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A Fuzzy Inventory Model With Lot Size Dependent Carrying / Holding Cost

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Abstract: *In the classical Harris Wilson inventory model al the cost associated with the formula was taken to be constant and which does not dependent on any quantity. In this paper we have taken Ordering cost, holding cost and order quantity all are triangular fuzzy numbers. Graded mean integration representation method is used for defuzzification. In this paper we consider an inventory model where the holding cost depends on order quantity. An algorithm is developed to find the economic order quantity along with numerical examples.*
Keywords: *Fuzzy inventory model, Triangular fuzzy numbers, Defuzzification*

I. Introduction:

Pricing and inventory policy are two important factors in success of business of different items. One of the challenging problems for the researchers is to study and analyze the inventory systems. Those systems cannot only reduce the costs, but also reduce stock outs and improve customer satisfactions.

The Harris – Wilson Formula [15] for determining the optimum lot size is the first and pioneering model in the inventory systems. The Basic EOQ model considers three type of costs like ordering or set up cost, holding cost and shortage cost. Several researchers deal with the modifications of these cost structures. Gupta [10] discussed an inventory model with lot size dependent ordering cost. Gordon [9], Emery [7], Arrow [2] have given so many worth results in this field.

The Harris Wilson formula for determining the optimum lot size, the quantity in which an item of inventory should be purchased or produces is

$$Q = \sqrt{\frac{2DA}{H}} \qquad \text{-------(1)}$$

where D – Demand, A – Ordering cost, H – Holding cost.

Our Endeavour is to introduce more realistic inventory model for which an algorithm is developed. Our proposed model is illustrated through numerical examples.

This Formula follows that,
(1) The optimal quantity minimizes the sum of the annual set up cost and holding cost
and
(2) The Total cost is

$$TC(Q) = \frac{QH}{2} + \frac{DA}{Q} \qquad \text{-----(2)}$$

i. e the costs considered are assumed to be constant. This body of research assumes that the parameters involved in the EOQ model, such as the demand and the purchasing cost, are crisp values or random variables. However, in reality, the demand and the cost of the items often change slightly from one cycle to another. Moreover, it is very hard to estimate the probability distribution of these variables due to a lack of historical data. Instead, the cost parameters are often estimated based on experience and subjective managerial judgment. Thus, the fuzzy set theory, rather than the traditional probability theory, is well suited to the inventory.

The paper developed by Karabi Dutta Choudhury and Sumit Saha [6], they considered all the costs as constant in their model. To make it more realistic we have taken ordering cost, holing cost and order quantity as triangular fuzzy numbers. Graded mean integration representation method is used for defuzzification.

II. Methodology

Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfy the following conditions, is a generalized fuzzy number $\tilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0, 1],

(ii) $\mu_A = 0, \ -\infty < x \le a_1$

(iii) $\mu_A = L(x)$ is strictly increasing on $[a_1, a_2]$

(iv) $\mu_A = w_A, \ a_2 \le x \le a_3$

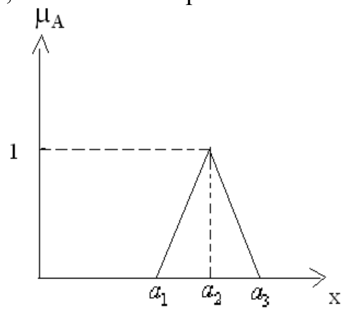
(v) $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$

(vi) $\mu_A = 0, \ a_4 \le x \le \infty$

Where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$; when $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$.

Triangular fuzzy number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined on R, is called the triangular fuzzy number, The membership function of $\tilde{A}$ is given by



$$\mu_A = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$

The Function Principle

The function principle was introduced by Chen [6] to treat fuzzy arithmetical operations. This principle is used for the operation for addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$

Where $T = (a_1b_1, a_1b_3, a_3b_1, a_3b_3), c_1 = \min T, c_2 = a_2b_2, c_3 = \max T$ if a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then $\tilde{A} \times \tilde{B} = (a_1b_1, a_2b_2, a_3b_3)$.

(iii) $-\tilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(iv) $\frac{1}{\widetilde{B}} = \widetilde{B}^{-1} = (\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$ where b_1, b_2, b_3 are all non zero positive real numbers, then the division of $\tilde{A}$ and $\widetilde{B}$ is $\frac{\widetilde{A}}{\widetilde{B}} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$

(v) For any real number K, $\begin{matrix} K\widetilde{A} = (Ka_1, Ka_2, Ka_3) if K > 0 \\ K\widetilde{A} = (Ka_3, Ka_2, Ka_1) if K < 0 \end{matrix}$

Graded Mean Integration Representation Method

Defuzzification of $\tilde{A}$ can be done by Graded Mean Integration Representation Method . If $\tilde{A}$ is a triangular fuzzy number and is fully determined by (a_1, a_2, a_3) then defuzzified value is defined as

$$\rho(A) = \frac{1}{2} \frac{\int_0^1 h[a_1 + h(a_2 - a_1) + a_3 - h(a_3 - a_2)]dh}{\int_0^1 h dh} = \frac{a_1 + 4a_2 + a_3}{6}$$

Model Development and Analysis:

In this model, we assume that the holding cost increases stepwise as the lot size increases. The following notations are used.

D – Total / Annual Demand

$\widetilde{A}$ - Fuzzy Ordering cost per order

$\widetilde{H}_j$ - Holding cost for the lot size $\widetilde{Q}_j$ if $q_{j-1} \le Q_j \le q_j$

Where j = 1, 2, 3, …..m. $q_0 = 0$ and $q_\infty = \infty$

Also assume $H_1 < H_2 < H_3 < \ldots\ldots H_m$.

For holding cost $\widetilde{H}_j$ Harris – Wilson EOQ is given by

$$\widetilde{Q}_j = \sqrt{\frac{2 \times D \times \widetilde{A}}{\widetilde{H}_j}} \qquad \text{-------(3)}$$

If $\widetilde{Q}_j$ does not lie within the interval [q_{j-1}, q_j] i. e is not order feasible, then the optimal lot size will be determined by

$$q_{j-1} \qquad if \quad Q_j \le q_{j-1} \qquad \text{-------(4)}$$
$$q_j \qquad if \quad Q_j \ge q_j \qquad \text{-------(5)}$$

With the known value of $\widetilde{Q}_j$ thus obtained from the equation (3), (4), and (5), $T\widetilde{C}(Q_j)$ can be calculated at the holding cost by

$$T\widetilde{C}(Q_j) = \frac{\widetilde{Q}_j \times \widetilde{H}_j}{2} + \frac{D \times \widetilde{A}}{\widetilde{Q}_j} \qquad \text{------(6)}$$

If $\widetilde{Q}_j$ is order feasible then $T\widetilde{C}(Q_j)$ will be the optimal cost, otherwise the value of $\widetilde{Q}_j$ thus obtained by equations (4) and (5), $T\widetilde{C}(Q_j)$ is calculated by (6). Thus among all the calculated values of $T\widetilde{C}(Q_j)$, the smallest value will be the optimal cost. After defuzzification of $\widetilde{Q}_j$ and $T\widetilde{C}(Q_j)$ we will get optimal TC (Q_{opt}) and the corresponding Q_j will be the optimal lot – size i. e Q_{opt}.

Algorithm

1. Input j, n = number of lot size
2. Set j = 1.
3. Calculate $\widetilde{Q}_j = \sqrt{\dfrac{2 \times D \times \widetilde{A}}{\widetilde{H}_j}}$
4. If $\widetilde{Q}_j$ is order feasible,
5. Calculate $T\widetilde{C}(Q_j) = \dfrac{\widetilde{Q}_j \times \widetilde{H}_j}{2} + \dfrac{D \times \widetilde{A}}{\widetilde{Q}_j}$. Go to Step 8.
6. If $\widetilde{Q}_j$ is not order feasible, then by equation (4) and (5), $\widetilde{Q}_j$ can be calculated and hence find $T\widetilde{C}(Q_j)$ by equation (6).
7. Set j = j + 1, until j <=n. Go to step 3
8. Defuzzify $\widetilde{Q}_j$ and $T\widetilde{C}(Q_j)$, then we will get Q_j and TC(Q_j).
9. Among all the calculated TC(Q_j), obtain the minimum value of TC(Q_j), and set
   TC(Q_{opt}) = min [TC (Q_j)].
10. Thus TC (Q_{opt}) obtained is the optimal cost and corresponding Q_j is the Q_{opt} the optimal lot – size.
11. Thus Q_{opt} and TC(Q_{opt}) are the required results.

Numerical Examples:
Example 1:

D = 1800

$\widetilde{A}$ = (340, 350, 360)

j	$\widetilde{Q}_j$	Range	Feasible $\widetilde{Q}_j$	$\widetilde{H}_j$	$T\widetilde{C}(Q_j)$	Q_j	TC(Q_j)
1	(100.99, 107.02, 113.84)	1 – 30	30	(100, 110, 120)	(10425.466, 11772.85, 13246.87)	107.15	11793.956
2	(93.5, 98.44, 103.9)	31 – 60	60	(120, 130, 140)	(11500.28, 12798.44, 14203.48)	98.52	12816.25
3	(87.46, 91.65, 96.21)	61 – 90	90	(140, 150, 160)	(12483.29, 13747.73, 15105.9)	91.71	13763.35
4	(80.263, 83.666, 87.31)	91 – 120	91	(170, 180, 190)	(13831.87, 15059.88, 16367.93)	83.71	15073.2
5	(76.345, 79.373, 82.589)	121 –150	121	(190, 200, 210)	(14662.97, 15874.51, 17159.64)	79.404	15886.775

For the lot size Q_{opt} = 30 units (approx) the optimal holding cost is TC(Q_{opt}) = Rs. 11793.956.

Example 2:

D = 1000

$\widetilde{A}$ = (340, 350, 360)

j	$\widetilde{Q}_j$	Range	Feasible $\widetilde{Q}_j$	$\widetilde{H}_j$	$\widetilde{TC}(Q_j)$	Q_j	$TC(Q_j)$
1	(75.277, 79.77, 84.85)	1 – 30	30	(100, 110, 120)	(7770.92, 8774.95, 9873.34)	79.87	8790.68
2	(69.69, 73.38, 77.46)	31 – 60	60	(120, 130, 140)	(8570.76, 9539.39, 10587.93)	73.445	9552.71
3	(65.19, 68.31, 71.71)	61 – 90	68.35	(140, 150, 160)	(9304.62, 10246.95, 11259.12)	68.35	10258.59
4	(59.82, 62.36, 65.08)	91 – 120	91	(170, 180, 190)	(10309.04, 11224.97, 12200.65)	62.39	11234.93
5	(56.9, 59.16, 61.56)	121 –150	121	(190, 200, 210)	(10928.51, 11832.16, 12790.69)	59.18	11841.31

For the lot size Q_{opt} = 30 units (approx) the optimal holding cost is $TC(Q_{opt})$ = Rs. 8790.68.

III. Conclusion:

In this paper the classical Harris – Wilson model has been extended with fuzzy holding cost depending on lot size. It is observed that if order quantity lies within the interval i. e order feasible it will give the optimal costs. And if no such value can be obtained which is order feasible then we can make order feasible by equation (4) and (5). From all the calculated values of the minimum value will give the optimal holding costs. We conclude that the optimal cost depends on demand required. The algorithm has been tested using a numerical example. The results show that the algorithms described in this paper perform well.

We have considered fuzzy nature of ordering cost, holding cost and order quantity. Even though the total cost in this model appears to be slightly higher than that in the classical model, this model is more suitable to real life situations. A trader can achieve his goal by adopting fuzzy nature.

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