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# On Self-Centered Fuzzy Cycles In Fuzzy Graphs

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**Abstract**

*In this paper, we proved that a complete, self-centered fuzzy graph can be embedded in a connected self-centered fuzzy graph. The criteria for a fuzzy cycle to be self-centered as well as its fuzzy radius is obtained. We also discussed some self-centered fuzzy cycles whose complements are also self-centered and proved that the fuzzy radius of the fuzzy cycle and its complement are same.*

***Keywords:*** *fuzzy distance, fuzzy cycle, fuzzy eccentricity, fuzzy radius, self-centered fuzzy graph*

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## I. Introduction And Preliminaries

Many authors have explored the characteristics of fuzzy graphs and their applications in various fields [1],[2]. The concept of distance in fuzzy graphs has also been widely investigated [3],[7]. Here, we established that a complete, self-centered fuzzy graph can be embedded in a self-centered fuzzy graph. The criteria for a self-centered fuzzy cycle are discussed, and the fuzzy radius of the fuzzy cycle is also obtained. There are fuzzy cycles whose complements are also fuzzy cycles and also, we established that both are self-centered and proved that the fuzzy radius of such fuzzy cycles and their complements are same.

**Definition 1.1. [4]** A fuzzy graph is an ordered triplet $G:(V,\sigma,\mu)$ where $V$ is a finite, non-empty vertex set, $\sigma: V \to [0,1]$ is a vertex membership function and $\mu : V \times V \to [0,1]$ is an edge membership function satisfying $\mu(u,v) \leq \sigma(u) \wedge \sigma(v),\ \forall\, u,v \in V.$ The relation $\mu$ is reflexive and symmetric.
**Definition 1.2. [5]** A fuzzy graph $H:(V,\tau,\upsilon)$ is called a partial fuzzy subgraph of a fuzzy graph $G:(V,\sigma,\mu)$, if $\tau(u) \leq \sigma(u)$, for all $u \in \tau^*$ and $\upsilon(u,v) \leq \mu(u,v)$, for all $u,v \in \upsilon^*$. $H:(V,\tau,\upsilon)$ a fuzzy subgraph of $G:(V,\sigma,\mu)$ if $\tau(u) = \sigma(u)$, for all $u,v \in \upsilon^*$ and $\upsilon(u,v) = \mu(u,v)$, for all $u,v \in \upsilon^*$.
**Definition 1.3. [6]** A weakest arc in $G:(V,\sigma,\mu)$ is an arc having minimum membership value among all arcs. A Path $P$ of length $n$ consists of a sequence of distinct vertices $u_0, u_1, u_2 \ldots u_n$ such that $\mu(u_{i-1},u_i) > 0$ for every $, i = 1,2,3,\ldots..n.$ The strength of this path is determined by the membership value of its weakest arc. If $u_0 = u_n$ and $n \geq 3$, then the path $P$ forms a cycle. Such a cycle is termed as a fuzzy cycle, if it includes more than one weakest arc. A fuzzy graph $G:(V,\sigma,\mu)$ is complete when, for all vertices $u,v \in V$, $\mu(u,v) = \sigma(u) \wedge \sigma(v)$.
**Definition 1.4. [6]** The complement of $G:(V,\sigma,\mu)$ is denoted by $G^c:(V,\sigma^c,\ \mu^c)$ where $\sigma^c = \sigma$ and $\mu^c = \wedge \{\sigma(x),\sigma(y)\} - \mu(x,y)$.

## II. Main Results

**Definition 2.1. [7]** Let $G:(V,\sigma,\mu)$ be a fuzzy graph. The fuzzy distance between two vertices $u$ and $v$ in $G$ is defined as

$$d_f(u,v) = \wedge \sum \{\wedge\left(\sigma(u),\sigma(v)\right) * \mu(u,v)\}$$

where $\wedge$ represents the minimum and $*$ represents the ordinary product. $d_f(u,v)$ satisfies the properties of a metric.
**Definition 2.2. [7]** Let $G:(V,\sigma,\mu)$ be a fuzzy graph. The fuzzy eccentricity $e_f(u)$ of the vertex $u$ in $G$ is defined as $e_f(u) = max\ \{d_f(u,v) : v \in V\}$. Fuzzy radius $r_f(G)$ of $G$ is the minimum of the fuzzy eccentricities of all the vertices in $G$. Fuzzy diameter $d_f(G)$ of $G$ is the maximum of the fuzzy eccentricities of all the vertices in $G$. If $e_f(u) = r_f(G)$, then the vertex $u$ is a fuzzy central vertex of $G$. If every vertex in $G$ is a fuzzy central vertex, then $G$ is self-centered. If $e_f(u) = d_f(G)$, then $u$ is a fuzzy peripheral vertex of $G$.

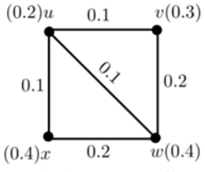


**Figure 1.** A Fuzzy Graph $G$.

**Example 2.3** Consider the fuzzy graph $G$ in figure 1. The fuzzy distance between two vertices is explained below.

$$d_f(u,v) = \wedge\begin{cases} 0.2 * 0.1 \\ (0.2 * 0.1) + (0.3 * 0.2) \\ (0.2 * 0.1) + (0.4 * 0.2) + (0.3 * 0.2) \end{cases} \Rightarrow d_f(u,v) = 0.02$$

$$d_f(u,w) = \wedge\begin{cases} 0.2 * 0.1 \\ (0.2 * 0.1) + (0.3 * 0.2) \\ (0.2 * 0.1) + (0.4 * 0.2) \end{cases} \Rightarrow d_f(u,w) = 0.02$$

$$d_f(u,x) = \wedge\begin{cases} 0.2 * 0.1 \\ (0.2 * 0.1) + (0.4 * 0.2) \\ (0.2 * 0.1) + (0.3 * 0.2) + (0.4 * 0.2) \end{cases} \Rightarrow d_f(u,x) = 0.02$$

$$d_f(v,w) = \wedge\begin{cases} 0.3 * 0.2 \\ (0.2 * 0.1) + (0.2 * 0.1) \\ (0.2 * 0.1) + (0.2 * 0.1) + (0.4 * 0.2) \end{cases} \Rightarrow d_f(v,w) = 0.04$$

$$d_f(v,x) = \wedge\begin{cases} (0.2 * 0.1) + (0.2 * 0.1) \\ (0.3 * 0.2) + (0.4 * 0.2) \\ (0.3 * 0.2) + (0.2 * 0.1) + (0.2 * 0.1) \\ (0.2 * 0.1) + (0.2 * 0.1) + (0.4 * 0.2) \end{cases} \Rightarrow d_f(v,x) = 0.04$$

$$d_f(w,x) = \wedge\begin{cases} (0.4 * 0.2) \\ (0.2 * 0.1) + (0.2 * 0.1) \\ (0.3 * 0.2) + (0.2 * 0.1) + (0.2 * 0.1) \end{cases} \Rightarrow d_f(w,x) = 0.04$$

Here, $e_f(u) = 0.02,\ e_f(v) = 0.04,\ e_f(w) = 0.04,\ e_f(x) = 0.04,\ r_f(G) = 0.02,\ d_f(G) = 0.04$

**Theorem 2.4** Every complete self-centred fuzzy graph $H$ can be embedded in a connected self-centered fuzzy graph $G$.

**Proof:** Consider the complete self centered fuzzy graph $H$ with $n$ vertices $v_1, v_{2,} \ldots\ldots, v_n$, such that $\sigma(v_i) = \begin{cases} p, & i \text{ is odd} \\ q, & i \text{ is even}, \end{cases}$ $p < q$ and $\mu(v_i, v_j) = p$, for all $i, j$ . Since $H$ is complete, all the vertices are connected to each other, $d_f(v_i, v_j)$ is same for all $i, j$ and therefore, $e_f(v_i) = p.p = p^2 = r_f(H)$, for all $i$.

Construct the new fuzzy graph $G$ by adding four new vertices $u_1, u_{2,} u_3, u_4$ to the fuzzy graph $H$ such that the fuzzy graph $G$ is in the sequential order $u_1 - u_2 - H - u_3 - u_4$. Join the vertex $u_1$ with $u_2$ and the vertex $u_3$ with $u_4$. Also join the vertices $u_1, u_{2,} u_3, u_4$ to each of the vertices $v_1, v_{2,} \ldots\ldots v_n$ in $H$. Put $\sigma(u_i) = p$ for all $i$, $\mu(u_1, u_2) = \mu(u_3, u_4) = \frac{p}{2}$, $\mu(u_i, x) = \frac{p}{2}$ for all $x \in H$ and for all $i$. Then clearly, we have $e_f(u_i) = e_f(v_i) = p^2 = r_f(G)$, for all $i$. Thus $G$ is self centered.

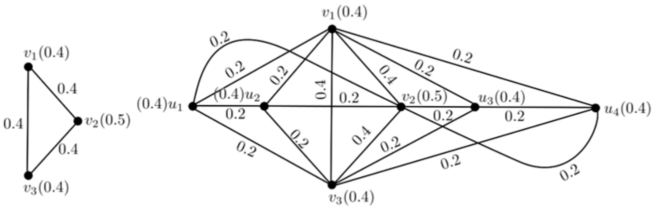


Figure 2. The Complete fuzzy graph $K_3$ and the fuzzy graph $G$

**Example 2.5** In figure 2, we have the complete self centered fuzzy graph $K_3$ with vertices $v_1, v_{2,} v_3$ such that $\sigma(v_i) = \begin{cases} 0.4, & i = 1,3 \\ 0.5, & i = 2 \end{cases}$ and $\mu(v_i, v_j) = 0.4$, for all $i, j$ .

Here, $e_f(v_i) = 0.16 = r_f(K_3)$, for all $i$.

Construct the fuzzy graph $G$ by adding four new vertices $u_1, u_{2,} u_3, u_4$ and put $\sigma(u_i) = 0.4$ for all $i$ and $\mu(u_1, u_2) = \mu(u_3, u_4) = 0.2$, $\mu(u_1, x) = \mu(u_2, x) = \mu(u_3, x) = \mu(u_4, x) = 0.2$ for all $x \in K_3$. Then $e_f(u_i) = e_f(v_i) = 0.16 = r_f(G)$, for all $i$ and thus $G$ is self centered.

**Theorem 2.6** Consider the fuzzy graph $G: (V, \sigma, \mu)$ with $G^* \approx C_n$, a cycle on $n$ vertices $v_1, v_2, \ldots v_n$. Then, for $0 < p < q \leq 1,\ 0 < a < b \leq 1,\ G$ is self centered in the following cases.

**Case 1.** When $n = 4k$ or $4k + 2,\ k = 1,2,3, \ldots..$

**(i)** $\sigma(v_i) = \begin{cases} p, & i \text{ is odd} \\ q, & i \text{ is even}, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} a, & i \text{ is odd} \\ b, & i \text{ is even}, \end{cases}$ $\mu(v_n, v_1) = b$

**(ii)** $\sigma(v_i) = \begin{cases} p, & i \text{ is odd} \\ q, & i \text{ is even}, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} b, & i\ is\ odd \\ a, & i\ is\ even, \end{cases}$ $\mu(v_n, v_1) = a$

**(iii)** $\sigma(v_i) = \begin{cases} q, & i\ is\ odd \\ p, & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} a, & i\ is\ odd \\ b, & i\ is\ even, \end{cases}$ $\mu(v_n, v_1) = b$

**(iv)** $\sigma(v_i) = \begin{cases} q, & i\ is\ odd \\ p & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} b, & i\ is\ odd \\ a, & i\ is\ even, \end{cases}$ $\mu(v_n, v_1) = a$

**Case 2.** When $n = 4k - 1, k = 1,2,3, \ldots..$

$$\sigma(v_i) = \begin{cases} p, & i\ is\ odd \\ q, & i\ is\ even, \end{cases}$$

$\mu(v_i, v_{i+1}) = \begin{cases} b, & i\ is\ odd \\ a, & i\ is\ even, \end{cases}$ $\mu(v_n, v_1) = b$

**Case 3.** When $n = 4k + 1, k = 1,2,3, \ldots..$

**(i)** $\sigma(v_i) = \begin{cases} p, & i\ is\ odd \\ q, & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} a, & i\ is\ odd \\ b, & i\ is\ even, \end{cases}$ $\mu(v_n, v_1) = a$

**(ii)** $\sigma(v_i) = \begin{cases} q, & i\ is\ odd \\ p, & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} a, & i\ is\ odd \\ b, & i\ is\ even, \end{cases}$ $\mu(v_n, v_1) = a$

The fuzzy radius $r_f(G) = \begin{cases} kp(a+b), \ n = 4k \ \text{ or } \ n = 4k+1, k = 1,2,3, \ldots... \\ p\{k(a+b) - a\}, \ n = 4k-1, k = 1,2,3, \ldots\ldots\ldots.... \\ p\{k(a+b) + a\}, \ n = 4k+2, \ k = 1,2,3, \ldots\ldots\ldots... \end{cases}$

**Example 2.7**

**Case 1**. Let $p = 0.4, \ q = 0.5, \ a = 0.2, \ b = 0.3$

**(i)** when $n = 4k$ or $4k + 2, \ k = 1,2.$

Consider the fuzzy cycle $C_n$ with vertices $v_i, i = 1,2,3, \ldots\ldots..n$.

Let $\sigma(v_i) = \begin{cases} p = 0.4, & i\ is\ odd \\ q = 0.5, & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} a = 0.2\ , & i\ is\ odd \\ b = 0.3, & i\ is\ even, \end{cases}$ $\mu(v_n, v_1) = b = 0.3.$

When $n = 4k, \ k = 1, \ e_f(v_i) = 0.2 = r_f(C_4)$ for all $i$ and hence $C_4$ is self centered.

Also, when $n = 4k, \ \ k = 2, e_f(v_i) = 0.4 = r_f(C_8)$ for all $i$ and hence $C_8$ is self centered.

When $n = 4k + 2, \ k = 1, \ e_f(v_i) = 0.28 = r_f(C_6)$ for all $i$ and hence $C_6$ is self centered.

Also, when $n = 4k + 2, \ k = 2, \ e_f(v_i) = 0.48 = r_f(C_{10})$ for all $i$ and hence $C_{10}$ is self centered.

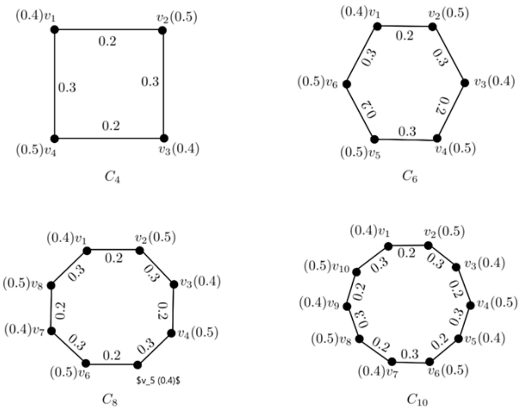


**Figure 3.** The fuzzy cycles $C_4$, $C_6$, $C_8$ and $C_{10}$

**(ii)** Let $\sigma(v_i) = \begin{cases} p = 0.4, & i\ is\ odd \\ q = 0.5\ , & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} b = 0.3, & i\ isodd \\ a = 0.2, & i\ is\ even, \end{cases}$ $\mu(v_n, v_1) = a = 0.2.$

Then $r_f(C_4) = 0.2,\ r_f(C_8) = 0.4, r_f(C_6) = 0.28,\ r_f(C_{10}) = 0.48$
Here, $C_4$, $C_8$, $C_6$ and $C_{10}$ are self centered.

**(iii)** Let $\sigma(v_i) = \begin{cases} q = 0.5, & i\ is\ odd \\ p = 0.4, & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} a = 0.2, & i\ is\ odd \\ b = 0.3, & i\ is\ even, \end{cases}\ \mu(v_n, v_1) = b = 0.3.$

Then $r_f(C_4) = 0.2,\ r_f(C_8) = 0.4,\ r_f(C_6) = 0.28,\ r_f(C_{10}) = 0.48$
Here, $C_4$, $C_8$, $C_6$ and $C_{10}$ are self centered.

**(iv)** Let $\sigma(v_i) = \begin{cases} q = 0.5, & i\ is\ odd \\ p = 0.4, & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} b = 0.3, & i\ is\ odd \\ a = 0.2, & i\ is\ even, \end{cases}\ \mu(v_n, v_1) = a = 0.2.$

Then $r_f(C_4) = 0.2,\ r_f(C_8) = 0.4,\ r_f(C_6) = 0.28,\ r_f(C_{10}) = 0.48$
Here, $C_4$, $C_8$, $C_6$ and $C_{10}$ are self centered.

**Case 2.** When $n = 4k - 1$, $k = 1,2$

Let $\sigma(v_i) = \begin{cases} p = 0.4, & i\ is\ odd \\ q = 0.5, & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} b = 0.3, & i\ is\ odd \\ a = 0.2, & i\ is\ even, \end{cases}\ \mu(v_n, v_1) = b = 0.3.$

When $k = 1,\ e_f(v_i) = 0.12 = r_f(C_3)$, for all $i$ and hence $C_3$ is self centered.
Also, when $k = 2,\ e_f(v_i) = 0.32 = r_f(C_7)$, for all $i$ and hence $C_7$ is self centered.

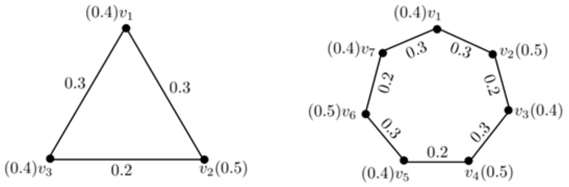


**Figure 4.** The fuzzy cycles $C_3$ and $C_7$

**Case 3.** When $n = 4k + 1$, $k = 1,2$

**(i)** Let $\sigma(v_i) = \begin{cases} p = 0.4, & i\ is\ odd \\ q = 0.5, & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} a = 0.2, & i\ is\ odd \\ b = 0.3, & i\ is\ even, \end{cases}\ \mu(v_n, v_1) = a = 0.2.$

When $k = 1,\ e_f(v_i) = 0.2 = r_f(C_5)$, for all $i$ and hence $C_5$ is self centered.
Also, when $k = 2,\ e_f(v_i) = 0.4 = r_f(C_9)$, for all $i$ and hence $C_9$ is self centered.

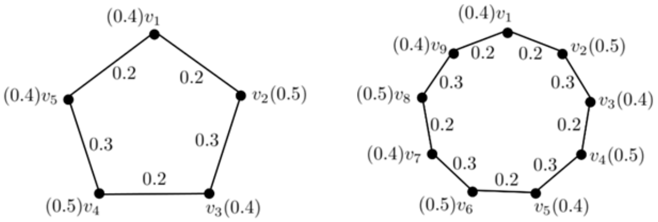


**Figure 5.** The fuzzy cycles $C_5$ and $C_9$

**(ii)** Let $\sigma(v_i) = \begin{cases} q = 0.5, & i\ is\ odd \\ p = 0.4, & i\ is\ even, \end{cases}$

$\mu(v_i, v_{i+1}) = \begin{cases} a = 0.2, & i\ is\ odd \\ b = 0.3, & i\ is\ even, \end{cases}\ \mu(v_n, v_1) = a = 0.2.$

Then, $r_f(C_5) = 0.2, r_f(C_9) = 0.4$. Here, $C_5$ and $C_9$ are self centered.

**Theorem 2.8** Consider the fuzzy graph $G: (V, \sigma, \mu)$ with $G^* \approx C_n$, a cycle on $n$ vertices $v_1, v_2, \ldots\ldots v_n$. Then $G$ is self centered with $\sigma(v_i) = p,\ 0 < p \leq 1,\ \mu(v_i, v_j) = a,\ 0 < a \leq 1$ for all $i, j$.

The fuzzy radius $r_f(G) = \begin{cases} 2kpa, n = 4k \text{ or } n = 4k + 1, k = 1,2,3, \ldots\ldots \\ (2k - 1)pa, n = 4k - 1, k = 1,2,3, \ldots\ldots\ldots\ldots \\ (2k + 1)pa, n = 4k + 2,\ k = 1,2,3, \ldots\ldots\ldots\ldots \end{cases}$

**Example 2.9** Let $\sigma(v_i) = p = 0.4$ and $\mu(v_i, v_j) = a = 0.2$ for all $i, j$

**Case 1.** When $n = 4k \; or \; 4k + 2, \; k = 1,2.$
When $n = 4k, k = 1$, $e_f(v_i) = 0.16 = r_f(C_4)$, for all $i$ and hence $C_4$ is self centered.
Also, when $n = 4k$, $k = 2$, $e_f(v_i) = 0.32 = r_f(C_8)$ for all $i$ and hence $C_8$ is self centered.
when $n = 4k + 2, \; k = 1, \; e_f(v_i) = 0.24 = r_f(C_6)$ for all $i$ and hence $C_6$ is self centered.
Also, when $n = 4k + 2, \; k = 2, \; e_f(v_i) = 0.40 = r_f(C_{10})$ for all $i$ and hence $C_{10}$ is self centered.

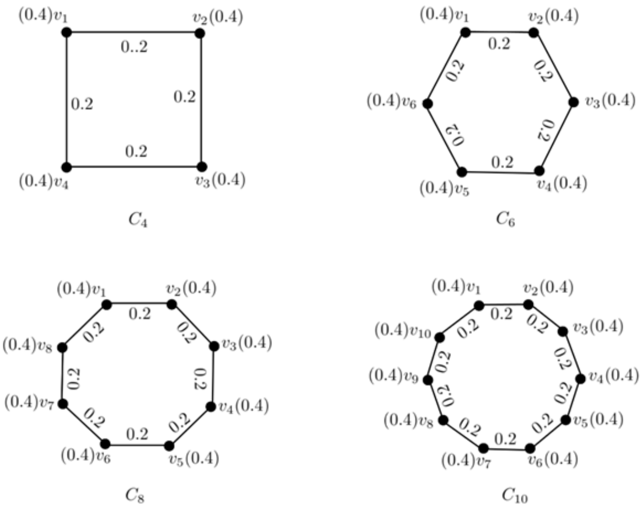


**Figure 6.** The fuzzy cycles $C_4$, $C_6$, $C_8$ and $C_{10}$

**Case 2.** when $n = 4k - 1, \; k = 1,2.$
When $k = 1$, $e_f(v_i) = 0.08 = r_f(C_3)$, for all $i$ and hence $C_3$ is self centered.
Also, when $k = 2$, $e_f(v_i) = 0.24 = r_f(C_7)$ for all $i$ and hence $C_7$ is self centered.
**Case 3.** when $n = 4k + 1, \; k = 1,2.$
When $k = 1$, $e_f(v_i) = 0.16 = r_f(C_5)$, for all $i$ and hence $C_5$ is self centered.
Also, when $k = 2, \; e_f(v_i) = 0.32 = r_f(C_9)$ for all $i$ and hence $C_9$ is self centered.
**Theorem 2.10** Consider the fuzzy graph $G: (V, \sigma, \mu)$ with $G^* \approx C_n$, a cycle on $n$ vertices, $n = 3,4,5$ whose complements are also fuzzy cycles by suitably choosing the membership value $p$ of the vertices and $a, b$ of the edges. Let $\sigma(v_i) = p$ for all $i$ and $\mu(v_i, v_{i+1}) = \begin{cases} a, & i \text{ is odd} \\ b, & i \text{ is even,} \end{cases}$ $\mu(v_n, v_1) = b, \; 0 < a < b \leq 1$. Then $G$ and $G^c$ are self- centered. The fuzzy radius $r_f(G) = r_f(G^c) = \begin{cases} k(a + b), & n = 4k \text{ or } n = 4k + 1, k = 1 \\ k(a + b) - a, & n = 4k - 1, \quad k = 1 \end{cases}$

**Example 2.11**

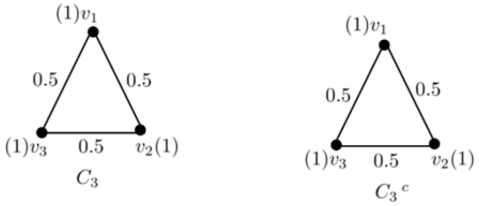


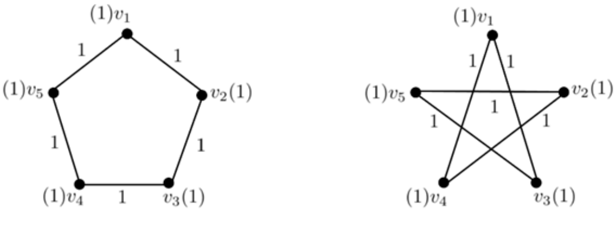
**Figure 7.** The fuzzy cycles $C_3$ and ${C_3}^c$

**Case 1.** For the fuzzy cycle $C_3$ in figure 7 with vertices $v_i, i = 1,2,3.$
Let $\sigma(v_i) = p = 1$, for all $i$,
$\mu(v_i, v_{i+1}) = a = b = 0.5, i = 1,2$ and $\mu(v_3, v_1) = 0.5.$
Then, $e_f(v_i) = 0.5 = r_f(C_3)$ for all $i$ and hence $C_3$ is self centered.
Here, ${C_3}^c$ is also a fuzzy cycle.
In ${C_3}^c$, $\sigma^c(v_i) = p = 1$, for all $i$,
$\mu^c(v_i, v_{i+1}) = 0.5, i = 1,2$ and $\mu^c(v_3, v_1) = 0.5.$
Then, $e_f(v_i) = 0.5 = r_f({C_3}^c)$ for all $i$ and hence ${C_3}^c$ is self centered.
Thus, $r_f(C_3) = r_f({C_3}^c) = 0.5.$

$C_4$ $\quad$ ${C_4}^c$

**Figure 8.** The fuzzy cycles $C_4$ and ${C_4}^c$

**Case 2.** Consider the fuzzy cycle $C_4$ in figure 8 with vertices $v_i, i = 1,2,3,4$.
Let $\sigma(v_i) = p = 1$ for all $i$,
$\mu(v_i, v_{i+1}) = \begin{cases} a = 0.5, i = 1,3 \\ b = 1. \quad i = 2 \end{cases}$ and $\mu(v_4, v_1) = b = 1$
Then $e_f(v_i) = 1.5 = r_f(C_4)$ for all $i$ and hence $C_4$ is self centered.
Here, ${C_4}^c$ is also a fuzzy cycle.
In ${C_4}^c$, $\sigma^c(v_i) = p = 1$ for all $i$,
$\mu^c(v_i, v_{i+1}) = 0.5, i = 1,3$ and $\mu^c(v_i, v_{i+2}) = 1, i = 1,2$
Then, $e_f(v_i) = 1.5 = r_f({C_4}^c)$ for all $i$ and hence ${C_4}^c$ is self centered.
Thus, $r_f(C_4) = r_f({C_4}^c) = 1.5$.



$C_5$ $\quad$ ${C_5}^c$

**Figure 9.** The fuzzy cycles $C_5$ and ${C_5}^c$

**Case 3.** Consider the fuzzy cycle $C_5$ in figure 9 with vertices $v_i, i = 1,2,3,4,5$.
Let $\sigma(v_i) = p = 1$ for all $i$,
$\mu(v_i, v_{i+1}) = a = b = 1$ for all $i$ and $\mu(v_5, v_1) = 1$
Then, $e_f(v_i) = 2 = r_f(C_5)$ for all $i$ and hence $C_5$ is self centered.
Here, ${C_5}^c$ is also a fuzzy cycle.
In ${C_5}^c$, $\sigma^c(v_i) = p = 1$ for all $i$.
$\mu^c(v_i, v_{i+2}) = 1, i = 1,2,3.$ $\mu^c(v_i, v_{i+3}) = 1, i = 1,2$
Then $e_f(v_i) = 2 = r_f({C_5}^c)$ for all $i$ and hence ${C_5}^c$ is self centered.
Thus, $r_f(C_5) = r_f({C_5}^c) = 2$.

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