

Study on active vibration control of random excitations on a piezoelectric beam using a tuned LQG controller

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Abstract: Objective is to study the effectiveness of a tuned Linear Quadratic Gaussian (LQG) controller for the active vibration suppression of random excitation on a smart beam. Aluminium beam with surface bonded PZT patches which are used both as sensor and actuator based on location. A group of PZT patches at the root of the beam were assumed as actuators in bimorph configuration and a single patch as sensor on the tip. LQG algorithm, the result of a modern state-space technique, was used in order to design a dynamic optimal regulator. The designed LQG regulators of the current study were evaluated via closed loop random vibration data fabricated (ergodic process assumed) by a suitable MATLAB code, in order to reduce the vibration level of the smart beam around null value. LQG controller designed address the random vibrations and the uncertainties in them with change of time and influence of predecessor excitations. The obtained time domain responses from finite element method have demonstrated that an LQG tuned controller successfully suppress the random excitations of the smart beam to a desired level.

Keywords: ergodic process, LQG algorithm, PZT, random vibration, smart beam

I. Introduction

A simple cantilever beam was used as the system whose dynamics was studied and active vibration control technique was applied to address the random vibration. The cantilever beam is given random excitations and the analogy of random excitations with the white Gaussian Noise control is taken into account and the control system was tuned with LQG algorithm. The simplest control algorithm that can be implemented to suppress the occurring vibrations in the system is direct feedback of the output parameter back into the system [1][2][3][4]. Measurable parameters like strain, displacement, velocity, acceleration, etc. are the commonly fed signals. This type of control is simple to implement and yet yields satisfactory results. PZT patches are used to sense the deformation by recording the electrical response and also as actuators by changing the electrical signal to vibrational excitations. Also the mechanical deformation of the piezoelectric material can be detected by its electrical response, hence the piezoelectric materials can be utilized as sensors as well. In this work, the system parameters were analysed through the free vibration simulation of the considered model consisting of one Lead-Zirconate-Titanate (PZT) patch producing the primary disturbance (the exciter), another PZT patch sensing the occurring disturbance (the sensor), and finally the first PZT patch in bimorph condition [5] suppressed the vibration (the actuator). As reported by Lim [6], the presence of patches shifts the natural frequencies of the passive structure to higher frequencies. Waghulde and Kumar [7] used piezoelectric material on a cantilever beam thereby making it smart. The placement of the piezo-sensors and actuators on the beam were determined through modal analysis similar to that reported by Tripathi and Gangadharan [1]. Nowadays, a widespread application area of piezoelectric materials is, using them as collocated actuator and sensor pair for active vibration control purposes. In this study, a least order possible LQG controller was tuned in MATLAB for modelling the vibration suppression of a smart beam, by assuming the PZT patches as sensor and actuator pairs [5]. The mathematical model of the deformation of a cantilever beam is provided by Premjyoti G. Patil [8] using Finite Element Method. Using the mathematical model, the beam deformation is plotted using MATLAB. Lucy Edery-Azulay, Haim Abramovich [9] described that the active damping is obtained by using an actuator and a sensor piezo-ceramic layer acting in closed-loop. Various random excitation cases were developed using MATLAB program to imitate the possible disturbances in environment and system is exposed to them. Finite Element Analysis and generalized equation of motion has been used in this paper. The study is purely based on a suitable MATLAB code developed.

II. Methodology

Finite Element Analysis of cantilever beam with dimensions of 500mm length, 30mm width and 3mm thickness shall be taken for study. The calculated natural Frequency of the structure and generated mode shapes (using the Finite elements constructed) is validated by comparison with ANSYS and theoretical formulae. The elements once approved shall be used for the study purpose with a suitable MATLAB code in which LQG optimisation technique is adapted and tune goals of control systems set accordingly.

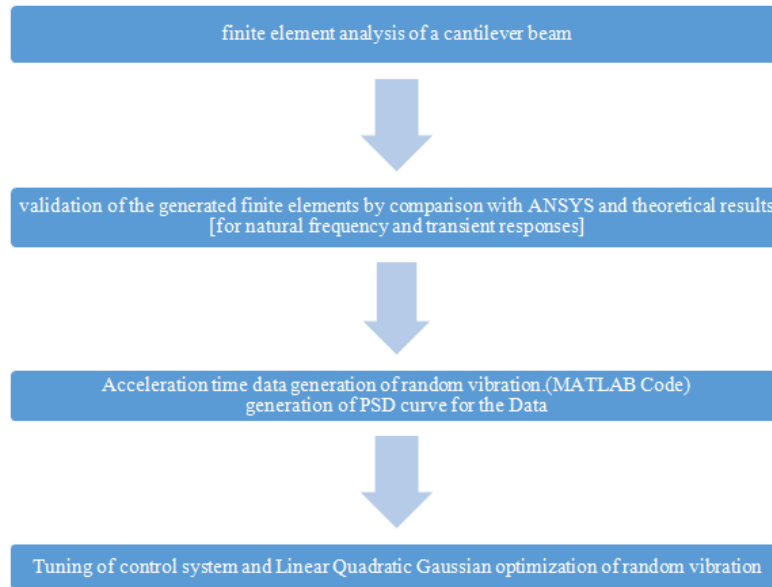


Figure 1: study on active vibration control of a piezoelectric beam

The smart beam given in Fig. 2 and Fig. 3 represent the geometry of considered model of the beam for the study purpose. Fig. 3 represents the PZT actuator/sensor in bimorph condition and its position. The actuator/sensor provide best result when the Piezo-electric patches are bonded near the fixed end [4]. The details of the smart beam along with the details of the PZT patches considered in this work are given in Table 1 [4].

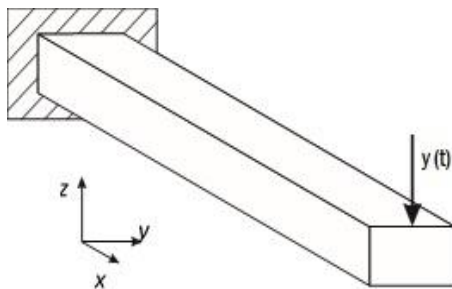


Figure 2: cantilever Beam

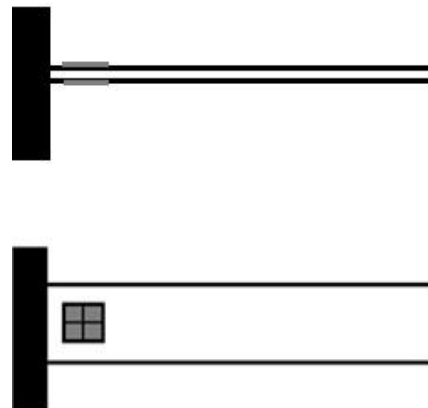


Figure 3: position of piezo patches [Bimorph condition]

Table 1 : Geometrical And Material Properties

Parameters	Cantilever Beam (Aluminium)	PZT Sensor / Actuator
Length	500 mm	10 mm
Width	30 mm	30 mm
Thickness	3 mm	0.6mm
Young's Modulus	$E = 6.9 \times 10^{10} \text{ N/m}^2$	$E = 6.66 \times 10^{10} \text{ N/m}^2$
Poison ratio	0.3	0.3
Density	$\rho = 2700 \text{ kg/m}^3$	$\rho = 7400 \text{ N/m}^2$
Piezoelectric Stress constant [PZT 5H]	8.5 Vm/N
Piezoelectric Strain constant [PZT 5H]	$265 \times 10^{-12} \text{ C/N}$
Damping Constants Used	$\alpha = 0.001$ $\beta = 0.0001$

For the study, the beam, sensor and actuator are taken as Euler-Bernoulli beam elements. Sensor and actuator layers are thin compared with the beam thickness. Cross-sections of beam, sensor and actuator remain plane and normal to the deformed longitudinal axis before and also after bending. The model has neutral axis of beam, sensor and actuator passing through the centroid and the polarization direction of the sensor and actuator layers is in the thickness direction (z-axis). The piezoelectric material is assumed homogeneous, transverse isotropic and elastic. Any kind of adhesive used in binding the Sensor/Actuator does not contribute to mass and stiffness of the smart beam elements.

1.1. LQG Design

LQG designs need a state-space model and the state-space realization of the smart beam model of form :

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}(\mathbf{u} + \mathbf{w}) \quad (1) \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}(\mathbf{u} + \mathbf{w}) + \mathbf{v} \quad (2) \end{aligned}$$

The regulator is tuned to regulate the output y to zero. The system is driven by process noise w and the control signal u , and regulator considers the noisy measurements $y_v = y + v$ to generate the desired control. In this study both w and v are modelled as white noise. Here, x is the state vector. A , B , C and D matrices are those obtained by the state space realization of the smart beam transfer function in the MATLAB code.

The transfer function from control input to tip position $y(t)$ is done using finite-element analysis. The actuator acts to give input as random excitations in the form of acceleration -time data generated using a MATLAB code. The input data is assumed to be normally distributed as the general representation of random probability distribution. Particular input data at a moment is given an uncertainty of 20% for its factors. Six predecessor excitations are assumed to have effect on the beam motion at a time in the analysis. The control system is tuned with LQG optimisation goals and the vibration suppression is analysed using suitable MATLAB code.

The tuning of LQG regulator comprise of an optimal state-feedback gain vector K and Kalman state estimator. In LQG control, the regulation performance is characterised by a performance criterion in quadratic form:

$$J\{\mathbf{u}\} = \int_0^{\infty} \{\mathbf{x}^T \mathbf{Q}\mathbf{x} + 2\mathbf{x}^T \mathbf{N}\mathbf{u} + \mathbf{u}^T \mathbf{R}\mathbf{u}\} dt \quad (3)$$

The weighting matrices Q , N and R are user specified. It define the balance between control effort and the regulation performance. In the design of LQG controller, a state-feedback law:

$$\mathbf{u} = -\mathbf{K}\mathbf{x} \quad (4)$$

Minimizes the cost function $J(u)$. The minimizing gain matrix K is obtained by solving algebraic Riccati equation. Finally, the design is completed by designing of the Kalman filter which is an optimal estimator when dealing with Gaussian white noise.

III. Results And Discussion

The Finite elements generated in MATLAB code in comparison with ANSYS and Theoretical obtained values as in Table 2 where in agreement within permissible errors. The discretisation of beam gives a valid model of the cantilever beam and acceptable response to PSD inputs.

Table 2: Comparison of Natural Frequencies Obtained Using the Finite Elements Generated

MODE	FREQUENCY (Theoretical, Hz)	FREQUENCY (MATLAB, Hz)	FREQUENCY (ANSYS, Hz)	%Error b/w MATLAB & Theoretical	%Error b/w ANSYS & Theoretical	%Error b/w ANSYS & MATLAB
1	4.892	4.8998	4.9407	0.1592	0.9857	0.8347
2	30.705	30.7062	30.958	0.0039	0.8172	0.8200
3	86.641	85.9784	86.708	0.7707	0.0773	0.8486
4	168.48	168.4837	170.02	0.0022	0.9058	0.9118
5	278.5	278.5173	281.32	0.0062	1.0024	1.0063
6	416.05	416.0611	420.74	0.0027	1.1147	1.1246

Fig.4 shows the closed loop response of the beam towards random excitation taken 6 excitations over time periods of 1.8 seconds. The excitation deflections dies out to a 10% value in next 5.8 seconds and next three sets

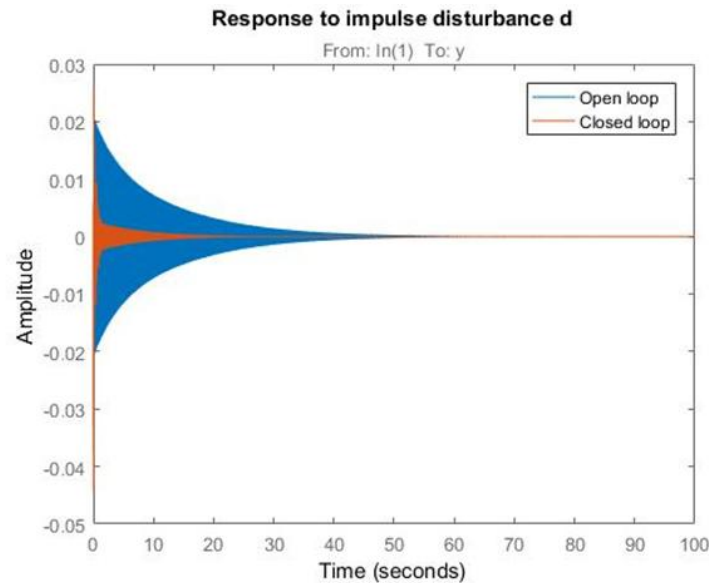


Figure 4: Response impulse random disturbances taken 6 data simultaneously

(of 1.8 seconds each) of excitations acts in that time. The analysis showed that a cyclic addressing of excitations taken six at a time can limit the deflection of the tip within 10% limit throughout.

IV. Conclusion

Vibration suppression of smart beams using the piezoelectric patch structure is presented in the paper. LQG optimization algorithm was used to tune the control system and address the acceleration time data and suppress the random vibrations imparted on the flexible cantilever beam. The control system tuning and Finite Element Analysis of vibration of beam was done using suitable MATLAB functions. The design of a second order LQG controller was presented for suppressing the random flexural vibrations of a smart beam. To design a controller with a low order is believed to have important advantages especially for commercial applications. The generated graphs obtained by using the active vibration control system for random excitations have demonstrated the validity of LQG systems and the sensor actuator bimorph condition for controlling random excitations.

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