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Comparative Study Of Dam Break Model For Different Flow Using Higher Order Method

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***Abstract**:*
In this research we can perform numerical simulation of shallow water (Saint-Venant) equations for dam break flow problem. The shallow water equations are system of hyperbolic partial differential equation. This study presents a comparative analysis of dam-break models under different flow regimes using a higher-order numerical method. The governing equations are solved in conservative form, and the numerical scheme is employed to improve solution accuracy while minimizing numerical diffusion and spurious oscillations. The performance of the proposed method is measured for different flow regimes based on water depth, velocity distribution. The comparative study demonstrates that the higher order-Lax Wendroff method method provides more accurate and stable solutions than the lower-order schemes. The findings indicate that the proposed scheme is an effective and reliable for simulating dam-break problems under different flow conditions and can be applied to flood forecasting, river hydraulics, and hydraulic engineering applications.
Keywords: *Dam-break problem, Shallow Water equations, Saint-Venant equations, Higher-order method, Flow regimes, Numerical simulation.*

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I. Introduction

A Mathematical Model is an abstract model that uses Mathematical language to describe the behavior of a system. Mathematical models are used particularly in the natural science and engineering disciplines (such as physics, biology, and electrical engineering) but also in the social sciences (such as economics, sociology and political science); physicists, engineers, computer scientists, and economists use mathematical models most extensively. We describe shallow water equations as mathematical model. The shallow water equations are system of hyperbolic partial differential equation. Most flows on the surface of the Earths, for examples in rivers, seas and the atmosphere, are Shallow water flows in which the horizontal length and velocity scales of interest are much larger than the vertical one's. The Mathematical formulation of this flows, the so called Shallow water equations, are already known for over a century. Shallow water dam break flow models are used in all kinds of applications such as flood warning system, impact of changes of water system and climate predictions. Dam break problems represent one of the most critical issues in hydraulic engineering due to their sudden release of water and the catastrophic impact on downstream areas. Accurate modeling of dam break flow is essential for flood prediction, risk assessment and the design of protective structures. Fayssal and Mohammed (2010) proposed a new simple finite volume method for the numerical solution of shallow water equations. Bellos et al. (2011) reported a two dimensional numerical methods for dam break problem by using the combination of finite element and finite difference method. Ahmed et al. (2013) developed Godunov type finite volume method for dam break problem. Bagheri and Das (2013) developed an implicit high order compact scheme for shallow water equations with dam break problem. Saiduzzaman & Ray (2013) examined some numerical methods for shallow water equations. Jafar and Samir (2015) have applied implicit method for dam break problem. Rahaman et al. (2017) have studied the finite difference method to simulate the water height and velocity through shallow water equations. Mgdalena et al. (2022) have examined the effect of shape and dimensions of the obstacle on downstream water height. In real life most of the differential equations are too difficult to solve analytically. Generally, analytical solutions are possible using simplifying assumptions. Numerical methods make it possible to obtain realistic solutions without the need for simplifying assumptions. Most non-linear PDEs are too complex and can only be solved using numerical methods. Our governing equations are non-linear system of PDEs. Therefore, there is a demand to investigate numerical methods for solving the governing equations. The proposed scheme is useful to solve to capture flow transition with sufficient nodal points.

Our intention is to investigate mathematical model numerically at different flow. The dam break problem using the shallow water equations is a classical test case in fluid dynamics and numerical methods. It models the sudden release of water due to the failure or removal of a dam using simplified assumptions of flow behavior. We implement the finite difference 2nd order methods for the numerical simulation of shallow water dam break flow equations. We compare the well-known qualitative and realistic phenomena of governing equations at subcritical flow, Subcritical upstream and Supercritical downstream strongly supercritical flow. This study is justified because dam break flows pose serious threats to life, property and infrastructure, making precise modeling a necessity. By comparing methods across different flow regimes, the research will highlight the most suitable approaches for practical applications. The analysis of dam break flow is important to capture spatial and temporal evolution of flood event and safety analysis.

II. Methodology

This study employs the one-dimensional **Shallow Water (Saint-Venant) equations** to simulate dam-break flows under different flow regimes.

Governing equation:

The governing equations are expressed as

$$\frac{\partial u}{\partial t}+\frac{\partial f(u)}{\partial x}=0 \qquad (1)$$

Where $u=\begin{pmatrix} h \\ v \end{pmatrix}$ is the vector of conserved variables. and

The flux vector is $f(u)=\begin{pmatrix} hv \\ \frac{1}{2}v^2+gh \end{pmatrix}$, which represents the transport of mass and momentum.

Mass conservation equation and Momentum equation are given below:

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(hv)=0$$

$$\frac{\partial v}{\partial t}+\frac{\partial}{\partial x}(\frac{1}{2}v^2+gh)=0 \qquad (2)$$

where $h(x,t)$, $v(x,t)$ represents the water depth and the fluid velocity respectively. g is the gravitational acceleration.

Initial and boundary conditions:

Initial water depth:

$$h(x,0)=\begin{cases} h_L & ,x<=l/2 \\ h_R & ,others \end{cases}$$

Initial velocity:

$$v(x,0)=v_0(x);\ a\le x\le b$$

Boundary condition: Neumann boundary

Discretization:

We study numerical methods for SWEs which are system of hyperbolic PDE and for this we investigate finite difference method (FDM). In this section, we present the discretization of the shallow water dam break flow model by finite difference formula, which leads to formulate explicit finite difference methods for the numerical solution of the governing equation as a nonlinear partial differential equation.

To begin with derivation of Lax Wendroff method, Consider the following Taylors series for $u(t+\Delta t,x)$.

$$u(t+\Delta t,x)=u(t,x)+\Delta t\frac{\partial u}{\partial t}+\frac{\Delta t^2}{2}\frac{\partial^2 u}{\partial t^2}+O(\Delta t^3) \qquad (3)$$

Where the time derivatives can be replaced by space derivatives using $u_t+(f(u))_x=0$. This has been done by so called Cauchy Kowalewski technique, which implies $\frac{\partial u}{\partial t}=-\frac{\partial f(u)}{\partial x}$

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial t}(-\frac{\partial f(u)}{\partial x})=\frac{\partial}{\partial x}(-\frac{\partial f(u)}{\partial t})$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}(-f'(u)\frac{\partial u}{\partial t})$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}[f'(u)\frac{\partial f(u)}{\partial x}]$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}(A(u)\frac{\partial f(u)}{\partial x})$$

where $A(u) \equiv \frac{\partial f(u)}{\partial u}$

Substitute the preceding expressions of time derivatives into the Taylors series of $u(t+\Delta t, x)$ to obtain

$$u(t+\Delta t, x) = u(t,x) - \Delta t \frac{\partial f(u)}{\partial x} + \frac{\Delta t^2}{2}\frac{\partial}{\partial x}(A(u)\frac{\partial f(u)}{\partial x}) + O(\Delta t^3)$$

$$\frac{\partial f(u)}{\partial x} = \frac{f(u_{i+1}^n) - f(u_{i-1}^n)}{2\Delta x} + O(\Delta x^2)$$

$$\frac{\partial}{\partial x}(A(u)\frac{\partial f(u)}{\partial x})(t^n, x_i) = \frac{A(u^n_{i+\frac{1}{2}})(f(u_{i+1}^n) - f(u_i^n)) - A(u^n_{i-\frac{1}{2}})(f(u_i^n) - f(u_{i-1}^n))}{\Delta x^2} + O(\Delta x^2)$$

The resulting method is as follows

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{2\Delta x}(f(u_{i+1}^n) - f(u_{i-1}^n)) + \frac{\Delta t^2}{2\Delta x^2}[A_{i+\frac{1}{2}}(f(u_{i+1}^n) - f(u_i^n)) - A_{i-\frac{1}{2}}(f(u_i^n) - f(u_{i-1}^n)) \quad (4)$$

Where $A_{i\pm\frac{1}{2}} = \frac{1}{2}(u_{i\pm1}^n + u_i^n)$

Stability (Courant–Friedrichs–Lewy) condition:

From shallow water equation: $u = \begin{pmatrix} h \\ v \end{pmatrix}$ and $f(u) = \begin{pmatrix} hv \\ \frac{1}{2}v^2 + gh \end{pmatrix}$,

$$A(u) = f'(u) = \begin{pmatrix} v & h \\ g & v \end{pmatrix}$$

Courant–Friedrichs–Lewy $(CFL) = \frac{|f'(u)|\Delta t}{\Delta x}$ (5)

Eigen values are $\lambda_1 = v - \sqrt{gh}$, $\lambda_1 = v + \sqrt{gh}$

Time step: $\Delta t = CFL \frac{\Delta x}{\max(\lambda_1, \lambda_2)}$, where $0 < CFL < 1$ (6)

In the next section, we apply the finite difference 2nd-order Lax wendroff methods for the numerical simulation of shallow water dam break flow equations. The obtained numerical solutions are validated with available analytical solutions. Computational domains with different conditions will be prepared to analyze the water depth and velocity.

III. Results And Discussion

Our governing equations are non-linear system of PDE's. Therefore, there is a demand to investigate numerical methods for solving the governing equations.

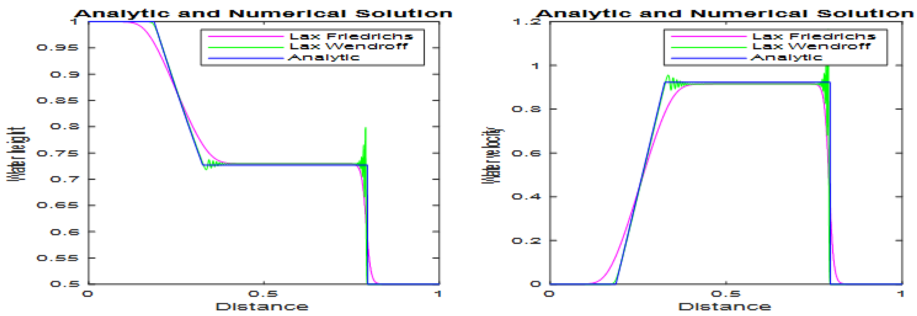

Fig. 1. Analytical and Numerical solution for subcritical flow

Fig.1 presents the comparison between the analytical solution and the numerical solutions obtained using the first-order Lax–Friedrichs scheme and the second-order Lax–Wendroff scheme for the dam-break problem. The elapsed time of Lax-Friedrichs scheme is 2.049 sec and the elapsed time of Lax- Wendroff scheme is1.787 sec. The left panel shows the water height profile, while the right panel illustrates the corresponding water velocity profile. The comparison demonstrates that the second-order Lax–Wendroff scheme achieves higher accuracy than the first-order Lax–Friedrichs scheme by significantly reducing numerical diffusion and providing better resolution of steep gradients. The numerical results are in good agreement with the analytical solution, confirming that the proposed second-order Lax–Wendroff scheme is effective for simulating dam-break flows under different flow regimes conditions. The minor oscillations observed near the discontinuities are a well-known characteristic of higher-order finite difference methods and remain within acceptable limits.

We apply the finite difference 2nd order Lax Wendroff method for the numerical simulation of shallow water dam break flow equations. We start by assuming that in both sides of the dam there are water with corresponding heights heights (i) Initial water height $h_L = 10m$, $h_R = 6m$ for subcritical, (ii) $h_L = 10m$, $h_R = 0.05m$ for Subcritical upstream and Supercritical downstream, (iii) $h_L = 10m$, $h_R = 0m$ for strongly supercritical flow and initial velocity $v(x,t) = 0m/s$, $h_L > h_R$ for three case. We estimate water height and water velocity for the test case of shallow water-dam break flow problem for different flow regimes flow. The results are shown in the following figures.

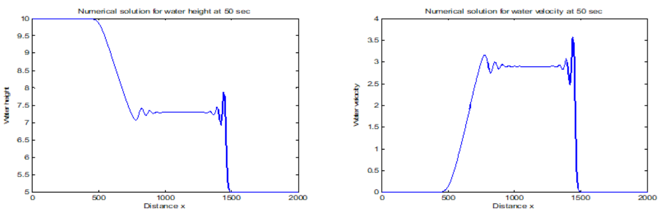

Fig. 2: Water height and water velocity for subcritical flow at 50 sec

Fig. 2 presents the numerical solution of the dam-break problem at **50 sec** obtained using the second-order Lax–Wendroff scheme. The left panel illustrates the water height profile, while the right panel shows the corresponding water velocity distribution. At 50 sec, the initial discontinuity has evolved into a propagating wave moving downstream. The water height decreases smoothly from the upstream reservoir level to an intermediate depth due to the formation of a rarefaction wave. The second-order Lax–Wendroff scheme resolves the shock reasonably well, although small oscillations appear near the discontinuity because of the dispersive nature of the method. Minor oscillations are observed near the discontinuity, but the overall velocity distribution remains physically consistent and agrees with the expected behavior of dam-break flows.

The numerical results demonstrate that the second-order Lax–Wendroff scheme accurately captures the essential characteristics of the dam-break process, including the rarefaction wave, the moving shock front, and the evolution of water depth and flow velocity. Although slight numerical oscillations are present near sharp gradients, the scheme provides satisfactory accuracy and good resolution of the flow features.

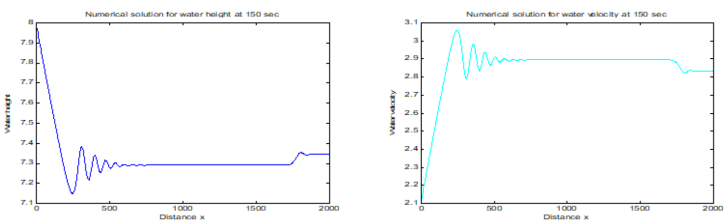

Fig. 3: Water height and water velocity for subcritical flow at 150 sec

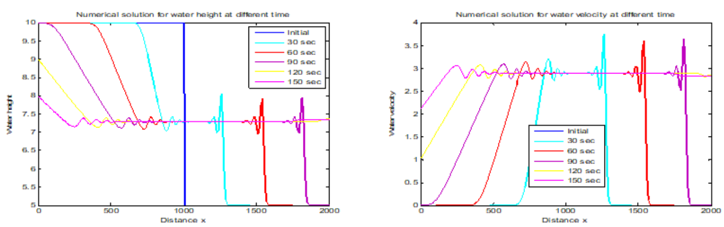

Fig. 4: Water height (Left) and water velocity (Right) for subcritical flow at different time

Fig. 4 illustrates the numerical solutions of the dam-break problem obtained using the second-order Lax–Wendroff scheme at different time levels. The left panel shows the evolution of the water height, while the right panel presents the corresponding water velocity profiles at 0, 30, 60, 90, 120, and 150 seconds. The numerical results demonstrate that the second-order Lax–Wendroff scheme successfully captures the temporal evolution of both water depth and flow velocity during the dam-break process. The method accurately predicts wave propagation and preserves the main characteristics of the shallow water flow, although minor oscillations are observed near sharp gradients.

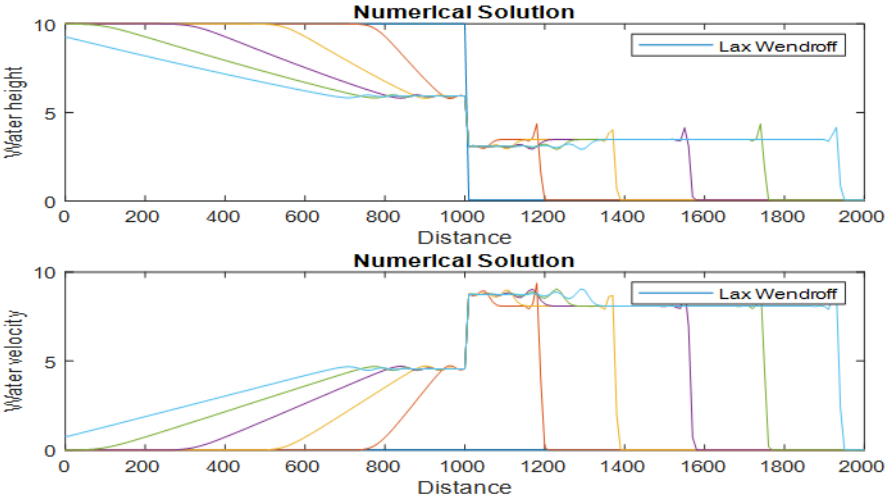

Fig. 5. Water height and water velocity for subcritical upstream and supercritical downstream flow.

The results demonstrate the formation of a rarefaction wave in the upstream region and a shock wave propagating downstream, which are consistent with the analytical characteristics of the shallow water equations. The upper panel illustrates the variation of the water height at different time levels. The lower panel shows the corresponding velocity profiles. The computed water height and velocity profiles remain stable throughout the simulation, although slight numerical oscillations can be observed near the discontinuities, which is a well-known characteristic of the classical Lax–Wendroff scheme. The numerical solution shows good agreement with the expected physical behavior of dam-break flows involving mixed subcritical and supercritical flow regimes.

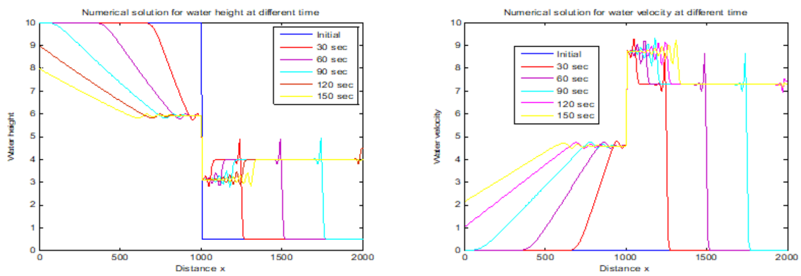

Fig. 6: Water height (Left) and water velocity (Right) for strongly supercritical flow at different time

Fig. 6 illustrates the numerical solutions of the dam-break problem obtained using the second-order Lax–Wendroff scheme at different time levels. The left panel shows the evolution of the water height, while the right panel presents the corresponding water velocity profiles at 0, 30, 60, 90, 120, and 150 seconds. We have seen that water height of the channel increases gradually towards the downstream direction and decreases gradually towards the upstream direction in Fig.4, Fig. 5 and Fig.6. we observed in Fig.4, Fig. 5 and Fig. 6 that water level drops in the upstream boundary and water level rises in the downstream boundary. We have also seen in figure that water velocity of the channel increases gradually towards in both directions after the dam break. We observed in figure that water velocity rises in both boundaries after the dam break. Therefore, the above realistic phenomenon is well described by our implementation.

IV. Conclusion

This study presented a numerical investigation of the one-dimensional shallow water (Saint-Venant) equations for the dam-break problem using a second-order Lax–Wendroff scheme. We compute water velocity, water height and the position of water front for each time step. Comparisons are carried out only for specific condition with respect to velocity and water height. 2nd order Lax Wendroff method is applied to solve the shallow water equations with dam break flow problem which has been presented. We have implemented the numerical scheme to simulate water height and water velocity for different flow regimes through the governing equations. The numerical results showed that the proposed scheme successfully captured the essential flow characteristics, including rarefaction waves, shock waves, and the temporal evolution of water depth and flow velocity. Comparisons with the analytical solution demonstrated that the second-order Lax–Wendroff scheme provides higher accuracy and significantly reduces numerical diffusion compared with the first-order Lax–Friedrichs method. The second-order Lax–Wendroff scheme is an efficient and reliable numerical method for solving shallow water dam-break problems. Future work will extend the present study to dam-break flows over variable bed topography.

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Conflicts of Interest: The author declares no conflict of interest.

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