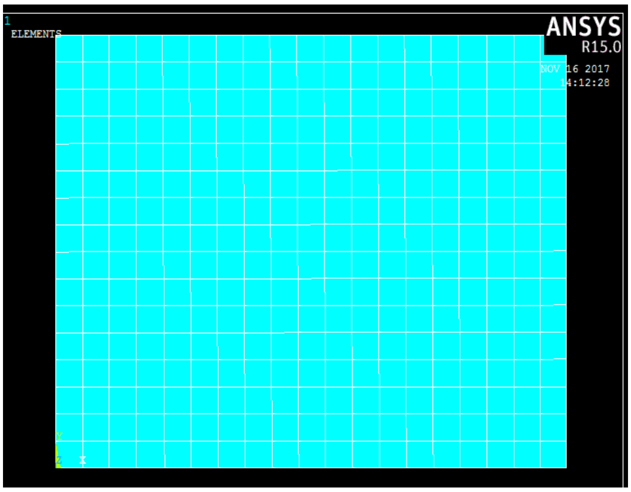

Fig. 6- A Meshed rectangular plate ready for analysis with ANSYS

III. Conclusions

Based on the results obtained from this study the following conclusions can be drawn.

i. The maximum stress occurs at corners/edges of holes for all the diameters of circular opening in the direction of the applied compressive force.

ii. The stress reduction factor increased with increase in the ratio of plate opening.

iii. The maximum stresses obtained from both the analytical and numerical method increased with increase in the ratio of plate opening

iv. The values of the maximum stress and stress reduction factors obtained from the present formulation at different points of opening ratio are almost identical with those obtained using numerical approach showing good reliability of the analytical models in the static analysis of plates with centre opening.

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