

## Applications of Headway Models

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**Abstract:** Headway data were collected for single lane traffic flows on a 2-lane, 2-way roadway. The hyperlang model was fitted into observed headway distributions. The composite exponential model (which is a reduction of the hyperlang model) was found to be a sound descriptor of observed headways for flows ranging from 170vph to 750vph, while the shifted negative exponential model was found to be suitable for low flows when most vehicles are in the free moving category. The parameters of the composite exponential model exhibited a discernable trend with traffic flow, thus a basis for rationally estimating model parameters, even for flows not monitored in the reported study, has been established. Other applications of headway models demonstrated are;

- Justification for (or otherwise) the provision of pedestrian crossing aids at study location.
- Predicting arrival patterns at a point
- Testing the randomness of traffic flow, and
- Timing of traffic signals

**Keywords:** traffic, headway, pedestrian, flow.

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### I. Introduction

Headways in a traffic stream are time spacing between successive vehicles as they pass a given point on the roadway. They influence drivers' choice of speeds and other maneuvers. At any point in the traffic lane the vehicular arrival rate is regulated by the sequence of headways between successive vehicles in the stream. Fitting of known mathematical models to measured headway distributions enable comparison of observed with theoretical headway distributions. The headway models are also useful in solving traffic problems on roadways.

### II. Objectives And Scope Of Study

The study objectives include the following:

- To choose suitable site within Sokoto environ where undisturbed headway data on two-lane, two-way roadways can be obtained.
- To collect reasonably representative headway data on the sites so selected.
- To analyse the headway data obtained and propose as well as fit them to the hyperlang model
- To explore if the parameters of the hyperlang model exhibit any trend with observed flows.
- To illustrate a few applications of the headway models on the route selected for monitoring.

This study is particularly designed to reflect the traffic situation in developing countries like Nigeria, where motor cycles contribute a reasonable proportion of traffic on urban roads.

### III. Field Surveys

After a preliminary survey in order to select an appropriate location that best promotes objective attainment for the study within Sokoto environ, a location along Maiduguri Road, was chosen for a detailed survey. A brief description of the location is as follows:

The location is opposite the last stretch of POLICE Officers' mess about 150m to the Teaching Hospital junction. The road itself is a dual carriageway paved with asphalt concrete still in good condition. At the point of observation, the road alignment is on tangent with a clear sight distance of approximately 250m. Vehicular traffic on the road is made up of passenger/private cars, and motor cycles with average proportions of 0.410, 0.435, and 0.150 respectively. There are no bus stops on either side of the study location; hence there is usually no stopping of vehicles. Most vehicles that ply this road travel at a speed of between 60 and 80 km/hr. The width of the road is 7.3m.

Headway data were collected with a pen recorder instrument connected to automatic traffic counter, while flows were simultaneously monitored by the same device. The flows captured ranged from 170vph to 750vph.

**IV. Fitting The Hyperlang Model To Observe Headway Distributions:**

The hyperlang model as derived by Dawson and Chimini (1968) is given as equation (1). The model is a linear combination of a translated exponential function and a translated erlang function. The exponential component of the distribution describes the free (unconstrained) headways in the traffic stream while the erlang component describes the constrained headways.

$$P(h \geq t) = (2 - a) \exp(-t - t_1)/T_1 - t_1 + a \exp(-k^t - t_2) + \sum k^t - t_2/T_2 - t_2/x \dots\dots\dots(1)$$

Where  $p(p \geq t)$  = probability that headway is greater than or equal to t.

a = proportion of constrained vehicles in the traffic stream.

(1 - a) = proportion of free vehicles in the traffic stream

t<sub>1</sub> = minimum headway of free vehicles

t<sub>2</sub> = minimum headway of constrained vehicles.

T<sub>1</sub> = average headway of free vehicles

T<sub>2</sub> = average headway of constrained vehicles

K = an index that indicate the degree of non randomness in the constrained headway distribution

The hyperlang model can reduce to a simple exponential function or to an erlang function and even to a composite exponential function as necessitated by the traffic situation. The model was fitted to observe headway distributions. From the kolmogorov smirnov (k - s) goodness of fit test result, the hyperlang model was judged as a suitable descriptor of observed headways for flows ranging from 170vph to 750 vph irrespective of whether or not motor cycles were in the traffic stream. Interestingly in all the cases in which the hyperlang model was found suitable, it reduced to the composite exponential model (i.e. k=1): Tables 1A and 1B contain the parameters of the hyperlang model.

**V. Fitting The Shifted Negative Exponential Model To Observed Headway Distribution:**

The shifted negative exponential model was fitted to observe headway distribution in a similar way as done for the hyperlang model. From the goodness of the fit test results, it was discovered that the model was only suitable for low flows for which most vehicles are in the free moving category.

**VI. Application Of Results**

The results obtained in this study are useful in;

- Estimating model parameters for flows not monitored in the reported study,
- Predicting vehicle arrival patterns at a point on the roadway.
- Justifying the need (or otherwise) for pedestrian crossing aids at study location
- Timing randomness of traffic flow
- Timing of traffic signal

These applications are demonstrated in the succeeding sections.

**6.1 Estimating Model Parameters For Flows Not Monitored In The Reported Study.**

In the course of this study, it was discovered that the model parameters exhibit a discernable trend with traffic flow;. Thus a basis for rationally estimating model parameters, even for flows not monitored in the reported study has been established.

Furthermore, it was discovered that the relationship of ‘a’ (the proportioned of constrained vehicles) with observed flows (q) is approximately linear. This is illustrated as the approximate relationship for cases where motorcycles were not included in the data is given as equation 2

$$i.e. q = 1.07 \times 10^{-3} - 0.06 \dots\dots\dots(2)$$

Where q is expressed in vehicles per hour, while that for data involving motorcycles is given as equation 3 i.e

$$a = 4.5 \times 10^{-4} + 0.13 \dots\dots\dots;(3)$$

Using equations (2) and (3) it is possible to estimate the values of “a” for flows not monitored in the study.

For example for a single lane traffic flow (q) of 900 vph which was not monitored in the reported study the proportion of constrained vehicles “a” for data excluding motorcycles would be about 90% while the proportion of free vehicles would be about 10%. This shows that at this flow, majority of the vehicles are constrained. Similarly the proportion of constrained vehicles for data including motorcycles would be 53.5% while proportion of free vehicles would be 46.5%.

The value of “a” becomes 1.00 (100%) when there are no free vehicles in the traffic stream, this occurs when flow  $q = 990$  vph for the case where motorcycles were excluded during data collection and 1930 vph for the case where motorcycles were include. This means that the flow that would produce congestion (traffic jam) in the two cases are 990 vph and 1930 vph respectively. This also suggests that a road which do not accommodates motorcycles have more capacity for vehicles than the ones which do not accommodate motorcycles which is quite true since motorcycles maintain shorter headways.

**6.2 Predicting Vehicles Arrival Patterns At A Point**

From the headway data, it was possible to estimate the rates at which vehicles arrive at the point of observation. This was one by applying the understanding that:

$$\text{Flow (vehicles per second)} = 1/h \dots\dots\dots(4)$$

Where h is average headway. See tables 1A and 1B for computed flows.

Perhaps a more useful application is that the results can be used for simulating arrival rates on the computer. Such an application required a random number generator and a simple illustration is given below. Let us take the case when observed headways could be described by the negative exponential model, we have:

$$f_H(h) = \alpha e^{-\alpha h} \dots\dots\dots(5)$$

Where  $f_H(h)$  = headways probability density function

$$F_H(h) = 1 - e^{-\alpha h} \dots\dots\dots(6)$$

Where  $F_H(h)$  = cumulative distribution function

To obtain the inverse function, assume that:

$$Z = f_H(h) \alpha e^{-\alpha h} \dots\dots\dots(6a)$$

$$Z/\alpha = e^{-\alpha h} \dots\dots\dots(6b)$$

$$\ln(Z)/\alpha = -h \dots\dots\dots(6c)$$

$$\text{Hence } G(Z) = h = -1/\alpha \ln Z/\alpha \dots\dots\dots(6d)$$

$$\text{(the inverse function)} \\ = -1/\alpha \ln(1-F) \dots\dots\dots(7)$$

If F, (1-F) are uniform in (0,1) we have the rule to find an exponential variate, therefore the in verse cumulative distribution function becomes

$$h = -1/\alpha \ln(u.r.v)$$

Where h = headways, is a parameter corresponding to flow in vehicles per second (v.p.s) and u.r.v. is a uniform random variate which is a unit of measurement. Hence using appropriate values of the uniform random variate and headways, arrival rates can be simulated on computer. Similarly using appropriate values of arrival rate and the uniform random variate, headways can be simulated.

**6.3 Testing The Randomness Of Traffic Flow:**

The values of K estimated in this study are measures of the degree of non-randomness of traffic flow in the constrained headway distribution. They were obtained by rounding off the values of the coefficient of variation of observed headways to the nearest integer. See tables 1A and 1B. The knowledge of the degree of non-randomness of traffic flow is very helpful in establishing traffic control warrants.

**64 Examining The Need Or Otherwise For Providing Pedestrian Crossing Aids At Study Location**

For safety reasons it is necessary to have a certain minimum number of safe gaps in traffic stream in one hour for pedestrian crossings. Different countries have different standards but 30 safe gaps is commonly used. A safe gap is normally considered as one where a pedestrian who begins to cross a roadway immediately after a vehicle passes completes the crossing before another vehicle arrives. Where this standard is not met remedial measures are considered. Hence, in order to verity whether there is need to provide pedestrian crossing aids at a specific location, the number of safe gaps in the traffic within one hour during the busy periods are normally computed. Such computations are facilitated by applying the results obtained in this study. The computations are now illustrated.

The minimum headway that can be used by a pedestrian to cross the traffic stream on a road of width W meters is given as  $h_{min} = W/V \dots\dots\dots(9)$

Where V is pedestrian walking speed in meters/sec. For this study W= 7.0m. Interestingly, an earlier study (Aliyu B.M, 1999) had established that a reasonable estimate for pedestrian walking speed around Sokoto is 1.39m/sec. substituting for W and V in equation 9, we have ;  
 $H_{min} = 7.0/1.39 = 5.04 \text{ secs} \dots\dots\dots(10)$

Assuming that the proposed composed exponential model for headway distribution at the maximum observed flow of 740vph is reasonably good then we have  
 $P(h > t) = (1-a) \exp \{-t - t_1\}/T_1 - t_1 + a \exp. (-t - t_2)/T_2 - t_2 \dots\dots\dots(11)$

(the parameters of the model are as previously defined).

Substituting for known values of model parameters in equation 10 (see table 1B), we have  
 $\text{Exp. } (P(h > t) = 0.519 \exp. \{-t - 2.25\}/5.757 + 0.481/ \{-t - 0.5\}/0.852$   
 $P(h > h(\min) + 0.481 = 0.519 \exp. \{-5.04 - 2.25\}/0.852 \dots\dots\dots$   
 $\text{Exp}\{5.04 - 0.5\}/0.852 \dots\dots\dots(13)$

$$= 0.3211$$

Also maximum number of minimum headways within one hour =3600/5.04.....(14)

$$\text{Number of safe gaps/hr} = 3600/5.04 (0.3211)^2 \underline{74} \dots\dots\dots(15)$$

In this illustration since the number of safe gaps within one hour is greater than 30 which is required by standard, there is no need for pedestrian crossing aids at the study location otherwise it would have been necessary to provide pedestrian crossing aids

**6.5 Timing Of Traffic Signal**

As earlier mentioned in section 3.0 of this paper the headway data were collected at a location in Sokoto. If it is found necessary to install a traffic signal at the junction the information obtained from the headway data collected in this study will be useful in timing the signal

A traffic signal is a control device often installed at road intersection with the objectives of creating orderliness, reduce total travel delay (especially when co-ordinated) and to some extent minimize accidents. To guide the selection of cycle timing, Webster, (1958), developed a formula for average delay per vehicle for a single approach to an intersection. This formula is empirical and applies to vehicles arriving at random at fixed cycle traffic signal

The formula is given as equation 16:

$$d = c \{1 - \alpha^*\} 2/2(1 - \alpha^*x) + x^2 / 2q(1 - x) - 0.65 (c)^{1/3}/q^2 x (2 + 5\alpha^*) q^2 \dots\dots\dots(16)$$

Where c = cycle time, i.e. one complete sequence of phases

$\alpha^* = g/c$ , i.e proportion of cycles time effectively green

Q = flow i.e average number of vehicles per second

S = saturation flow;

$X = q/\alpha s$  .i.e degree of saturation

$$\text{Also } C_o = 1.5L + 5 / 1 - Y \dots\dots\dots(17)$$

$C_o$  = optimum cycle that minimizes delay

$Y = y_i$  over the whole intersection where y is maximum q/s for each phases

L = total lost time/cycle

=  $nl + R$  where n is the number of phases, l is the average lost time/phase and R is the time during each cycle when every signal shows red (or red amber together ).

From equation (17), it could be seen that in order to select appropriate cycle timing for a traffic signal the estimation of “q” which is the average number of vehicles per second and “s” which is the saturation flow is very important, “q” can be computed from the headway data by using the formula

$$q = 1/h_s \text{ vps} \dots\dots\dots(4)$$

While “s” can be estimated by using the formula

$$s = 1/h_{s\_vps} \dots\dots\dots(18)$$

Where  $h_s$  = saturation headway.

From the foregoing it is evident that the information obtained from the headway data on a particular approach is useful in timing traffic signal on the approach

### VII. Conclusions

- i. The composite exponential model was found to be a sound descriptor of observed headways for flows ranging from (170vph to 750vph) irrespective of whether or not motorcycles were in the traffic stream
- ii. The parameters of the composite exponential distribution were found to exhibit discernable trend with traffic flows thus a basis for rationally estimating model parameters even for flows not monitored in the reported study has been established.
- iii. Information derived from the headway data and their distribution are also useful in:-
  - Predicting arrival patterns of vehicles at a point
  - Testing the randomness of traffic flow
  - Justifying the need or otherwise for pedestrian crossing aids at pedestrian ingress point
  - Timing of traffic signals

### VIII. Recommendations

The applications of headway models illustrated in this study should be substantiated and evaluated for a wider range of traffic flow and roadway conditions

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### NOTATION

a	= proportion of constrained vehicles in the traffic stream
K	= cycle time
Co	= optimum cycle time
Cv	= coefficient of variation
F <sub>H</sub> (h)	= headways probability density function
h	= Average Headway
h (min)	= minimum headway that can be used by a pedestrian to cross the traffic stream
hs	= saturation headway
K	= An index that indicates the degree of non-randomness in the constrained headway distribution
L	= total lost time/cycle
p (h ≥ t)	= probability that headway is greater than or equal to t.
q	= vehicles arrival rate
s	= saturation flow
t	= time
t <sub>1</sub>	= minimum headway of free vehicle
t <sub>2</sub>	= average headway of constrained vehicle
u.r.v	= uniform random variate
v	= pedestrian walking speed in metres/second
w	width of road (m)
x	degree of saturation
y	= eyeline over the whole intersection where y is maximum q/s for each phase
α	= parameters of the negative exponential distribution corresponding to arrival rate in vehicles per second.
α*	= proportion of cycle time effectively green.

**Table 1a Hyperlang Model Parameters When Motorcycles Were Not Considered During Headway Data Collection**

Obser- Equiva- Ved lent	h secs	cv	k	t <sub>1</sub> secs	t <sub>2</sub> secs	T <sub>1</sub> secs	T <sub>2</sub> secs	(1-a)	a
Flow flow (vph) (vph)									
168 172	20.963	0.815	1	2.500	0.750	23.674	1.450	0.878	0.122
268 265	13.587	1.065	1	2.500	0.750	17.152	1.467	0.773	0.227
368 369	9.761	1.130	1	2.000	0.500	16.745	1.244	0.549	0.451
280 281	12.819	1.218	1	2.250	0.750	17.807	1.417	0.696	0.304
460 455	7.906	1.218	1	2.250	0.750	11.267	1.442	0.658	0.342
570 570	6.305	1.506	1	2.500	0.750	11.046	1.511	0.504	0.496
660 645	5.578	1.731	1	3.500	0.750	14.240	1.778	0.305	0.695

**Table 1b Hyperlang Model Parameters When Motorcycles Were Considered During Headway Data Collection**

Obser- Equiva- Ved lent	h secs	cv	k	t <sub>1</sub> secs	t <sub>2</sub> secs	T <sub>1</sub> secs	T <sub>2</sub> secs	(1-a)	a
Flow flow (vph) (vph)									
228 221	16.272	1.386	1	2.250	0.750	21,190	1.536	0.750	0.250
420 414	8.704	1.182	1	2.000	0.500	11.993	1.305	0.692	0.308
632 637	5.651	1.167	1	2.000	0.500	8.210	1.284	0.631	0.369
736 749	4.807	1.304	1	2.250	0.500	8.007	1.352	0.519	0.481
404 393	9.152	0.994	1	2000	0.500	11.523	1.217	0.770	0.230
480 473	7.603	1.081	1	2.500	0.500	11.036	1.535	0.639	0.361
580 571	6.304	1.319	1	3.000	0.500	11.337	1.611	0.483	0.517