

Car Dynamics using Quarter Model and Passive Suspension, Part II: A Novel Simple Harmonic Hump

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Abstract : *A novel simple harmonic speed hump is presented in this work. The dynamics of a quarter-car model are investigated to reach the conditions of ride comfort when using the novel simple harmonic hump. The study assumed passive car suspension elements of linear characteristics. It covers car crossing speed between 5 and 30 km/h, simple harmonic hump of length between 3 and 9 m and height between 60 and 120 mm. A ride comfort diagram is presented using MATLAB simulation using the quarter-car model allowing the design of the simple harmonic hump for any desired hump-crossing speed in the range 5 to 30 km/h .*

Keywords: *Car dynamics , quarter-car model , Passive suspension system , Novel simple harmonic humps , Ride comfort, Hump design diagram.*

I. Introduction

Weber and Braaksma (2000) stated that vehicles can transverse circular humps of 75-100 mm height and 3.7 m length safely at speeds of 25 to 30 km/h [1]. Krylov (2001) studied the ground-borne vibrations generated by road vehicles crossing road humps, and speed cushions used for traffic calming. He generated analytical results and compared with existing experiments [2]. Ibrahim, Karim and Atif (2003) presented the results of a study on the performance of road humps used in Malaysia to reduce vehicle speed. They related the performance of the road hump to the locational characteristics, roadside development and vehicle type [3]. Johnson and Nedzesky (2004) compared speed humps, speed slots and speed cushions traffic reducing devices. Their study covered 12-22 ft asphalt speed humps, 14 ft prefabricated speed humps, 22 ft speed slots and 10 ft speed cushions [4]. Hessling (2008) quoted that comfort changes with car speed and improper design of road humps leading to a risk of injury. He listed the tools of dynamic measurement systems used in the measurement of vehicle dynamics when crossing a speed hump [5]. Mao and Koorey (2010) studied the effects of traffic calming devices on traffic volume, speeds and crashes in urban local streets. They found that crashes reduced by 10-20 %, but traffic volumes and speeds were reduced [6].

Namee and Witchayangkoon (2011) introduced crossroad speed table to reduce crossroad rear-end collisions. They focused on investigation of the vehicle speed control devices installed at crossroads to control traffic speed [7]. Ben-Edigbe and Mashros (2012) investigated the highway capacity loss due to speed reduction using road humps. They mentioned that a 75 mm hump would reduce speed to 20 km/h on average. They concluded their work by saying that although road humps are an effective mechanism for vehicle speed reduction, the resulting highway capacity loss is significant [8]. Rosli and Hamsa (2013) investigated the effects of road humps on traffic volume and noise level in a residential area in Kuala Lumpur [9]. Kanjanavapastit and Thitinaruemit (2013) stated that a road speed hump can cause an accident, and they proposed a technique to estimate speed hump profile using a quarter car model. They used accelerometers located on the unsprung mass to measure the axial and vertical accelerations and used MATLAB Simulink to estimate the speed hump profile [10].

Silva and Vasconcelos (2014) emphasized the use of speed humps for their ease of construction / installation and their efficiency in reducing vehicle speeds. They developed a speed profile model using database and hierarchical multiple regression techniques providing speeds on the approach and exit of isolated speed humps [11]. Garcia-Pozuelo et. al. (2014) developed a simulation program using MATLAB taking into account the vehicle dynamics, hump geometry and vehicle speeds. Their proposed tool was expected to provide useful information to set guidelines for the proper design and installation of speed humps [12]. Hassaan (2014) examined the dynamics of a car crossing a circular hump for sake of maintaining ride comfort for the passengers. He considered passive suspension elements and a specific circular hump. He examined the effect of suspension damping in the range 1 to 15 kNs/m and car speed in the range 5 to 25 km/h [13].

II. Analysis

2.1 Quarter Car Model

A quarter-car model consists of the wheel and its attachments, the tire (of visco-elastic characteristics), the suspension elements and quarter the chassis and its rigidly connected parts. Fig.1 shows a line diagram of a

car quarter physical model [14].

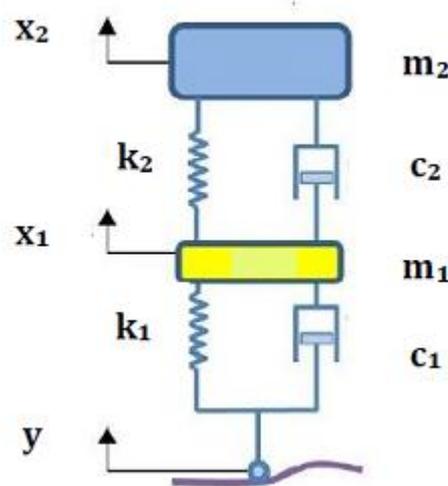


Fig.1 Quarter-car physical model [14].

The parameters of the quarter-car model according to Florin, Ioan-Cosmin and Liliana are considered in this analysis except for the suspension damping coefficient c_2 . Their parameter are given in Table 1 except the damping coefficient of the suspension which is set by the author [14].

Table 1: Quarte-car model parameters [14].

Parameters	Description	Value
k_1 (kN/m)	Tire stiffness	135
c_1 (kNs/m)	Tire damping coefficient	1.4
m_1 (g)	Un-sprung mass	49.8
k_2 (kN/m)	Suspension stiffness	5.7
c_2 (kNs/m)	Suspension damping coefficient	5
m_2 (g)	Sprung mass	466.5

2.2 Model Input

The input is the irregularity of the road. It may take various shapes. It can be random roughness or standard humps to force drivers to reduce their vehicle speeds (say) in residential areas (speed hump). Fig.2 shows what is known as a circular hump [4].



Fig.2 Circular hump [4].

A circular hump has the dimensions:

Height : h

Length : L

The point now is: Is the hump in Fig.2 a circular or a simple harmonic hump ?. This is because they are very close in geometry. A circular hump is simply a cylinder sector. In the x-y plane it has the mathematical model (derived by the author):

$$y = \sqrt{\{R^2 - (0.5L - x)^2\}} - R\cos\alpha \tag{1}$$

where: R = hump radius,

L = hump length

α = hump sector angle between terminal radii

x and y are the coordinates of any point on the hump from the hump starting point.

The expression of 'simple harmonic hump' is a new expression used by the author. The expression is extracted from cam-follower kinematics where the simple harmonic motion is one of the motion given to cam-followers [15].

The profile of a simple harmonic hump is defined by the equation:

$$y = h \sin \omega t \quad \text{for } 0 \leq t \leq T \tag{2}$$

Where: h = hump height (maximum y)
 ω = displacement angular frequency
 T = time corresponding to L

The angular frequency ω is related to the displacement wave period τ through:

$$\omega = 2\pi / \tau = \pi / T \tag{3}$$

This is simply because the period τ is twice the interval T .
 The time T is related to the car speed V (km/h) through:

$$T = L / (1000V / 3600) = 3.6L / V \quad \text{s} \tag{4}$$

Combining Eqs.3 and 4 gives ω as:
 $\omega = \pi V / (3.6L) \quad \text{rad/s}$

Eqs.1 and 2 are used to draw the profile for both circular and simple harmonic humps. A MATLAB generated the hump profile for a 100 mm height and 3 m length as shown in Fig.3.

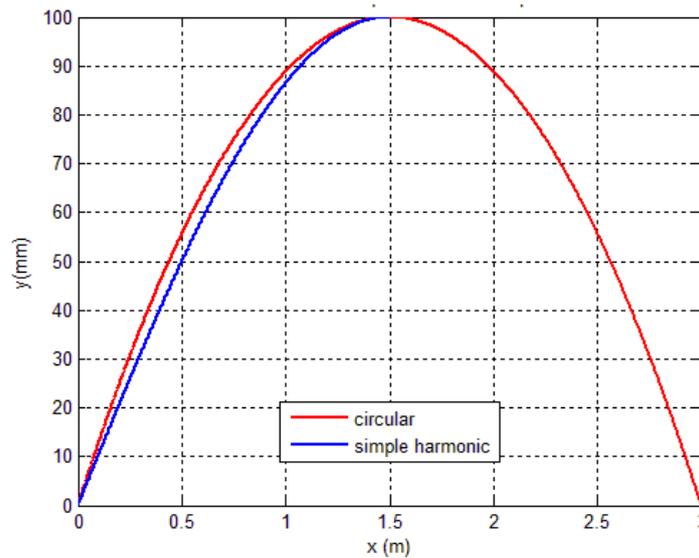


Fig.3 Circular and simple harmonic humps.

As depicted from Fig.3, both profiles have:

- Same starting point.
- Same end point.
- Same maximum point.
- Same going down half.
- Deviation in the going up half (about 5 % maximum).

2.3 Mathematical Model

Writing the differential equation of the unsprung and sprung masses of the quarter-car model yields the following two equations:

$$m_1 x_1'' + (c_1 + c_2) x_1' - c_2 x_2' + (k_1 + k_2) x_1 - k_2 x_2 = k_1 y + c_1 y' \tag{5}$$

$$m_2 x_2'' - c_2 x_1' + c_2 x_2' - k_1 x_1 + k_2 x_2 = 0 \tag{6}$$

The state model of the dynamic system is driven from Eqs.5 and 6 as follows:

- State variables: z_1, z_2, z_3 and z_4 . The state variables are related to the masses displacements x_1, x_2 and velocities x_1', x_2' as:

$$\begin{aligned} z_1 &= x_1 & , & & z_2 &= x_1' \\ z_3 &= x_2 & , & & z_4 &= x_2' \end{aligned} \tag{7}$$

- Output variable:

The output variable of the quarter-car model is the sprung mass motion, x_2 . It is related to the state variables through:

$$x_2 = z_3 \tag{8}$$

- State model:

Combining Eqs.5, 6 and 7 gives the state model of the quarter-car model as:

$$z_1' = z_2 \tag{9}$$

$$z_2' = (1/m_1) \{k_1 y + c_1 y' - (c_1 + c_2)z_2 - (k_1 + k_2)z_1 + k_2 z_3\} \tag{10}$$

$$z_3' = z_4 \tag{11}$$

$$z_4' = (1/m_2) \{c_2 z_2 - c_2 z_4 + k_2 z_1 - k_2 z_3\} \tag{12}$$

III. Quarter-Car Model Dynamics

- The state model of this dynamic problem is linear since the suspension parameters are assumed constant (linear characteristics).
- MATLAB is used to solve this problem using its command "ODE45" [16,17].
- The sprung mass motion is excited by the hump displacement only, i.e. zero initial conditions are set in the solution comment.
- Time span is set to twice the half-period, T of the hump.
- The car speed is changed in the range: 5 to 30km/h when crossing the hump with 2.5 km/h increment.
- The height of the simple harmonic hump is changed in the range: 60 to 120 mm with 20 mm increment.
- The length of the simple harmonic hump is changed in the range: 3 to 9 m with 1 m increment.
- The purpose of this research was to emphasise the effect of the dimensions of the simple harmonic hump on the sprung mass displacement and the ride comfort in terms of the maximum sprung-mass acceleration in m/s^2 .

3.1 Sprung-mass Displacement

The displacement of the sprung-mass as generated by MATLAB for a car velocity of 25 km/h, hump height of 5 m, besides the system parameters in Table I and the hump height range stated before is shown in Fig.4.

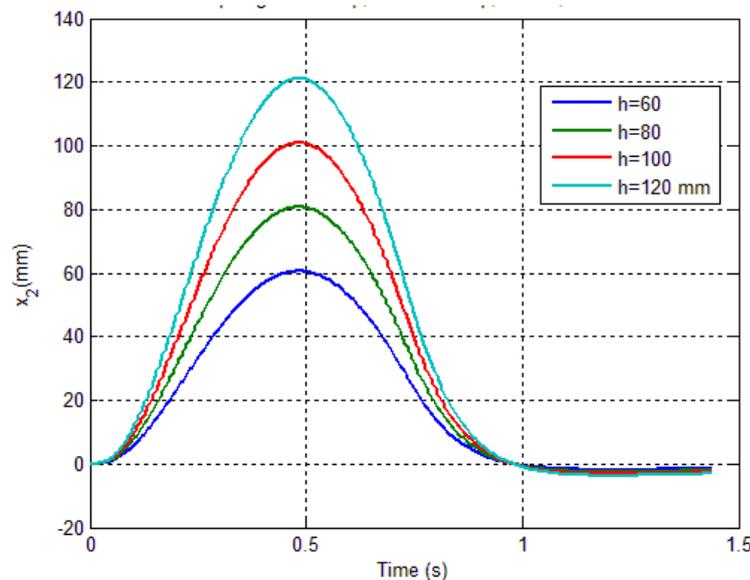


Fig.4 Sprung-mass displacement for $V = 25$ km/h.

3.2 Sprung-mass Maximum and Minimum Displacements

- As clear from all the sprung-mass response of the quarter model as shown in Fig.4, the displacement reaches a maximum value then drops to a minimum value as the car crosses the hump.
- The maximum and minimum displacements of the sprung-mass depend on the hump dimensions for a specific car speed.
- Figs.5 and 6 illustrate graphically this relation obtained using the MATLAB commands "max" and "min" respectively.

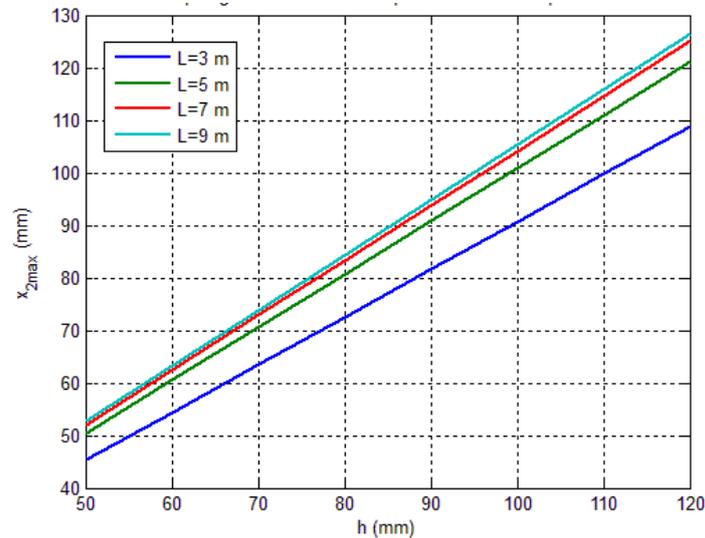


Fig.5 Sprung-mass maximum displacement at $v = 25$ km/h.

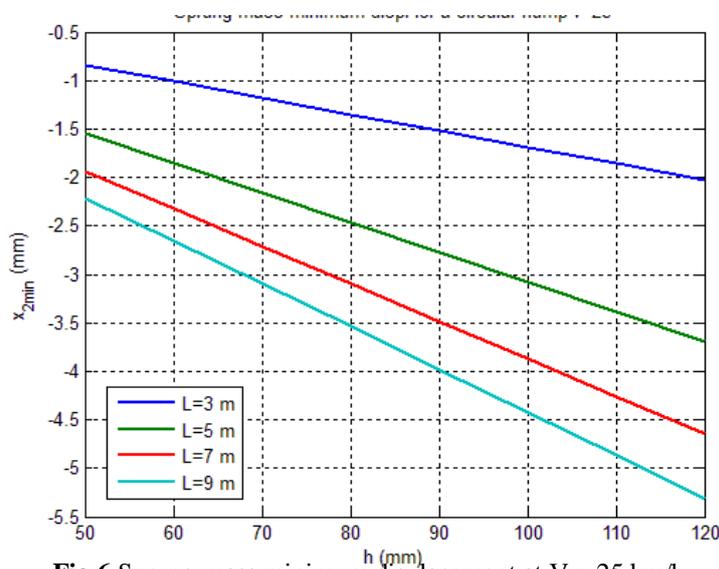


Fig.6 Sprung-mass minimum displacement at $V = 25$ km/h.

3.3 Sprung-mass Acceleration

The sprung-mass acceleration is the second derivative of its displacement with respect to time.

- The MATLAB command "diff" to differentiate the x_2 - t response twice producing the acceleration.
- Doing this, it didn't give any useful information.
- The author tried to overcome this pug by fitting an 8th order polynomial to the displacement time response, then differentiated this polynomial analytically yielding the sprung-mass acceleration.
- A sample result of this procedure is shown in Fig.7 showing the effect of the dimensions of circular hump on car dynamics when crossing the hump with 25 km/h speed.

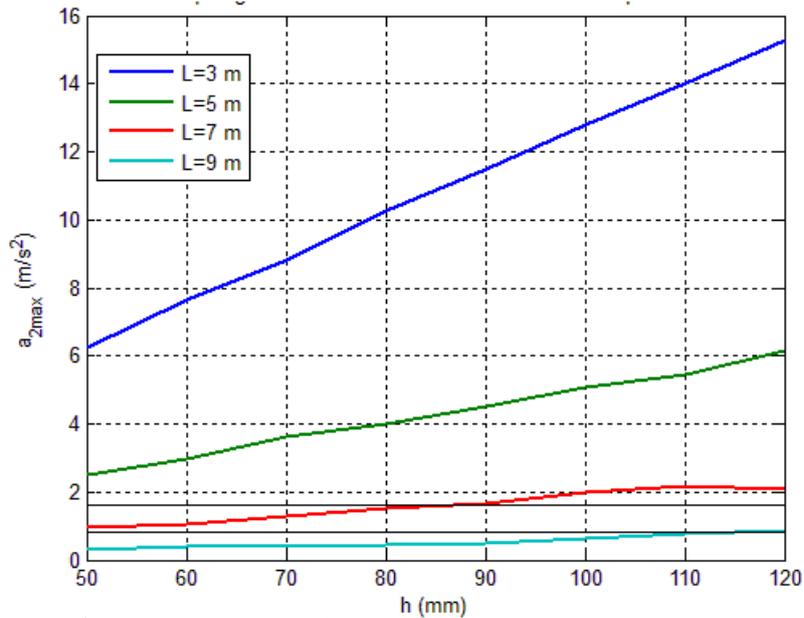


Fig.7 Sprung-mass maximum acceleration at $V = 25$ km/h .

- The maximum acceleration of the sprung-mass at certain car speed depends on the hump dimensions
- With a 25 km/h car speed a simple harmonic hump of 9 m length has maximum accelerations within the ride comfort range of < 0.8 m/s² according to ISO 2631 [18].
- Any other value of hump length < 9 m is expected to produce uncomfortable.

3.4 Maximum Car Speed for Ride Comfort

According to ISO 2631, the ride comfort range starts from 0.8 m/s² [18]. Imposing this limit on the car dynamics of a quadratic-car model when passing a simple harmonic hump of the dimensions stated in section III gives an estimation for the maximum car speed when passing the hump for accepted ride comfort. This maximum car speed is given graphically in Fig.8 against hump length for hump height in the range: $60 \leq h \leq 120$ mm.

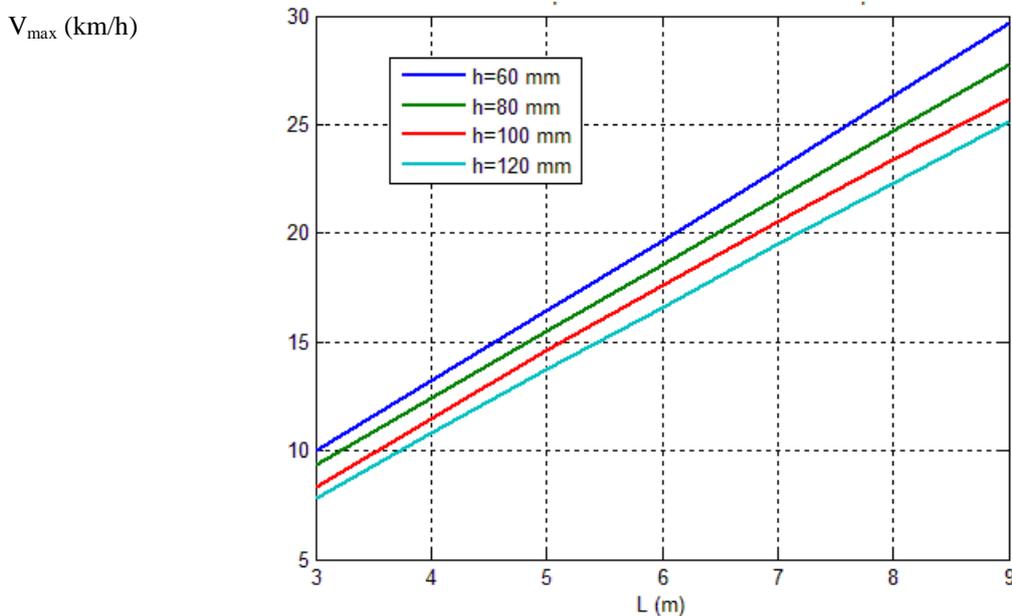


Fig.8 Car maximum speed for ride comfort across the simple harmonic hump

IV. Conclusion

- A quarter-car model with passive elements was used in this study to investigate the car dynamics during passing a new proposed simple harmonic hump.
- The damping coefficient of the suspension was kept constant at 5 kNs/m.
- Car speed between 5 and 30 km/h was considered during crossing the simple harmonic hump.
- Hump length between 3 and 9 m was considered.
- Hump height between 60 and 120 mm was considered.
- The undershoot decreased as the suspension damping coefficient increased.
- Ride comfort was considered through investigating the maximum unsprung mass acceleration during crossing the hump.
- The simulation of the quarter-car model using MATLAB showed that it is possible to reach a car speed of 30 km/h when crossing the simple harmonic hump if the hump is designed with 9 m length and 60 mm height.
- A diagram was provided allowing the design parameters of the simple harmonic hump for desired hump crossing speeds in the range 7.5 to 30 km/h.
- A 25 km/h hump crossing speed can be safely used for a simple harmonic hump of (9 m, 120 mm), (8.6 m, 100 mm), (8.1 m, 80 mm), (7.6 m, 60 mm) hump dimensions.
- A 20 km/h hump crossing speed can be safely used for a simple harmonic hump of (7.2 m, 120 mm), (6.75 m, 100 mm), (6.5 m, 80 mm), (6.1 m, 60 mm) hump dimensions.
- A 15 km/h hump crossing speed can be safely used for a simple harmonic hump of (5.4 m, 120 mm), (5.15 m, 100 mm), (4.75 m, 80 mm), (4.55 m, 60 mm) hump dimensions.
- A 10 km/h hump crossing speed can be safely used for a simple harmonic hump of (3.7 m, 120 mm), (3.5 m, 100 mm), (3.2 m, 80 mm), (3 m, 60 mm) hump dimensions.

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Dedication

- I dedicate this work to the sole of late Prof. Ahmed Ezzat Professor of System Dynamics at the Department of Mechanical Design & Production, Faculty of Engineering, Cairo University in the 1960's.
- Prof. Ezzat taught me System dynamics courses between 1968 and 1970.
- He was the reason to love this specialization and join his department in 1970 as teaching assistant and conduct research in this field up to now.

Biography

Galal Ali Hassaan



- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations , Mechanism Synthesis and History of Mechanical Engineering.
- Published 10's of research papers in international journal and conferences.
- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.