The Effect of Imbalance on the Performance of Unpaced Production Line – A Mathematical Modeling Approach

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Abstract: *In this study a five station unpaced production line with Exponentially distributed operation time with no buffer capacities is consider. A practical situation where five operation times are to be assigned on five station is investigated. Assuming that precedence restriction are minimal the operation can be permitted on stations. A number of data set with different kind of imbalance and with different degree of imbalance have been used for arriving at important conclusions. Operations on the production rate of the line is studied.This result in 60 different assignment operation on stations. The effect of such ordering on production rate is studied by calculating the percent improvement in production rate that is possible by following the best assignment over that with worst assignment. It is seen that the effect is highly significant at moderate degree of imbalance.*
**Keywords:** *Unpaced Production/Assembly line, Production rate, degree of imbalance, five station*

Notation

X_i = Mean operation time chosen on station i, where i = 1, 2, 3, 4, 5
μ_i = Mean operation time assigned on station i, where i = 1, 2, 3, 4,5
D = Degree of imbalance = $(X_{max} - X_{min})$
R = Production rate
R_H = Best Production rate

R_L = Worst Production rate
E = Percentage difference between best & worst Production rate
Δ = small time interval
P_{ij} = steady state probability at any station, where i, j = 1, 2, 3,
$P_{ij}(t)$ = transient state probability at any station where i, j = 1, 2, 3
P_{ijklm} = steady state probability at any station where i, j, k, l, m = 1, 2, 3
$P_{ijklm}(t)$= transient state probability at any station where i, j, k, l, m = 1, 2, 3
α_i = Mean operation rate of station i, where i = 1, 2, 3, 4, 5

I. Introduction

An unpaced Production line consists of a series of workstations arranged in work sequence. There may or may not be buffer space of limited capacities between workstation. The raw job enters the first station and passes from station to station till the work is completed on the last station. On each station the assigned task is completed and the job is passed on the next buffer if there is space available. In case no buffer space, the job is passed on the next station, provided the station is free Fig.1
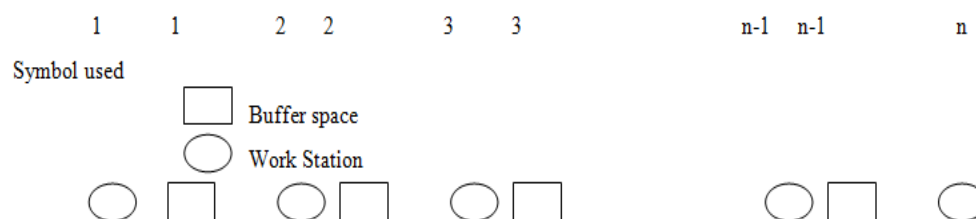
Fig.1. structure for the unpaced Production line

Several researchers on unpaced production line, Hiller & Boiling (1966) showed that unpaced line, with exponentially distributed operation line, deliberately unbalanced in a particular way gave production rate that was higher than then balanced line.They show that if end station were assigned work loads requiring higher

mean operation time and central stations were assigned work loads requiring less operation time the production rate may be improved over that of the balanced line. They called it bow phenomenon. EL – Rayah (1979) Proved that this is fact was true. Several researches on unpaced production line to find efficiency of balanced line or to find the unbalancing strategy to achieve maximum production rate.

These researchers are Yamashina and Okamura (1983), Hillier & so (1993), Pike & martin 1994, Akbay (1996).

In the present study attempt has been made to determine, whether a particular order of stations is better than another ordering of station, if station workload remains fixed and unbalanced.

The study has been carried out on a 5 – station unpaced Production line. The station times are assumed to be exponentially distributed due to ease of mathematical modeling. There are no buffer capacities in between the station. A work piece can leave the line only from the last station and that no work piece can move backward in the line. Processing times are independent across the workstation.

II.   Problem Formulation & Methodology

A hypothetical process to be completed on a five station unpaced production line has to be broken into 5 operation each to be performed on a station. A data set

$X = \{X_1, X_2, X_3, X_4, X_5\}$ is chosen mean operation time and the values are assigned to stations.

X_i, i= 1,2,3,4,5 is arranged in ascending order, the sum of all X_i's being equal to 5 & μ_i, i=1,2,3,4,5 is the mean operation time assign on station i.

D is Degree of Imbalance is the difference between highest mean operation time and lowest mean operation time irrespective of which stations have been assigned to mathematically D= $X_{max} - X_{min}$.

μ_i represents ordering permuted by computer program such as

$\mu_1 = X_1, \mu_2 = X_2, \mu_3 = X_3, \mu_4 = X_4, \mu_5 = X_5$

$\mu_1 = X_1, \mu_2 = X_2, \mu_3 = X_3, \mu_4 = X_5, \mu_5 = X_4$

………………

Each ordering has a mirror image also and since the production rate is forward and reverse direction is same, there are actually 60 orderings in all. For the same degree of imbalance irrespective of ordering the production rate has been obtained.

The effect of imbalance is to find the effect of ordering on the percentage difference (E) between best (R_H) and worst production rate R_L.

The percentage difference E is then calculated as

E = $(R_H - R_L)$ *100 / R_L

Where R_H = best Production rate

R_L = Worst Production rate

The value of E will be used to convince that ordering of task results in significant difference in production rate and substantial improvement in line performance can be achieved by judicially selecting the ordering of tasks.

III.   Mathematical Modeling

Before the development of model following assumption have been made

(a)   There are always jobs available for the first station so that first station is never starved

(b)   There is always space available after the last station so that the last station is never blocked.

(c)   All operation times are exponentially distributed

(d)   There are no buffer space between stations

(e)   The system reaches a steady state of statistical equilibrium after sometime, and the interest in this study lies in steady state only.

(f)   The time required for the transfer of the job in between the station is negligible.

Exponential distribution property is known as forgetfulness or lack of memory. Let us define an event to be the completion of processing of work piece on a station. Than it states that on a particular station of the production line, process on a work piece begins at time 't$_0$' then at any time t, the probability that the process on that work piece will end during a small time interval Δ, where Δ is very small, is independent of the magnitude of (t-t$_0$). Let probability density function (pdf) of an exponential distribution is

f(t) = $\alpha e^{-\alpha t}$; t ≥ 0 …………………..(i)

This equation represent an exponential distribution with a mean of 1/ α time unit.

Where α is operation rate at the station; which means that on an average α work piece are processed on the station per unit time. The probability that an operation at this station will end in a small interval Δ is equal to

α Δ irrespective of times elapsed since its beginning. Similarly the probability that the operation at that station will not end in time interval Δ is equal to (1- α Δ).

Since each station may be in one of the three states, namely working, starved or blocked. These states are coded as follows.

1- starved
2- blocked
3- working

System states are formed by combining station states and the probability of various system states are determined by forming probability equations.

Since model of 5 -station line consist of large number of system states and equation, the whole procedure of modeling is described with the help of a 2-station line.

III.A. Development Of A Model For 2 – Station Line

Let

ij – system state where i is the state of station 1 and j is the state of station 2, i= 1,2,3 j = 1,2,3 station state

$dP_{ij}(t) / dt$ --- first derivative of $P_{ij}(t)$ with respect to t.

α_i = Mean operation rate of station i, $\alpha_i = 1/ \mu_i$

the total system states that can be formed with two station can be given as

11, 12 , 13 ,21 ,22 , 23 , 31 , 32 , 33 using assumption only valid states are

23, 31 , 33

since Δ is very small, the usual assumption that two or more events cannot occurs during Δ is taken here as well.

Let us see how can the probability that, at time (t+Δ), the system is in state 23 be found out. This can occur if.

Either

The system was in state 23 at time t AND nothing changed during Δ

OR

The system was in state 33 at time t AND the first station completed its operation during Δ

The above can be written in the form of state transition in the following way

To 23   FROM   23/X   33/1

Where the notation ij/K represents that the system was in state ij at time t AND Δ station K completed its operation. The notation ij/x denotes that the system was in state ij at time 't' AND nothing changed during Δ

Similar argument may be given to arrive at various system state at time (t+ Δ). All system state transition for a two station line are shown in table1.

Table :1

TO	FROM		
23	23/X	33/1	
31	31/X	33/2	
33	33/X	33/2	33/1

The following three equation[6] can now be written for $P_{ij}(t+ \Delta)$ for all system states.

$$P_{23}(t+ \Delta) = P_{23}(t) (1- \alpha_2 \Delta) + P_{33}(t) \alpha_1 \Delta (1- \alpha_2 \Delta)\ldots\ldots\ldots\ldots(ii)$$

$$P_{31}(t+ \Delta) = P_{31}(t) (1- \alpha_1 \Delta) + P_{33}(t)\alpha_2 \Delta (1- \alpha_1 \Delta)\ldots\ldots\ldots\ldots(iii)$$

$$P_{33}(t+ \Delta) = P_{33}(t) (1- \alpha_1 \Delta) (1- \alpha_2 \Delta) + P_{23}(t) \alpha_2 \Delta + P_{31}(t) \alpha_1 \Delta\ldots..(iv)$$

Ignoring higher power of Δ and recalling the fact that

$$dP_{ij}(t) / dt = \lim_{\Delta \to 0} [P_{ij}(t+\Delta) - P_{ij}(t)] / \Delta \ldots\ldots\ldots\ldots..(v)$$

The equation (ii) to (iv) can be arranged to give three differential equation as follows

$$dP_{23}(t) / dt = - \alpha_2 P_{23}(t) + \alpha_1 P_{33}(t) \ldots\ldots\ldots..(vi)$$
$$dP_{31}(t) / dt = - \alpha_1 P_{31}(t) + \alpha_2 P_{33}(t) \ldots\ldots\ldots..(vii)$$
$$dP_{33}(t) / dt = - (\alpha_1 + \alpha_2) P_{33}(t) + \alpha_2 P_{23}(t) + \alpha_1 P_{31}(t) \ldots\ldots(viii)$$

These equations are in time independent steady state solution. Therefore the probabilities stabilize to a constant value, all derivative tends to zero.

$$dP_{ij}(t) / dt = 0 \ldots\ldots\ldots\ldots(ix)$$

Therefore these equations are

$$- \alpha_2 P_{23} + \alpha_1 P_{33} = 0 \ldots\ldots\ldots..(x)$$
$$- \alpha_1 P_{31} + \alpha_2 P_{33} = 0 \ldots\ldots\ldots..(xi)$$

$$- (\alpha_1 + \alpha_2)\ P_{33}\ + \alpha_2\ P_{23} + \alpha_1\ P_{31} \ldots\ldots\ldots(xii)$$

The probabilities are mutually exclusive. Therefore the sum of all probabilities is equal to 1.0.

$$P_{23} + P_{31} + P_{33} = 1.0 \ldots\ldots\ldots\ldots\ldots.(xiii)$$

Consider any two equations from (x) to (xii) and (xiii). The system of simultaneous linear equation can be solved to find the probability of all system states the production rate of the line can be obtained from any station of the line because in steady state the production rate is equal to across all the station. Thus

R = (Probability that station 1 is in working state) * α_1

R = (Probability that station 2 is in working state) * α_2

Therefore

$$R = (P_{31} + P_{33}) * \alpha_1 \ldots\ldots\ldots\ldots (xiv)$$
$$R = (P_{23} + P_{33}) * \alpha_2 \ldots\ldots\ldots(xv)$$

Solving the system of equation (x) to (xiii) and using (xiv) or (xv), the production rate of the line is obtained in closed form as

$$R = \alpha_1 * \alpha_2\ (\alpha_1 + \alpha_2) / (\alpha_1^2 + \alpha_2^2 + \alpha_1 * \alpha_2)\ldots\ldots\ldots\ldots(xvi)$$

III.b. Model for 5 Station line

A 5 -station line can be modeled with similar consideration as given above for 2-station line. In this case ijklm system state. Where i, j, k, l, m are states of the station 1 to 5 respectively. Where i = 1,2,3 ; j = 1,2,3 ; k = 1,2,3 ; l= 1,2,3 ; m = 1,2,3

p_{ijklm} – steady state probabilities that the system is in state ijklm

the total system states for 5 station line will be 55. Which are given below

22223	22231	22233	22311	22313
22323	22331	22333	23111	23113
…..	……	……..	…….	…….

Etc…..   33333

The state transition are

Table :2

TO	FROM					
22223	222223/X	22233/4	22323/3	23223/2	32223/1	
….	…..	…..	…….	……..	…….	
….	…..	……	…….	…….	…….	
33333	33333/X	22223/5	22231/4	22313/3	23133/2	31333/1

The 55 equation are

$$- \alpha_5\ P_{22223} + \alpha_4\ P_{22233} + \alpha_3\ P_{22323} + \alpha_2\ P_{23223} + \alpha_1\ P_{32223} = 0$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..$$
$$-(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)\ P_{33333} + \alpha_5\ P_{22223} + \alpha_4\ P_{22231} + \alpha_3\ P_{22313} + \alpha_2\ P_{23133} + \alpha_1\ P_{31333} = 0$$

Sum of all Probabilities being equal to one. The Production rate can than found by adding all such probabilities where any station i is working and multiplying the sum by α_i.

A closed form solution for production rate R will be difficult to obtain. Therefore a computer program in C was developed to solve the simultaneous linear equation numerically by Gaussian[7] elimination to obtain production rate R for specific value of

μ_i , i = 1,2,3,4,5

IV. Results And Discussion

The kind of imbalance and the data sets considered for each data set the production rates have been calculated for all be orderings and percent difference 'E' between highest and lowest production rate have been obtained for each degree of imbalance D

Table Data set of distribution & Result

S.N.	Data Set (Work Load Distribution)	Degree of imbalance D	RH	RL	E%
1.	Linear				
	{0.1, 0.55, 1.0, 1.45, 1.9}	1.8	0.4859	0.3694	18.0
	{0.4, 0.7, 1.0, 1.3, 1.6}	1.2	0.469	0.4131	13.53
	{0.7, 0.85, 1.0, 1.15, 1.3}	0.6	0.4884	0.4566	6.96
	{0.9, 0.95, 1.0, 1.05, 1.1}	0.2	0.4902	0.4786	2.42
2.	Random				
	{0.1, 0.15, 0.25, 0.5, 4}	3.9	0.2498	0.2468	1.22
	{0.1,0.3,0.6,2.0,2.0}	1.9	0.4022	0.3293	22.14

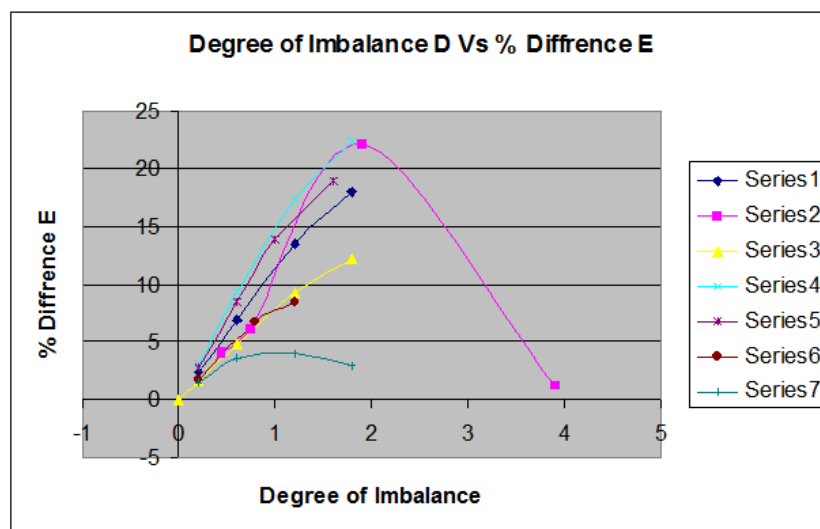|					
	{0.82,0.83,0.84,0.9,1.6}	0.79	0.4694	0.4472	4.92
	{0.7,1.0,1.05,1.1,1.15}	0.45	0.4885	0.4674	4.51
	{0.7,1.0,1.1,1.1,1.1}	0.45	0.4875	0.4686	4.03
3	Three values are equal to 1.0 and other values are different				
	{0.1,1.0,1.0,1.0,1.9}	1.8	0.4396	0.3918	12.20
	{0.4,1.0,1.0,1.0,1.6}	1.2	0.4689	0.4290	9.30
	{{0.7,1.0,1.0,1.0,1.3}	0.6	0.48590.	0.4635	4.83
	{0.9,1.0,1.0,1.0,1.1}	0.2	4884	0.4805	1.64
	{1.0,1.0,1.0,1.0,1.0}	0.0	0.4858	0.4858	0.0
	{2.0,2.0,2.0,2.0,2.0}	0.0	0.2429	0.2429	0.0
4.	Two equal values are higher & other three values are equally lower				
	{0.28, 0.28, 0.28, 2.08, 2.08}	1.8	0.391	0.319	22.3
	{0.52, 0.52, 0.52, 1.72, 1.72}	1.2	0.446	0.379	17.31
	{0.76, 0.76, 0.76, 1.36, 1.36}	0.6	0.485	0.444	9.31
	{0.92, 0.92, 0.92, 1.12, 1.12}	0.2	0.491	0.477	3.04
5.	Two equal values are lower & other three values are equally higher				
	{0.04, 0.04, 1.64, 1.64, 1.64}	1.6	0.4078	0.343	18.89
	{0.40, 0.40, 1.40, 1.40, 1.40}	1.0	0.4556	0.4	13.90
	{0.64, 0.64, 1.24, 1.24, 1.24}	0.6	0.4788	0.4416	8.42
	{0.88, 0.88, 1.08, 1.08, 1.08}	0.2	0.4885	0.4571	2.82
6.	Four equal values are higher & other one value is lower				
	{0.04, 1.24, 1.24, 1.24, 1.24}	1.2	0.4501	0.4151	8.43
	{0.36, 1.16, 1.16, 1.16, 1.16}	0.8	0.4727	0.443	6.70
	{0.84, 1.04, 1.04, 1.04, 1.04}	0.2	0.4879	0.4795	1.75
7.	Four equal values are lower & other one value is higher				
	{0.64, 0.64, 0.64, 0.64, 2.44}	1.8	0.3799	0.3691	2.93
	{0.76, 0.76, 0.76, 0.76, 1.96}	1.2	0.4331	0.4165	3.99
	{0.88, 0.88, 0.88, 0.88, 1.48}	0.6	0.4763	0.4599	3.57
	{0.96, 0.96, 0.96, 0.96, 1.16}	0.2	0.4874	0.48	1.54

Fig. 2. Degree of imbalance Vs % Different E

The behavior of D Vs E is seen in fig2. It is seen that for a certain kind of imbalance and in general, the effect of production rate increases as degree of imbalance increases initially and than decrease. At a particular level of degree of imbalance increases, the effect of production rate is increases because the mean operation time on the station is not much higher than other station in the production line. Therefore the one station mean operating time effect the other station mean operating time. Therefore best and worst ordering have high difference on production rate. However it is also evident that the degree of imbalance has no specific universal relationship with the effect on the production rate.

The case when the effect become very low at very high degree of imbalance can be understood by the following reasoning. When D is very high, one of the station has very large mean operation time compared to all others. This makes the station with the maximum mean operation time so slow that other station have hardly any effect on production rate. Therefore the best and worst orderings have very little difference in production rate.

V.   Conclusion

The ordering of operation, by assigning different operations of the same data set to different station of the line, affects the production rate significantly. The percent difference between best and worst ordering was sometimes as high as 22%.

It may be concluded that, in general, the effect of ordering increases as the degree of imbalance increases from low to moderate. But as the degree of imbalance increase further the effect starts stabilizing and then decrease. At very high degree of imbalance the effect of ordering is minimal. This is due to the fact that when degree of imbalance is high, one station becomes so slow that other station have very tittle effect on Production rate thus ordering also has little effect.

The general trends of the variation of production rate with degree of imbalance was similar for all kind of imbalance. However, it is observed that the degree of imbalance is not a very good parameter to assess the effect of ordering on production rate universally. Nevertheless, in a specific kind of imbalance it may be used as an effective measure.

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