

## **Prediction of static response of Laced Steel-Concrete Composite beam using effective moment of inertia approach**

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**ABSTRACT:** *Laced Steel-Concrete composite (LSCC) system is a new form of Steel-Concrete Composite (SCC) system developed recently by the authors[1]. From the experimental studies conducted on LSCC beam, it is found that the system possess enormous ductility with support rotation of about 13°. From finite element analysis, the support rotation achieved by LSCC beam is found to be around 20° and tensile plate failure mode is observed. In this study, analytical model for determining the load displacement behaviour of LSCC beams under monotonic loading by using effective moment of inertia is proposed. Validation of the proposed analytical model is carried out by using the numerical analysis results. Proposed model gives an initial estimate of load and deformation capacity of LSCC beams.*

**Keywords** - *Effective moment of inertia, finite element analysis, steel-concrete composite construction, shear connector, strut and tie model*

### **I. INTRODUCTION**

LSCC system comprises of thin steel cover plates provided with perforations, through which reinforcements are introduced and held in position with the help of cross rods and in filled with concrete between the cover plates as shown in Fig. 1. Reinforcing member consists of continuously bent rods known as lacing, which transfer the force between steel and concrete. This system is devoid of welding due to particular arrangement of lacings being inserted through the silts at appropriate places and made to stay intact by using cross rods.

Preliminary studies have been conducted to understand its basic characteristics under static or quasi-static loading. Experimental investigations on LSCC beams have been carried out by the authors [1] and it has been observed that it possesses large ductility and rotational capacity. This makes it suitable in structures subjected to suddenly applied loads such as due to blast, impact. The support rotation of the tested LSCC beam specimens are found to be nearly 13° for maximum mid-span displacement of about 170 mm. Due to certain limitations in the test set-up, the failure mechanism of the LSCC beam specimen could not be ascertained from the experiments.

Finite element analysis (FEA) on LSCC beam has also been carried out by the authors to determine the actual deformation capacity and mode of failure [2]. Finite element model has proven to be effective in terms of predicting load-deflection response, post peak behaviour and failure mode of LSCC beam. Through FEA, tension plate failure has been observed as the failure mode. The maximum support rotation and ductility index of LSCC beam was observed to be about 20° and 32 respectively.

Experimental investigations give actual behaviour of the LSCC structural components. But, it is highly tedious job to carry out experiments of complex structural systems such as LSCC and only few representative specimens can be tested experimentally. Though FEA can be used to analyse complicated structure, finite element models are time consuming to create and verify. The aim of developing analytical solutions is to reduce the test specimens to obtain the response of specified problem, realising the fact that conducting tests are costly and time consuming.

Few analytical solutions for estimation of displacement for SCC elements are already available. Double Skin Composite (DSC), a form of SCC construction consists of two steel plates and a group of shear stud connectors (welded to either top or bottom steel plate). Bending deflection has been calculated through pure bending theory and shear deflection has been obtained through the concept of effective shear modulus. However, deflections are relied upon shear stiffness determination from experimental data [3]. Bi-steel is another form of SCC construction, consisting of two steel plates, a concrete core and a group of shear

connectors (welded to both plates). Equivalent steel beam approach with steel modulus has been adopted to calculate the bending stresses and deflection due to bending, slip, shear [4]. Liew and Sohel [5] investigated a composite structures comprising of lightweight concrete core sandwiched in between two steel plates which are interconnected by J-hook connectors. Deflection is computed using the procedure given by McKinley and Boswell [3] for DSC elements. Equation for estimating the effective moment of inertia of fiber reinforced concrete beam taking effect of elastic modulus of rebar, reinforcement ratio, level of loading into consideration, on the basis of genetic algorithm and experimental results are proposed by Roohollah and Esfahani [6]. Effective moment of inertia approach for predicting the deflection of concrete beams reinforced with twisted bamboo cables taking effect of compressive strength, reinforcement ratio into account, are given by Akmaluddin and Pathurahman [7]. In this paper, an analytical model based on effective moment of inertia approach is proposed to obtain the load-displacement response of LSCC beams.

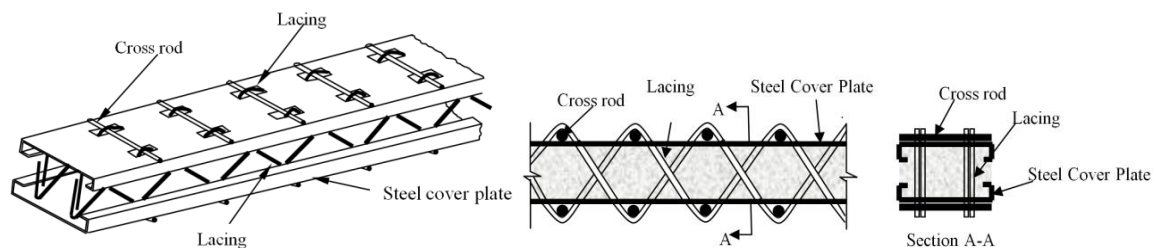


Figure 1: (a) Isometric view of the LSCC configuration; (b) Cross-section of LSCC system

## II. STRUCTURAL BEHAVIOUR OF LSCC SYSTEM

The strength of bond that exist between steel cover plate and concrete core, transfer of force between the cover plates and concrete core are the two governing factors to consider for designing a SCC system [5]. Most importantly composite action requires the sufficient transfer of load between the concrete and steel.

In LSCC beams, the lacings together with crossrods are connected to the cover plates, and their primary function is to resist both transverse and longitudinal shear and to provide resistance against outwards local buckling of the top plate when the beam is subject to loading. Lacings are the main components that mainly transfer the load between the plates, even after the core concrete is completely disintegrated. Cross rod also plays a significant role in transferring forces and preventing local buckling of steel cover plates apart from holding the lacings in position. The steel cover plates confines the concrete and are used to prevent spalling of concrete core.

Bottom steel cover plate will be in tension, while top steel cover plate will be in compression. The lacings which are connected to the top and bottom plates will be in tension, and the resistance to compression is also provided by virtual concrete strut as shown in Fig. 2.

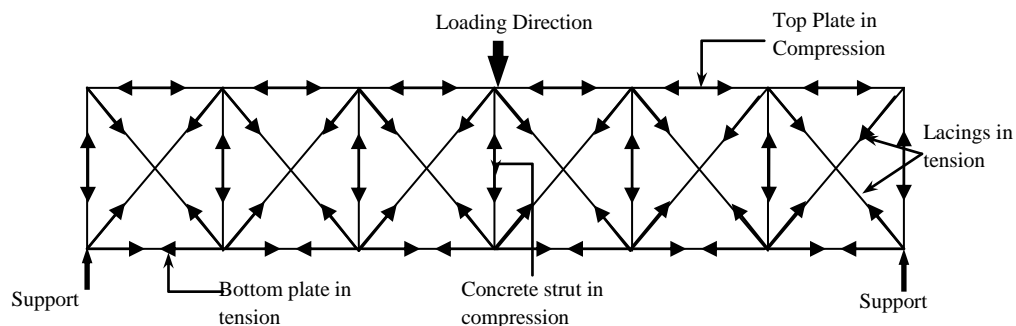


Figure 2: Strut and tie model of LSCC system

### III. LOAD-DISPLACEMENT RESPONSE

Load-displacement response of structural members gives an indication of ductility possessed by them. Analytical solution is developed to determine the load-displacement responses of LSCC beam-type elements having equal plate thickness and loaded in four-point bending. Load-displacement response is obtained using the equation 1. Equation for estimating the effective moment of inertia of LSCC beam is proposed. Proposed analytical model is validated by comparing the results with the numerical analysis results reported in literature [2].

The maximum deflection at the centre of a beam in a four-point bending arrangement can be calculated using the formula:

$$\delta_{\max} = \frac{PL_a}{48E_c I_e} (3L^2 - 4L_a^2) \quad (1)$$

where L is the span of the beam, P is the total concentrated load applied on beam, load P/2 is applied at a distance  $L_a$  from the support,  $I_e$  is the effective moment of inertia of the beam section beyond cracking stage and  $E_c$  is the modulus of elasticity of concrete.

### IV. EFFECTIVE MOMENT OF INERTIA

Different equations for effective moment of inertia of various forms of reinforced concrete beams are available in literature. In this study, a new equation for estimating the effective moment of inertia of LSCC beams is proposed taking into account of effect of level of loading, angle of lacing, thickness of cover plate and compressive strength of concrete.

From the values of load applied and its corresponding mid-span displacement, the values of effective moment of inertia can be obtained using equation 1 as follows:

$$(I_e) = \frac{PL_a}{48E_c \delta} (3L^2 - 4L_a^2) \quad (2)$$

The effective moment of inertia for concrete suggested by Branson [8],  $I_e$ , is as follows:

$$I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \quad (3)$$

where  $M_{cr}$  is the cracking moment;  $M_a$  is the applied moment;  $I_{cr}$  is the moment of inertia of the transformed section after cracking, and  $I_g$  is the moment of inertia of the gross section. The tension stiffening component in Branson's equation 3 depends on the ratio of gross-to-cracked moment of inertia  $I_g/I_{cr}$ , and increases substantially with  $I_g/I_{cr}$  ratios greater than 3 [6]. Ratio of  $I_g/I_{cr}$  is calculated to be more than 5 for LSCC beams. Therefore, Equation 1 will result in the deflection values which are of lesser than the actual ones of LSCC beams and suggested that the theoretical evaluation of  $I_e$  need to be modified.

Equation proposed by Fikry and Thomas [9] for effective moment of inertia is given by

$$I_e = I_{cre} + (I_g - I_{cre})e^{\varphi} \quad (4)$$

where  $I_{cre}$  is the modified moment of inertia of section in fully cracked section,  $\varphi$  is a function depending upon level of loading and reinforcement ratio. And

$$I_{cre} = (\alpha + \beta n \rho) \frac{1}{2} b d^3$$

$$\varphi = - \frac{M_a L_{cr}}{M_{cr} L} \rho \quad (5)$$

where  $\alpha$ ,  $\beta$  are the constants depending on the reinforcement ratios  $n\rho$ ,  $L_{cr}$  is the cracked length of the beam.

Above set of equations are developed by Fikry and Thomas [9] using the experimental data available. Modifications to these set of equations are developed to be used in for different forms of concrete beam in the recent years. From the previous study it was concluded that the amount and nature of reinforcement in a beam section will significantly affect the effective moment of inertia of beam. While the concrete compressive

strength did not straight forward affect the effective moment of inertia, but indirectly influences the cracked moment of inertia and cracked moment.

As previously discussed, reinforcements are provided in the form of bent rods known as lacings in case of LSCC beams. Therefore this study aims to propose the effective moment of inertia in order to be valid for LSCC beams. Experimental test data of LSCC beams giving load deflection relationship is available [1]. Finite element model developed is validated using the experimental results available [2]. Numerical and experimental results are found to match very well. LSCC beams possessing lacings inclined at an angle of 30°, 45°, 60°, plate of thickness of 2mm, 3mm, 4mm, concrete of grade 20MPa, 35MPa, 50MPa are considered. The beam section is 300mmx150mm and a span of 2400mm. Gross and cracked moment of inertia values are significantly influenced by varying the thickness of cover plate. It is decided to consider the effect of thickness of cover plate on the value of  $I_{cre}$  and  $I_{ge}$ . By plotting the ratio ( $I_{cre}/I_{cr}$ ) against the cover plate thickness as shown in Fig. 3(a) which gives the best fit of the data and provide modified formula for  $I_{cr}$ . From the regression analysis,

$$I_{cre} = (0.8485t - 1.5455)I_{cr} \tag{6}$$

where, t is the thickness of cover plate in mm. Similarly plotting ratio ( $I_{ge}/I_g$ ) against the cover plate thickness as shown in Fig. 3(b) gives

$$I_{ge} = (0.0385t + 0.8846)I_g \tag{7}$$

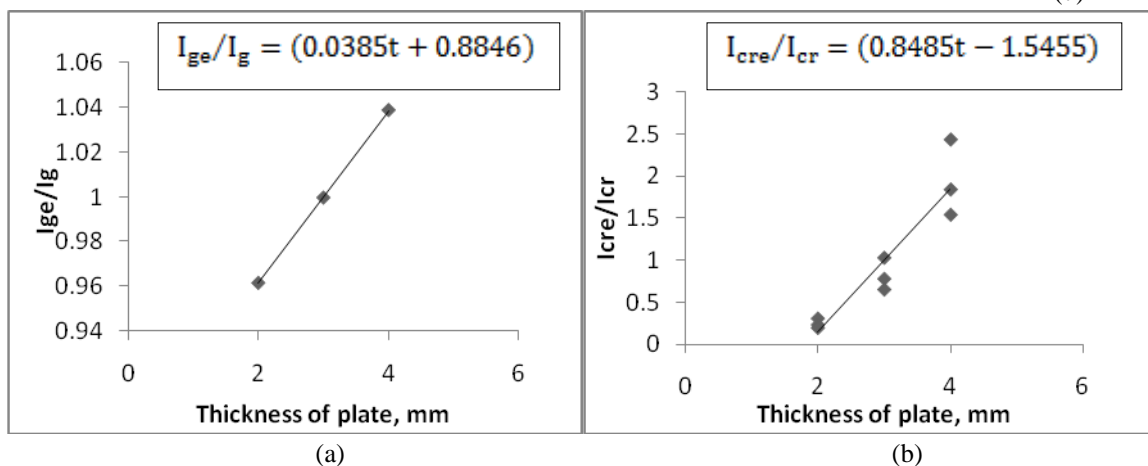


Figure 3: Regression analysis for  $I_{cre}$  and  $I_{ge}$  computation

From equation 4, rearranging the effective moment of inertia expression, function  $\varphi$  is obtained as

$$\varphi = \ln \left[ \frac{I_e - I_{cre}}{I_{cre} - I_{ge}} \right] \tag{8}$$

$\varphi$  values divided by  $(M_a/M_{cr})$  defines P and plotted against angle of lacing as shown in Fig. 4. From regression analysis, this provides equation as given by

$$\varphi = - \left( \frac{M_a}{\mu M_{cr}} \right) (0.0009586\omega + 0.08161) \tag{9}$$

where  $\omega$  is the angle of lacing,  $\mu$  is constant depending upon the angle of lacing as given in Table 1.

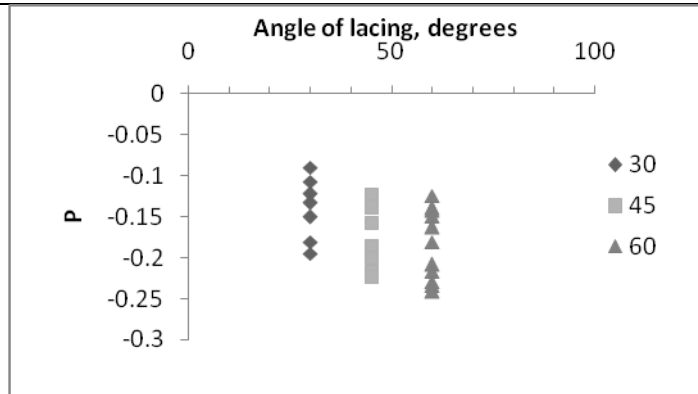


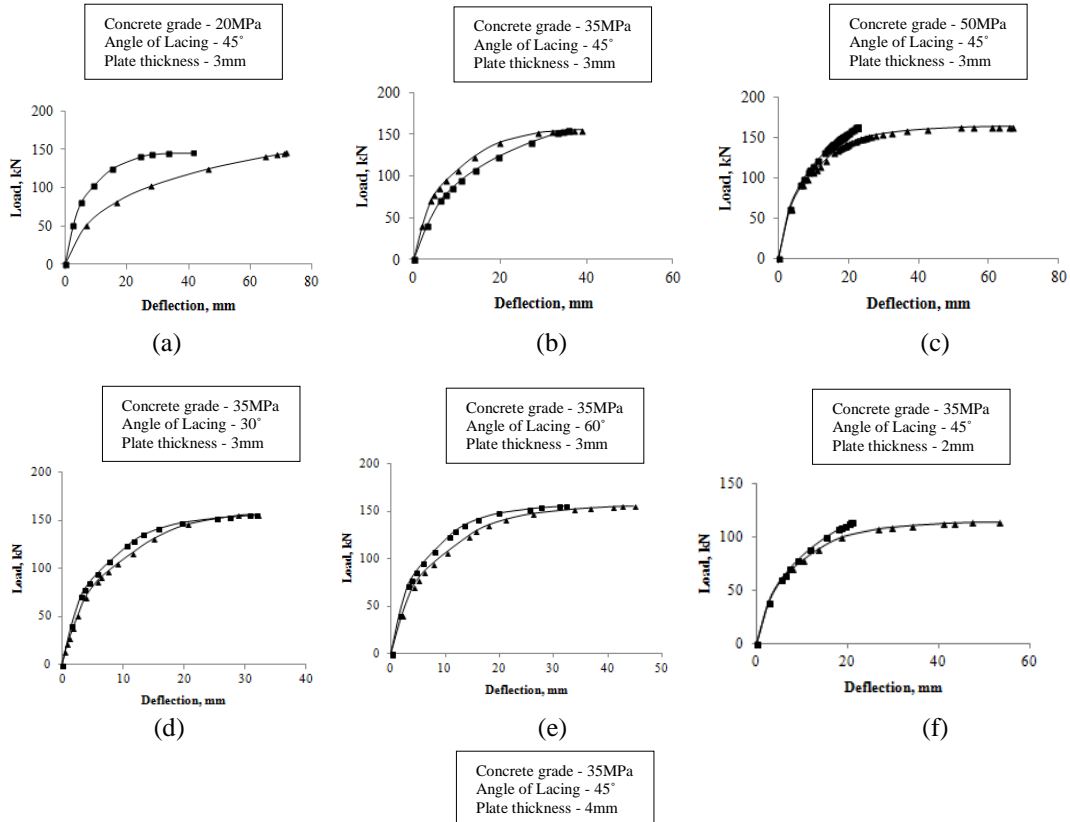
Figure 4: Regression analysis for  $\phi$  computation  
 Table 1: Constant  $\mu$  values for varying lacing angles

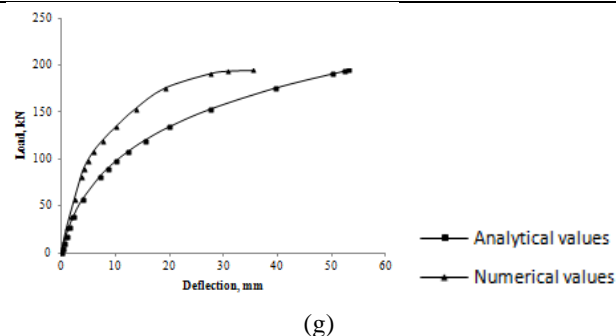
Angle of lacing (in degrees)	$\mu$ value
30	0.1
45	0.5
60	0.8

Finally, a proposed model for calculating  $I_e$  is given by equation 4 with modified  $I_{cre}$ ,  $I_{ge}$  and  $\phi$ .

## V. VALIDATION

Load-Displacement response of LSCC beams with different lacing angle, plate thickness and compressive strength of concrete obtained using the proposed approach are compared using the numerical results and are presented in Fig. 5.





(g)  
Figure 5: Load-displacement responses of LSCC beams

From the load-displacement response, it is observed that deflection predicted using proposed effective moment of inertia is matching appropriately with the deflection values obtained numerically. In case of LSCC beams with 45° lacing, 50MPa grade of concrete, 3mm plate thickness and 45° lacing, 35MPa grade of concrete, 2mm plate thickness deflection values are found to be deviating after the load level of around 100kN. The difference between the numerical and analytical results may be due to the fact that linear fit is adopted in calculating the cracked moment of inertia.

## VI. CONCLUSIONS

Effective moment of inertia approach is adopted in determining the load-deflection response of LSCC beams under monotonic loading. In general, large volume of data are required to evaluate the generalized formula for effective moment of inertia of LSCC beam. However, with limited numerical results available an attempt has been made to find out the effective moment of inertia, which can be used to obtain deflection. To obtain more comprehensive conclusion, further research is needed to verify this model.

## VII. Acknowledgements

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## REFERENCES

- [1] N. Anandavalli, N. Lakshmanan, G.M. Samuel Knight, Nagesh R. Iyer, and J. Rajasankar, Performance of Laced Steel-Concrete Composite (LSCC) beams under monotonic loading, *Engineering Structures*, 41, 2012, 177-185.
- [2] A. Thirumalaiselvi, and N. Anandavalli, Numerical investigations on static response of laced steel-concrete composite slabs, *Proceedings of Intl. conf. on advances in civil, structural and mechanical engineering, Hong Kong*, 2013, 37-41.
- [3] B Mc Kinely, and L.F. Boswell, Behaviour of double skin composite construction, *Journal of Construction Steel Research*, 58(10), 2002, 1347-1359.
- [4] M. Xie, N. Foundoukos, and J.C. Chapman, Static tests on steel – concrete- steel sandwich beams, *Journal of Construction Steel Research*, 63(6), 2007, 735-750.
- [5] J.Y.R. Liew, and K.M.A. Sohel, Lightweight steel – concrete –steel sandwich system with J-hook connectors, *Engineering Structures*, 31(5), 2009, 1166-1178.
- [6] S. Roohollah Mousavi, and M. Rza Esfahani, Effective moment of inertia prediction of FRP-reinforced concrete beams based on experimental results, *Journal of composites for construction*, 16, 2012, 490-498.
- [7] Akmaluddin, and Pathuahman, Effective moment of inertia for predicting the deflection of concrete beams reinforced with twisted bamboo cables, *International Journal of Civil & Environmental Engineering*, 13(3), 2012, 6-13.
- [8] D.E. Branson, Instantaneous and Time-dependant deflections of simple and continuous reinforced concrete beams, *HPR Report No.7, Part 1, Alabama Highway Dept. Bureau of Public Roads, AL*, 1965.
- [9] A.M Fikry, and C. Thomas, Development of a model for the effective moment of inertia of one-way reinforced concrete elements, *ACI Structural Journal*, 95(4), 1988, 445-455.