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Implementation Of Geometrical Nonlinearity in FEAST^{SMT}

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Abstract:*Analysis of the structures used in aerospace applications is done using finite element method. These structures may face unexpected loads because of variable environmental situations. These loads could lead to large deformation and inelastic manner. The aim of this research is to formulate the finite elements considering the effect of large deformation and strain. Here total Lagrangian method is used to consider the effect of large deformation. After deriving required relations, implementation of formulated equation is done in FEAST^{SMT}(Finite Element Analysis of Structures - Substructured and Multi-Threading). .Newton-Raphson method was utilized to solve nonlinear finite element equations. The validation is carried out with the results obtained from the Marc Software***.**

Keywords– *Geometrical nonlinearity, Newton-Raphson scheme, Total Lagrangian formulation,*

I. Introduction

Nonlinearity is natural in physical problems. In fact the linear assumptions made are only valid in special circumstances and usually involve some measure of "smallness", for example, small strains, small displacements, small rotations, small changes in temperature, and so on. The finite element equilibrium equation derived for static analysis is $[K]\{x\} = \{F\}$.The fig 1 indicates the relation between force and displacement. In linear analysis [K] is constant, that means if the force doubles, the displacement (and stresses) should be double. This relation may not be linear in all cases. In these situations, nonlinear analysis may be required.

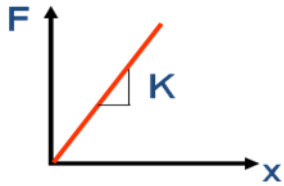


Fig.1: Force Displacement Diagram.

In nonlinear static analysis, the stiffness [K] is dependent on the displacement {x};
The resulting force-displacement curve may be nonlinear, as shown in fig 2. A nonlinear analysis is an iterative solution because the relationship between load applied in the structure [F] and corresponding displacement (x) is not known beforehand. Also no time dependent effects are considered.

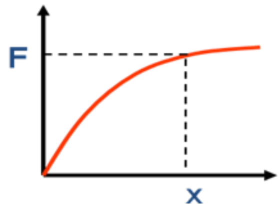


Fig.2 : Force Displacement Diagram

Non-linear behaviour admits a wide variety of phenomena, possibly interacting with one another, and each perhaps difficult to formulate. It is fortunate that the linear model provide satisfactory approximation for many problems of practical interest such as collapse or buckling of strut due to sudden load overload, progressive damage behaviour due to long lasting severe loads etc. Non-linear problem pose a difficulty of describing phenomena by realistic mathematical and numerical models and difficulty of solving non-linear equation.

In structural mechanics, type of non-linearity include the following

- Material non-linearity: Material nonlinearities occur when the stress-strain or force-displacement law is not linear, or when material properties change with the applied loads.
- Contact non-linearity: In which a gap between adjacent parts may open or closed, the contact area between the parts changes as the contact force changes or there is a sliding contact with frictional forces.
- Geometrical non-linearity: Geometric nonlinearities involve nonlinearities in kinematic quantities such as the strain-displacement relations in solids. Such nonlinearities can occur due to large displacements, large strains, large rotations, and so on

The term large deflection[2] is somehow misleading since problems falling in this category need not have actual deformations which are large. In fact, they can be as small as those in linear cases. However, for the strain-displacement relationships to be accurate they must include the appropriate higher order nonlinear terms which are taken to be small and negligible in the linear theory. Nonlinear solid mechanics theory[1] is certainly more complex than the corresponding linear theory. Consequently, the application of the nonlinear theory to physical problems can lead to complicated mathematical problems. In addition, analytic solutions are very limited for the geometrical nonlinear problems in solid mechanics.

II. Total Lagrangian Method And Newton- Raphson Method

2.1 Total Lagrangian Method

There are different methods[4] available for non-linear analysis such as total lagrangian method, updated lagrangian method depending on whether there exists large or small strain, deflection and rotation. Also the method depends type non-linearity. In this report geometrical non-linearity is considered. Since the report deals with implementation of a brick element with small strain and large deformation, total lagrangian method[1] is used for the analysis and Newton-Raphson method for iteration.

1.Equation of motion

$$\int_{0_V} {}^{t+\Delta t}_{0}S_{ij}\, \delta\, {}^{t+\Delta t}_{0}\varepsilon_{ij}\, d^0V = {}^{t+\Delta t}R;\ \text{where, } {}^{t+\Delta t}_{0}S_{ij} = \frac{{}^0\rho}{{}^{t+\Delta t}\rho}\, {}^{0}_{t+\Delta t}x_{i,m}\, {}^{t+\Delta t}\tau_{mn}\, {}^{0}_{t+\Delta t}x_{j,n};$$

$$\delta\, {}^{t+\Delta t}_{0}\varepsilon_{ij} = \delta\, \frac{1}{2}\left({}^{t+\Delta t}_{0}u_{i,j} + {}^{t+\Delta t}_{0}u_{j,i} + {}^{t+\Delta t}_{0}u_{k,i}\, {}^{t+\Delta t}_{0}u_{k,j}\right)$$

2.Incremental decompositions

(a) Stresses

$${}^{t+\Delta t}_{0}S_{ij} = {}^{t}_{0}S_{ij} + {}_{0}S_{ij}$$

(b)Strains

$${}^{t+\Delta t}_{0}\varepsilon_{ij} = {}^{t}_{0}\varepsilon_{ij} + {}_{0}\varepsilon_{ij};\ {}_{0}\varepsilon_{ij} = {}_{0}e_{ij} + {}_{0}\eta_{ij}$$

$${}_{0}e_{ij} = \frac{1}{2}\left({}_{0}u_{i,j} + {}_{0}u_{j,i} + {}^{t}_{0}u_{k,i}\, {}_{0}u_{k,j} + {}_{0}u_{i}\, {}^{t}_{0}u_{k,j}\right);\ {}_{0}\eta_{ij} = \frac{1}{2}\left({}_{0}u_{k,i} + {}_{0}u_{k,j}\right);$$

3.Equation of motion with incremental decompositions

Noting that $\delta\, {}^{t+\Delta t}_{0}\varepsilon_{ij} = \delta_0 \varepsilon_{ij}$ the equation of motion is

$$\int_{0_V} {}_{0}S_{ij}\, \delta_0 \varepsilon_{ij}\, d^0V + \int_{0_V} {}^{t}_{0}S_{ij}\, \delta_0 \eta_{ij}\, d^0V = {}^{t+\Delta t}R - \int_{0_V} {}^{t}_{0}S_{ij}\, \delta_0 e_{ij}\, d^0V$$

$${}_{0}S_{ij} = {}_{0}C_{ijrs}\, {}_{0}e_{rs},\ \delta_0 \varepsilon_{ij} = \delta_0 e_{ij}$$

4.Linearization of equation of motion

Using the approximations ${}_{0}S_{ij} = {}_{0}C_{ijrs}\, {}_{0}e_{rs},\ \delta_0 \varepsilon_{ij} = \delta_0 e_{ij}$, we obtain as appxoximate equation of motion

$$\int_{0_V} {}_{0}C_{ijrs}\, {}_{0}e_{rs}\, \delta_0 e_{ij}\, d^0V + \int_{0_V} {}^{t}_{0}S_{ij}\, \delta_0 \eta_{ij} = {}^{t+\Delta t}R - \int_{0_V} {}^{t}_{0}S_{ij}\, \delta_0 e_{ij}\, d^0V$$

2.2 Newton Raphson Method

Nonlinear solutions require several iterations. The actual relationship between load and displacement is not known beforehand. Consequently , a series of linear approximations with correction is performed. The fig 3 is a simplified explanation of the Newton-Raphson method[2].

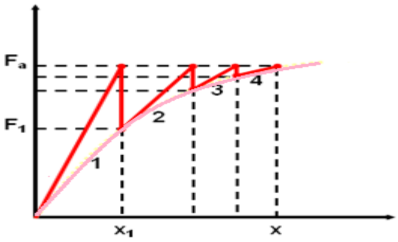


Fig.3: force displacement diagram

- In the Newton- Raphson Method, the total load F_a is applied in iteration 1. The total load Fa is applied in iteration 1.The result is x_1. From the displacements, the internal forces F_1 can be calculated. If $F_a \neq F_1$, then the system is not in equilibrium. Hence a new stiffness matrix (slope of red line) is calculated based on the current conditions. The difference of F_a-F_1 is the out-of-balance or residual forces. The residual forces must be 'small' enough for the solution to converge.
- This process is repeated until F_a – F_i. In this example, after iteration 4, the system achieves equilibrium and the solution is said to be converged.

III. Formulation Of Solid Elment

1.Incremental strains

$$\frac{1}{2}({}_0u_{i,j}+{}_0u_{j,i})+\frac{1}{2}({}_0u_{i,j}+{}_0u_{j,i}+{}_0^tu_{k,i}\,{}_0u_{k,j}+{}_0u_{i}\,{}_0^tu_{k,j});\quad i = 1,2,3;\ j = 1,2,3;\ k = 1,2,3;\ {}_0u_{j,i} = \frac{\partial u_i}{\partial\, {}^0x_i} \quad (1)$$

2. Linear strain-displacement transformation matrix

Using ${}_0e = {}_0^tB_L\,\hat{u}$; where ${}_0e^T = [{}_0e_{11} \quad {}_0e_{22} \quad {}_0e_{33} \quad 2\,{}_0e_{12} \quad 2\,{}_0e_{23} \quad 2\,{}_0e_{31}]$; (2)

$\hat{u}^T = [u_1^1 \quad u_2^1 \quad u_3^1 \quad u_1^2 \quad u_2^2 \quad u_3^2 \quad . \quad . \quad . \quad u_1^N \quad u_2^N \quad u_3^N]$; ${}_0^tB_L = {}_0^tB_{L_0} + {}_0^tB_{L_1}$ (3)

$$ {}_0^tB_{L_0} = \begin{bmatrix} {}_0h_{1,1} & 0 & 0 & {}_0h_{2,1} & . & . & . & 0 \\ 0 & {}_0h_{1,2} & 0 & 0 & . & . & . & 0 \\ 0 & 0 & {}_0h_{1,3} & 0 & . & . & . & {}_0h_{N,3} \\ {}_0h_{1,2} & {}_0h_{1,1} & 0 & {}_0h_{2,2} & . & . & . & 0 \\ 0 & {}_0h_{1,3} & {}_0h_{1,2} & 0 & . & . & . & {}_0h_{N,2} \\ {}_0h_{1,3} & 0 & {}_0h_{1,1} & {}_0h_{2,3} & . & . & . & {}_0h_{N,1} \end{bmatrix} \quad (4)$$

$${}_0h_{k,j} = \frac{\partial h_k}{\partial\, {}^0x_j}\ ;\ u_j^k = {}^{t+\Delta t}u_j^{\,k} - {}^tu_j^{\,k}\ ;\ l_{ij} = \sum_{k=1}^{N} {}_0h_{k,j}\, {}^tu_i^k \quad (5)$$

3.Nonlinear strain- displacement transformation matrix

$${}_0^tB_{NL} = \begin{bmatrix} {}_0^t\tilde{B}_{NL} & \tilde{0} & \tilde{0} \\ \tilde{0} & {}_0^t\tilde{B}_{NL} & \tilde{0} \\ \tilde{0} & \tilde{0} & {}_0^t\tilde{B}_{NL} \end{bmatrix};\ \tilde{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ \text{where }\ {}_0^t\tilde{B}_{NL} = \begin{bmatrix} {}_0h_{1,1} & 0 & 0 & {}_0h_{2,1} & . & . & . & {}_0h_{N,1} \\ {}_0h_{1,2} & 0 & 0 & {}_0h_{2,2} & . & . & . & {}_0h_{N,2} \\ {}_0h_{1,3} & 0 & 0 & {}_0h_{2,3} & . & . & . & {}_0h_{N,3} \end{bmatrix} \quad (6)$$

4. Second Piola-Kirchhoff stress matrix and vector

$${}_0^tS = \begin{bmatrix} {}_0^t\tilde{S} & \tilde{0} & \tilde{0} \\ \tilde{0} & {}_0^t\tilde{S} & \tilde{0} \\ \tilde{0} & \tilde{0} & {}_0^t\tilde{S} \end{bmatrix};\ \tilde{0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (7)$$

$${}_0^t\hat{S}^T = [{}_0^tS_{11} \quad {}_0^tS_{22} \quad {}_0^tS_{33} \quad {}_0^tS_{12} \quad {}_0^tS_{23} \quad {}_0^tS_{31}];\quad \text{where }\ {}_0^t\tilde{S} = \begin{bmatrix} {}_0^tS_{11} & {}_0^tS_{12} & {}_0^tS_{13} \\ {}_0^tS_{21} & {}_0^tS_{22} & {}_0^tS_{23} \\ {}_0^tS_{31} & {}_0^tS_{32} & {}_0^tS_{33} \end{bmatrix} \quad (8)$$

IV. Implementation

- Addition of incremental strains
- Addition of Linear strain-displacement transformation matrix
- Addition of nonlinear strain- displacement transformation matrix

- Addition of Second Piola-Kirchhoff stress matrix and vector.

V. Validation Problems

5.1 Solid Element

Material properties:
Young's modulus, E = 1e8 N/mm^2,Poisson's ratio = 0.3
Boundary and loading conditions:

- Fully fixed condition (Ux = Uy = Uz = 0) on one face of the solid and (Uy = Uz = 0) on opposite face as shown and a point Load (Fx) of 1.5e8N on four nodes as shown

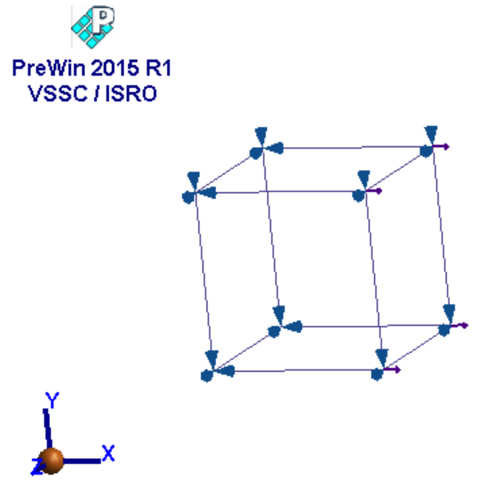


Fig 4: Modelling in FEAST^{SMT}

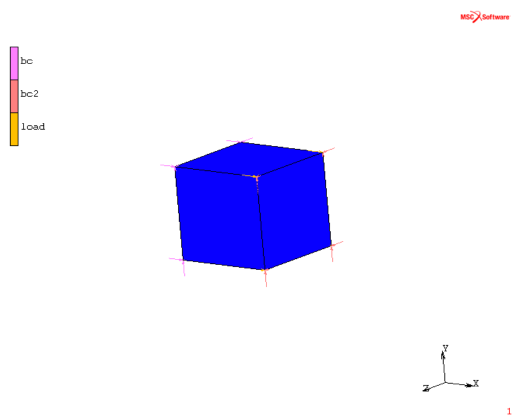


Fig 5: Modelling in MARC

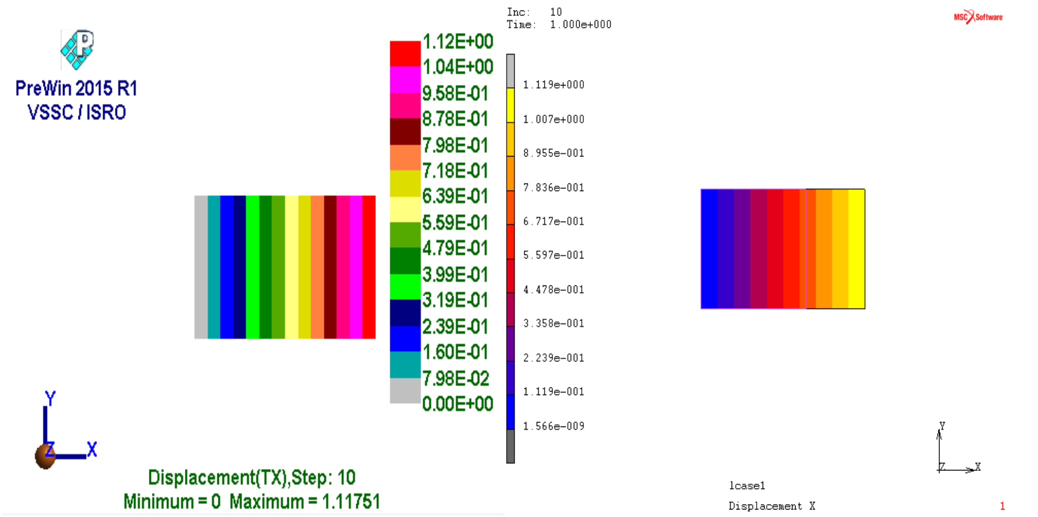


Fig 6: Displacement contours of non-linear analysis from FEAST^{SMT} and MARC

LOAD	DEFLECTION in mm	
	FEAST^{SMT}	MARC
1	0.194302	0.193783
2	0.349164	0.349032
3	0.491028	0.481077
4	0.603111	0.597259
5	0.706461	0.701749
6	0.800946	0.797186
7	0.888453	0.885357
8	0.97014	0.967539
9	1.0469	1.04468
10	1.11943	1.11751

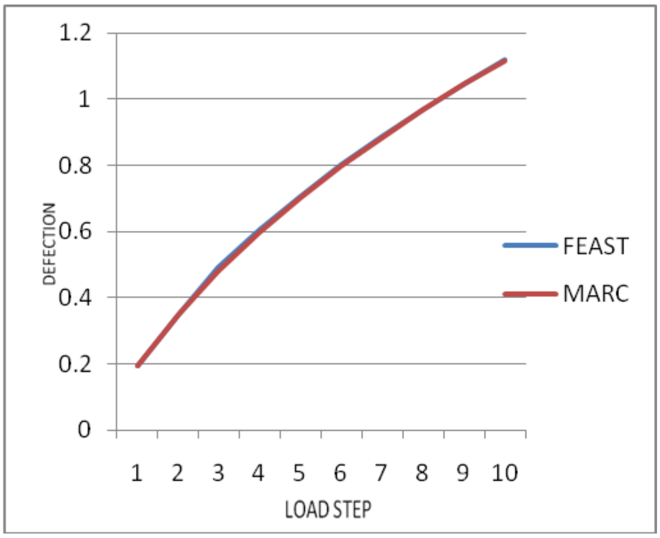


Fig 7: Force-Displacement Curve

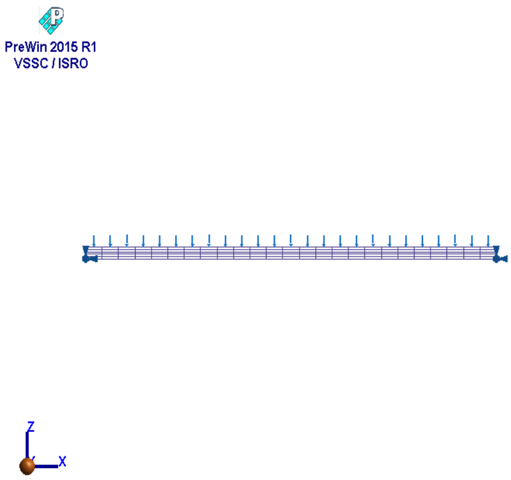
5.2 Simply Supported Beam

Material properties:

Young's modulus, E = 1e8 N/mm^2; Poisson's ratio = 0.3

Boundary and loading conditions:

- Simply supported condition (Ux = Uy = Uz =0) on both ends of the beam
- A udl of 1000N/m on top face as shown



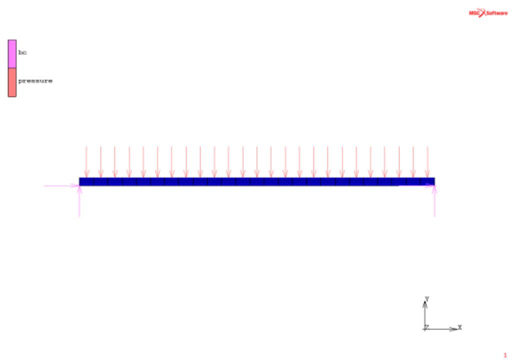


Fig 8: Modelling in FEAST^{SMT} **Fig 9:** Modelling in MARC

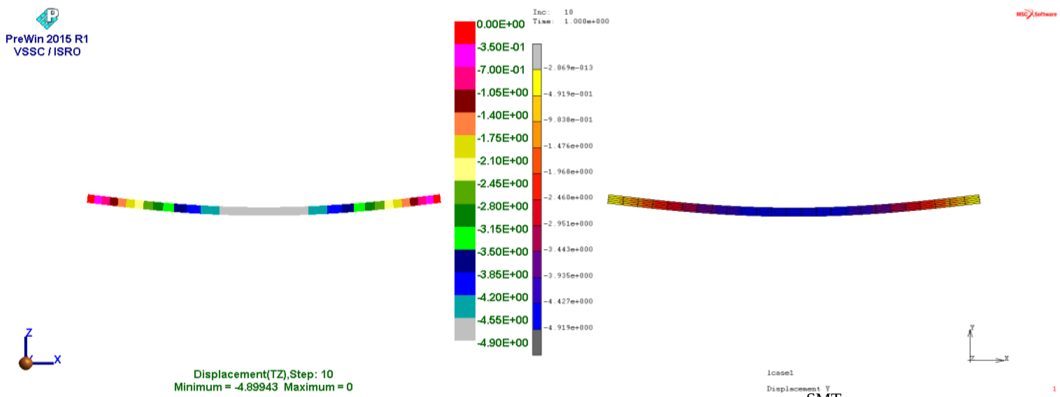


Fig 10: Displacement contours of non-linear analysis from FEAST^{SMT} and MARC

LOAD	DEFLECTION in mm	
	FEAST^SMT	MARC
1	0.376326	0.3785
2	0.90927	0.9099
3	1.81396	1.752
4	2.89533	2.9443
5	3.52412	3.604
6	3.93534	3.964
7	4.24553	4.269
8	4.4977	4.519
9	4.71198	4.732
10	4.89943	4.919

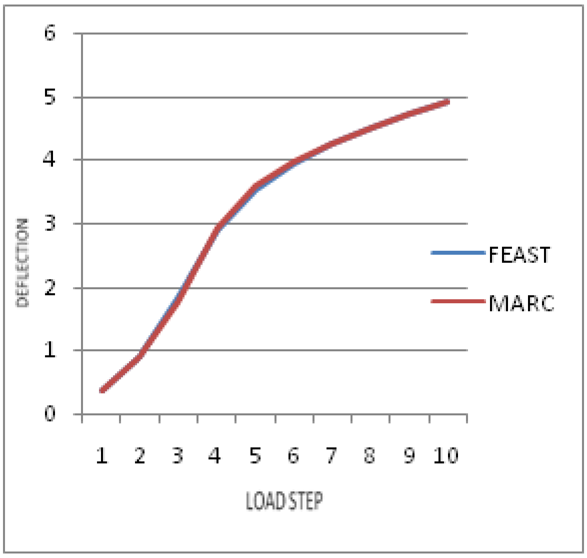


Fig 11: Force-Displacement Curve

VI. Conclusion

The paper consists of implementation of geometrical non-linearity in FEAST^SMT. The formulation of total lagrangian method and Newton-Raphson Method are implemented and the method is validated by using a solid finite element for simply supported beam with udl and the analysis of a pressure vessel. The results were compared with the analytical solution and the results obtained using the Marc software. The results are matching well between FEAST^SMT and the Marc software as well as with the analytical solution.

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