An Analytical Study Of A Deteriorating EOQ Inventory System With Seasonal Demand And Preservation Technology Under Inflationary Conditions

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Abstract
This study proposes a novel EOQ-based inventory model for deteriorating items under seasonal demand Variation, inflation, and investment in preservation technology Demand is modeled time- dependent Seasonal function while deterioration is treated as controllable through preservation efforts, Creating a cost-benefit trade-off between technology investment and spoilage reduction. The time value of money and inflation are incorporated using a discounted Cash flow approach to reflect realistic economic condition. A Comprehensive total Cost function is formulated including Ordering, holding, deterioration, preservation investment and inflation-adjusted costs. The objective is to determine the optimal replenishment Cycle and preservation inventory cost. Analytical and numerical result indicate that preservation technology significantly under fluctuating demand and rising economic Conditions. The model establishes a bridge between technological intervention and classical inventory theory, enabling adaptive and economically resilient policies for deteriorating items in dynamic market condition.
Keywords: *Perishable inventory System, Seasonal demand, Preservation technology optimization, Inflation-adjusted costs, EOQ model, Deterioration control*

I.   Introduction

Inventory management serves as a critical Component of operational planning in Supply chains, directly influencing profitability, Service level, and resource utilization In Competitive and uncertain markets, firms must balance ordering, holding, and shortage costs while responding to dynamic Customer demand. This challenge becomes even more complex when products are Subject to physical decay, financial fluctuations, and behavior uncertainty. As a result, traditional inventory assumptions are Often inadequate for representing modern logistics environments. Inventory Controls for perishable and time-Sensitive products has therefore attached considerable attention due to its direct impact on cost efficiency and service performance. Unlike Classical inventory system that assume Stable demand and non-decaying items, many real-life goods such as food products, medicines, Chemicals, and fashionable Commodities lose value over time. Early foundational work by Ghare and Schrader [7] first incorporated deterioration into inventory modeling, demonstrating how decay influences replenishment decisions. Later Covert and Philip [3] extended this Concept by analyzing Variable deterioration rates, laying the ground work for modern perishable inventory theory.

As market evolved, researcher recognized that demand is rarely constant. Studies such as Donaldson [4] and Silver and Meal [19] emphasized the importance of time-Vary demand, while later works incorporated seasonal demand patterns to better represent industries where Consumption fluctuates periodically. These models showed that ignoring Seasonal effect may lead to suboptimal replenishment policy and increased holding or shortage costs. Another major development in inventory research has been the consideration of inflation and the time value of money with rising price levels affecting purchasing and holding Costs, Buzacott [2] and Mishra[14] introduced inflation-adjusted inventory models using discounted Cash flow techniques.

Subsequent Contributions, including Wee and Law [11], highlighted that inflation significantly alters optional order Cycle, especially for long planning horizons. However, many of these Studies did not incorporate deterioration control or seasonal demand simultaneously. In recent year, attention has shifted toward and preservation or protective technology investment as a means of reducing deterioration. Researcher such as Hsu et al. [9] and Yang [21] modeled deterioration as a controllable parameter, demonstrating that investment in preservation can reduce spoilage and improve profitability. These study established a Cost-benefit trade-off between technology expenditure and reduction in decay, but they typically assumed constant demand and ignored inflationary influences.

Customer behavior during shortages has also been extensively studied. Classical works by Nahmias [15] and Ravid [17] considered complete or partial backlogging based on waiting time. Later, Wu et al. and Singh and Sharma [18] introduced price and time-dependent backlogging, Showing that Customer willingness to wait decreases when waiting time increases or selling price ricers. Despite these improvements, most separately, without integrating them into a hybrid framework a long side deterioration and inflation. Storage capacity constraints from another practical concern in inventory System. Studies such as Mandal and Pal [13] and Khanra et al [10] examined limited storage space and its impact on order quantity decisions. However, these works generally did not incorporate preservation investment or inflation-adjusted cost structure. A Careful review of the existing literature reveals that while significant progress has been made in studying deterioration Seasonal demand, preservation technology, inflation, and Customer backlogging behavior, these factor are often analyzed independently very limited research has developed a unified analytical framework that simultaneously integrated seasonal demand variation, Controllable deterioration through preservation investment, inflationary effect, and hybrid backlogging.

To address this gap, the present study develops a Comprehensive EOQ-type inventory model that captures these interacting Complexities within a single optimization framework. The objective is to determine optimal replenishment and preservation Strategies that minimize total cost while reflecting realistic economic and market conditions.

Assumption:
1. The inventory System considers a single deteriorating item
2. The planning horizon is infinite, and replenishment is instantaneous with zero lead time.
3. The demand rat is seasonal and time dependent, expressed as $D(t) = a + b\sin(\omega t), \quad a > b > 0$
4. Items in stock are subject to constant deterioration.
5. Preservation technology is adopted to reduce the effective deterioration robe.
6. The inflation rate is constant, and all costs are adjusted by the time value of money.
7. Shortage are not permitted; hence backlogging is not considered in this model
8. Replenishment occurs exactly when inventory level reaches Zero.
9. Ordering Cost, holding Cost, purchasing cost, and preservation technology cost are known and deterministic
10. The objective is to minimize the total average inventor Cost per unit time.

Notation
$I(t)$ = Inventory level at time t
$D(t)$ = Seasonal demand rate
a = Average demand rate
b = Seasonal demand amplitude
ω = Seasonal frequency
θ = Deterioration rate without preservation
θ_p = Effective deterioration rate with preservation technology
C_0 = Ordering cost per Cycle
C_h = Holding cost per unit per unit time
C_p = Purchasing Cost per unit
C_{pt} = Preservation technology Cost
r = Inflation rate
T = Replenishment Cycle length
Q = order quantity
TC = Total average cost per unit time.

Analytical Expression:
Context I: During the inventory accumulation stage $[0, t_1]$ the inventory occur due to demand Fulfillment and deterioration, consequently, it is modeled by the differential equation,

$$\frac{dI(t)}{dt} = -D(t), \quad 0 \le t \le t_1 \qquad (1)$$

To boundary condition $I(0) = I_0$

The solution of eqn (1) is

$$I(t) = I_0 - at + \frac{b}{\omega}(\cos(\omega t) - 1) \qquad (2)$$

Consequently, the overall cost per replenishment cycle is composed in following ways:-

1. Holding Cost Per Cycle

$$H_c = C_h \int_0^{t_1} I(t)\, dt$$

$$= C_h \int_0^{t_1}[I_0 - at + \frac{b}{\omega}(\cos(\omega t) - 1)dt]$$

$$= C_h[\int_0^{t_1} I_0 dt - \int_0^{t_1} at\, dt + \frac{b}{\omega}\int_0^{t_1}(\cos \omega t - 1)\, dt$$

$$= C_h[I_0 t_1 - \frac{at_1^2}{2} + \frac{b}{\omega}\left(\frac{\sin \omega t_1}{\omega} - t_1\right)] \qquad = C_h[I_0 t_1 - \frac{at_1^2}{2} + \frac{b}{\omega}\sin \omega t_1 - \frac{b}{\omega}t_1$$

2. Deteriorating Cost

$$D_c = C_d \int_0^{t_1} \theta I(t)\, dt$$

$$= C_d \int_0^{t_1} \theta \left[I_0 - at + \frac{b}{\omega}(\cos(\omega t) - 1)dt \right. \qquad = C_d \theta \int_0^{t_1}[I_0 - at + \frac{b}{\omega}\cos(\omega t) - 1]d$$

$$= C_d\theta \left[\int_0^{t_1} I_0\, dt - \int_0^{t_1} at\, dt + \frac{b}{\omega}\int_0^{t_1}(\cos(\omega t) - \right.$$

$$\left. 1\right) dt$$

$$= C_d\theta[I_0 t_1 - \frac{at_1^2}{2} + \frac{b}{\omega}\sin \omega t_1 - \frac{b}{\omega}t_1$$

$$= C_d\theta[I_0 t_1 - \frac{at_1^2}{2} + \frac{b}{\omega^2}\sin \omega t_1 - \frac{b}{\omega}t_1]$$

3. Purchasing Cost

$$P_c = C_p \int_0^T D(t)\, dt$$

$$= C_p \int_0^T a + b\sin(\omega t)\, dt \qquad = C_p[\int_0^T a\, dt + \int_0^T b\sin(\omega t)\, dt]$$

$$= C_p[aT + \frac{b}{\omega}(1 - \cos(\omega t)] \qquad = C_p[aT + \frac{b}{\omega}(1 - \cos(\omega t)]$$

4. Ordering Cost

$$O_c = C_0$$

5. Preservation Technology Cost

$$PTC = C_{pt}$$

Total Cost per Cycle

$$T_{cycle} = H_{cycle} + D_{cycle} + P_{cycle} + O_{cycle} + PTC_{cycle}$$

$$= C_h\left[I_0 t_1 - \frac{at_1^2}{2} + \frac{b}{\omega}\sin \omega t_1 - \frac{b}{\omega}t_1\right] +$$

$$C_d\theta\left[I_0 t_1 - \frac{at_1^2}{2} + \frac{b}{\omega^2}\sin \omega t_1 - \frac{b}{\omega}t_1\right] +$$

$$C_p[aT + \frac{b}{\omega}(1 - \cos(\omega t)] + C_0 + C_{pt}$$

Numerical Example

Consider an inventory system characterized by data provided in units is assumed. Then a = 60, $\omega = 1, \theta = 0.03, O_c = 250, C_p = 15, C_h = 4,$
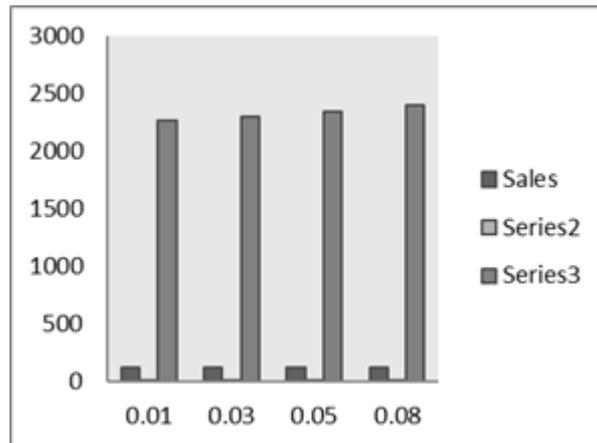
$C_{pt} = 180, T = 2, Q = 121.42$

$, TC = 2306.46$

*outcome of adjustment in the rate of deterioration θ

Index

θ	Q	T	TC
0.01	121.42	2	2270.05
0.03	121.42	2	2306.46
0.05	121.42	2	2345.21
0.08	121.42	2	2401.88

Deterioration rate θ
*Impact of (θ) on Total cost

Deteriorating rate θ
Parametric Analysis

Index I demonstrate the impact of the deterioration rate θ on the optimal Cycle time T and total cost TC. It is clearly observed that an increase in θ leads to a consistent rise in total cost due to higher inventory spoilage. The optimal cycle time T also adjust moderately as θ increases, reflecting the system's effort to balance deterioration and replenishment decisions. Overall, the model is highly sensitive to Change in the deterioration rate, indicating that effective preservation strategies are essential to control total inventory cost under seasonal demand and inflationary Conditions.

II.   Conclusions

This research develops a comprehensive analytical EOQ framework for deteriorating items by jointly incorporating, Seasonal demand variation, preservation technology investment, and inflationary effects. The model demonstrates that demand Seasonality and economic fluctuations substantially influence optimal replenishment policies and overall cost performance. By treating preservation technology as a strategic decision variable, the study establishes a structured balance between technological investment and deterioration control.

The finding reveals the proactive investment in preservation significantly enhances cost efficiency and stabilizes inventory decisions under inflation-Sensitive and dynamically changing market conditions. The proposed framework not only Strengthens classical EOQ theory but also advances it toward a more realistic, technology-drives and economically resilient inventory management approach suitable for modern supply chains.

References
[1].   Abad, P.L. (2000): Optimal Lot Size For A Perishable Good Under Condition Of Neither Production And Partial Backordering And Lost Sale. Computer & Industrial Engineering, Pp457-465
[2].   Buzacott (1975): Deteriorating Inventory Model With Variable Holding Cost And Price Dependent Time Varying Demand. Operational Research Quarterly, 26(3), 553-558
[3].   Covert, R.P. & Philip. GC. (1973). An EOQ Model For Items With Weibull Distribution Deterioration. Alle Transactions, 5(4), 323-326
[4].   Donaldson, W.A (1977). Inuentory Replenishment Policy For A Linear Bend In Demand: An Analytical Solution. Operational Research Quarterly, 28(3), 663-670
[5].   Dye, CY. And Ouyang, L.Y (2005): An EOQ Model For Perishable Item Under Stock Dependent Selling Rate And Time-Dependent Partial Backlogging. European Journal Of Operation Research, 163, PP776-783
[6].   Donselaar Et Al. (1996): A Model For Exponentially Decaying Inventory. Journal Of Industrial Engineering
[7].   G.Hare, P.M & Schrader, G.F (1963): A Model For Exponentially Decaying Inventory. Journal Of Industrial Engineering, 14, 238-243
[8].   G.P. Samantha, Ajanta Roy (2004): Production Inventory Model With Deteriorating Item And Shortage. Yugoslav Journals Of Operation Research, 14, Number 2, Pp 219-230
[9].   Hsu,P.H, Wee H.M &. Teng H.M (2010). Preservation Technology Investment For Deteriorating Inventory. International Journal Of Production Economics, 124(2), 388-394
[10].   Khanra, S.S, Ghosh, S.K. & Chaudhri, K.K (2011): An EOQ Model For Deteriorating Items With Limited Storage Capacity And Shortages. International Journal Of System Science, 42(11), 1937-1947
[11].   Law, S.T & Wee. H.M (1999): Replenishment And Pricing Policy For Deteriorating Items Taking Into Account The Time Value Of Money. International Journal Of Production Economics 60-61,213-220-
[12].   Misra, R.B (1975). A Note On A EOQ Model With Inflation. Naval Research Logistics Quarterly, 22(3), 551-556

[13]. Mandal, B. N & Pal A.K. (1998): Order Level Inventory System With Limited Storage Space. International Journal Of Production Economic, 54(1), 55-64
[14]. Mak, K.L(1982): A Production Lot Size Inventory Model For Deteriorating Item. Computer & Industrial Engineering, 614) PP309-317.
[15]. Nahmias, S. (1982): Perishable Inventory Theory. A Review Operation Research, 30(4), 680-708
[16]. Pal, Ak Bhunis & Mukherjee R.N (2006); Optimal Lot Size Of Model For Deteriorating Item With Demand Rate Dependent On Displayed Stock Level (DSL) And Partial Back Ordering, European Journal Of Operational Research, 175, PP 977-991
[17]. Ravid. B (1987). Inventory Control With Partial Backlogging IIE Transactions, 19(4) 389-395
[18]. Sharma And Singh (2003): Two Warehouse Inventory Models With Ramp Type Demand Rate, Partial Backlogging And Weibull Deterioration Rate. European Journal Of Operational Research, 192 PP 79-92.
[19]. Silver, E.A & Meal 4.C (1973): A Heuristic For Selecting Lot Size Requirements For Deterministic Time-Varying Demand. Production And Inventory Management 14(2), 64-74.
[20]. Vinod Kumar Mishra, Lal Sahab Singh (2010): Deteriorating Inventory Model With Time Dependent Demand And Partial Backlogging Applied Mathematical Science, Vol.4 No 72, Pp 3611-3619
[21]. Yang, H.L (2012): Inventory Model With Investment Is Preservation Technology For Deteriorating Item. Applied Mathematical Modeling, 36(9), 4181-4191