

Stochastic Petri-Net Simulation Model For Evaluation Of Election Voting Processes In Federal Capital Territory (FCT), Abuja, Nigeria

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Abstract:

This work develops a Stochastic Petri Net (SPN) simulation model, enhanced with Colored Petri Nets (CPNs) and queuing theory, to evaluate and improve the election voting process across the six Area Councils of the Federal Capital Territory (FCT), Abuja, Nigeria. Persistent electoral challenges such as prolonged voter queues, excessive waiting times, and widespread disenfranchisement motivated the need for a robust modeling framework. The study specifically models the accreditation and voting phases, using empirical data obtained through field observations during elections. Key performance parameters, including voter arrival rates (λ), service rates (μ), average waiting times, queue lengths, and system utilization (ρ) were analyzed across different polling units. Simulation results identified significant congestion and resource strain in densely populated councils such as Abuja Municipal (AMAC) and Kuje, revealing critical inefficiencies in staff deployment and polling unit capacity. Scenarios involving optimized staff allocation and improved service protocols demonstrated measurable improvements in throughput and reduction in wait times. The findings underscore the suitability of SPN-based modeling for diagnosing operational bottlenecks in electoral systems and for informing strategic planning. The study recommends the adoption of simulation tools in electoral operations to support real-time decision-making, improve resource planning, and enhance voter experience. Future work may incorporate biometric verification delays and multi-stage voting behaviors for more comprehensive modeling.

Keywords: *Stochastic Petri-Net (SPN), Voters, Election, queuing theory, voter arrival rate.*

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I. Introduction

In democratic societies, the efficiency and transparency of election processes are critical to the legitimacy of governance and public trust. In Nigeria, however, elections are frequently characterized by systemic inefficiencies, logistical setbacks, and cases of voter disenfranchisement, particularly in urban centers such as the Federal Capital Territory (FCT), Abuja. Despite reforms by the Independent National Electoral Commission (INEC), challenges persist, including long queues, delayed accreditation, malfunctioning electoral technology, and overcrowded polling units [1]. These issues not only compromise the quality of elections but also discourage voter participation, further weakening democratic engagement.

Modern-day electoral management demands analytical tools capable of modeling the randomness, concurrency, and complex interactions that define voting processes. In recent years, Stochastic Petri Nets (SPNs) have gained prominence as suitable modeling frameworks for analyzing such systems, especially in contexts where time-dependent and probabilistic events occur simultaneously [2]. As an extension of classical Petri Nets, SPNs allow researchers and electoral bodies to simulate various stages of voting—from voter arrival to ballot casting and result collation under conditions of uncertainty.

Several studies have emphasized the usefulness of simulation modeling in public systems management, including elections. Colored Petri Nets (CPNs), an advanced form of SPNs, enable more detailed modeling by categorizing voters and resources using token attributes, offering insights into system bottlenecks and potential interventions [3]. When integrated with queuing theory, these models provide a robust framework for analyzing voter flow, queue lengths, waiting times, and polling staff efficiency, all of which are crucial for improving electoral logistics [4]. In the Nigerian context, research has traditionally focused on political and sociological aspects of elections, with limited attention paid to computational or mathematical modeling. However, emerging scholarship is beginning to bridge this gap. For instance, [5] advocate for data-driven election planning as a means of enhancing transparency and operational efficiency. Similarly, [6] argue that electoral simulations could support INEC in testing logistical strategies before Election Day, thereby minimizing costly disruptions.

International comparisons reveal a growing trend toward simulation-assisted electoral planning in developing democracies. Countries like Ghana and Kenya have explored simulation tools for modeling voter behavior and polling station dynamics to reduce congestion and improve service delivery [7]. These examples demonstrate that integrating stochastic models into election planning is not merely academic but also practically viable and policy-relevant.

This paper, therefore, seeks to develop a Stochastic Petri Net simulation model enhanced with Colored Petri Nets and queuing theory to evaluate the election voting processes across six Area Councils in Abuja, FCT. By simulating voter dynamics and operational parameters, the study aims to identify inefficiencies and recommend data-informed strategies for improving electoral performance. The outcomes are expected to inform electoral reforms and contribute to a more efficient, transparent, and trustworthy voting process in Nigeria.

II. Material And Methods

Research Design

This study adopts a quantitative simulation-based approach using Stochastic Petri Nets (SPNs) and Colored Petri Nets (CPNs) to model and analyze the dynamics of the election voting processes within six Area Councils in the FCT: Abaji, AMAC, Bwari, Gwagwalada, Kuje, and Kwali. The methodology incorporates principles of queuing theory, discrete event simulation, and performance metrics evaluation.

Primary data were collected through field observation of polling units during the 2023 Nigerian general elections. Parameters gathered include the following; number of accredited voters and actual voters per unit, voter arrival patterns, service times at accreditation and voting booths (measured per APO I and APO II), balloting time and staffing levels (number and roles of polling officers) per polling units. These were complemented by structured interviews with electoral officers and public INEC datasets.

Formulation of the Model

The stochastic petri Net (SPN) is mathematically represented as a five (5) tuple, $SPN = (P, T, F, W, M_0)$, where $P = \{p_1, p_2, \dots, p_m\}$ is a finite set of places.

$T = \{t_1, t_2, \dots, t_n\}$ is a finite set of transitions.

$F \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs connecting places and transitions.

$W: F \rightarrow \{1, 2, 3, \dots\}$ is the weight function for arcs.

$M_0: P \rightarrow \{0, 1, 2, 3, \dots\}$ is the initial marking. (distribution of tokens across places)

The initial marking in a Petri net represents the initial distribution of tokens across the places, reflecting the state of the system at the start of the process. In the context of this research, the initial marking corresponds to the number of voters at each stage of the election process at time $t = 0$. The stochastic behavior is governed by a Continuous-Time Markov Chain (CTMC) derived from the SPN. Transition rates in the CTMC are determined by the service rates (μ) and arrival rates (λ).

The stochastic petri-net (SPN) model is formulated to represent the election voting process in Abuja Metropolis. This involves defining the elements of the SPN, including places, transitions, tokens, and arcs, along with the stochastic parameters governing the system.

Places represent the states or conditions in the voting process, this include key stages like Arrival, Accreditation, Voting Booth). In our model we have the following places:

P_1 : Voters waiting to enter the polling station

P_2 : Voters undergoing identification (i.e. biometric verification)

P_3 : Voters receiving their ballot papers.

P_4 : Voters casting their votes

P_5 : Voters exiting the polling station or waiting to watch the counting and collation process.

In petri net theory, transition model events that cause tokens (representing voters) to move between places. In other words, transitions simulate the flow of tokens (voters). In the formulation of the model, we have the following transitions;

T_1 : Entry into the polling station, T_2 : Completion of voter Identification, T_3 : Insurance of ballot papers.

T_4 : Casting of votes,

T_5 : Exit from the polling station

Tokens represent individual voters. The number of tokens in a place indicates the number of voters in that stage of the process. Arcs connect places to transitions and transitions to places, defining the flow of voters through the system. Each arc may have a weight indicating the number of tokens required for a transition to fire.

The voting system was modeled using Colored Petri Nets (CPNs) based on the workflow observed in Figure 1 and 2 below:

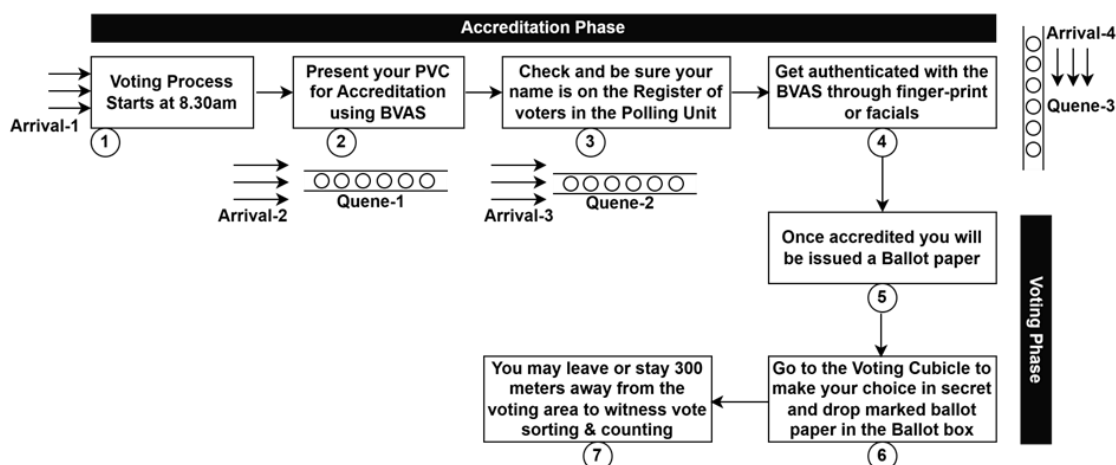


Figure 1 General Voting Process for each Polling unit (Voting Station).

The voting procedures during a general election can be represented by a stochastic petri model. First we describe the steps involve in accreditation phase and the voting process in a general election as shown in Figure 1 above.

Step 1: At the Polling unit, each voters arrive the voting station and join the queue. An INEC official - Assistant Polling Officer III (APO III) will check if voters are at the correct polling unit and also confirm if the PVC presented belong to the voter. If satisfied, direct the voter to the Assistant Polling Officer I (APO I) who does the verification and the statistics collation.

Step 2: The APO I (verification and Statistics Officer) shall request for the voters Permanent Voters Card (PVC) and confirm if it is genuine using the BVAS. The voter will be asked to place his/her finger on the BVAS. The purpose of this is to make sure the photograph and the details of the voter match the voter's e-register.

Step 3: The verified voter shall then present himself/herself to the APO II who shall request for the voter's PVC, check the list of registers voters to confirm that the voters' name, details and the Voters Identification Number (VIN) are contained on the list of registered voters. If the name is on the list, the APO II will tick the left side of the name of the voter and apply indelible ink to the cuticle of the specified finger-nail on the left hand and offer the voters an accreditation tag bearing the signature of the presiding officer (PO).

Step 4: The Presiding Officer (PO) will stamp, sign and endorse the ballot paper. The ballot paper will be given to the voter rolled with the printed side inwards and directed to the voting cubicle (voting booth).

Step 5: A voter is now expected to cast his/her vote. The voter stain his/her fingers with the ink provided and mark the box for his/her preferred candidate of political party. Thereafter he/she roll the ballot paper in the manner he/she was given and flatten it.

Step 6: The voter leaves the voting cubicle and proceed to drop the ballot paper in the ballot box in full view of people at the polling unit.

Step 7: The voter leaves the voting station/polling unit or wait 300 meters away in an orderly and peaceful manner to watch the counting process up to the time of declaration of the winner.

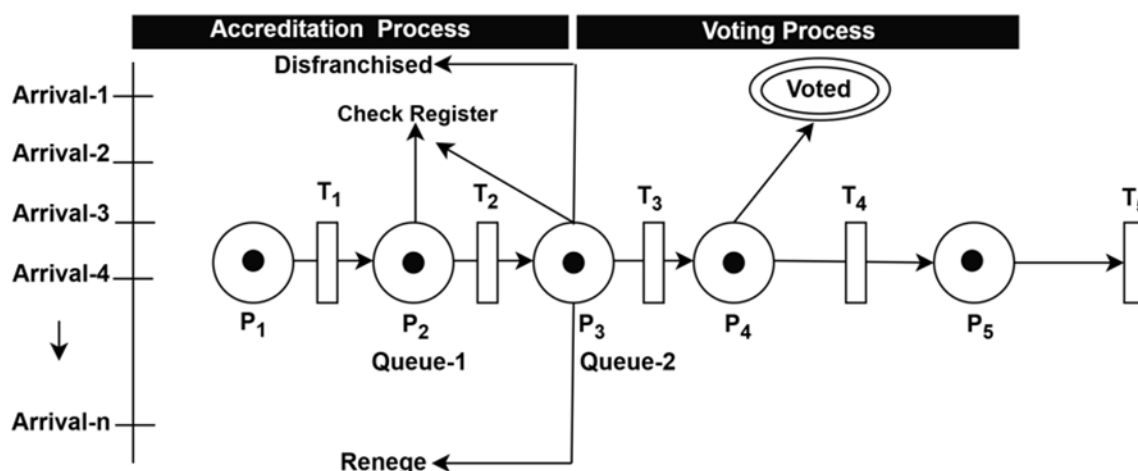


Figure 2 A Stochastic Petri Net model for Accreditation and Voting Process in an Election.

In the development of the model, timing functions were introduced using exponential distributions to represent arrival and service processes. Also CPN Tools software was employed for model implementation, validation, and simulation.

Queuing Model and Analytical Framework

We now derive the mathematical equations for the stochastic Petri Net model given in Figure 1 and Figure 2 based on the number of places and transition provided in the model formulation. The objective is to compute the following stochastic parameters such as arrival rates, service rates, transition probabilities, average waiting times, queue lengths, throughput, and utilization rates, as well as define the optimization problem and the associated objective function.

Let λ_p denote the arrival rate for place P_i , specifically λ_{P_i} and defined by

$$\lambda_{P_i} = \frac{\text{Accredited Voters}}{\text{Total Time Available (minutes)}} = \frac{\text{Accredited Voters}}{\sum_{i=1}^{6pm} T} \quad (1)$$

The service rate (μ) is the rate at which transition from one place to the other. For example let μ_{T_1} represent the service rate of voters entering the polling station. Similarly, each transition T_i has its associated μ_{T_i} . Generally, it is the average number of voters serviced per minutes and computed as the inverse of the total time taken by APO I and APO II for either accreditation or voting.

$$\mu_{T_i} = \frac{\text{Total Number of APO Officers}}{\sum_{i=1}^2 T} = \frac{2}{\text{APO I Time} + \text{APO II Time}} \quad (2)$$

Let the Probability of a token (voter) moving from P_i to P_{i+1} via T_i is given by

$$P(T_i) = \frac{\mu_{T_i}}{\sum_{j=1}^n \mu_{T_j}} \quad (3)$$

where $\sum_{j=1}^n \mu_{T_j}$ represents the sum of service rates for all transitions.

The waiting time $W(P_i)$ for voters in place P_i can be derived using Little's Law

$$W(P_i) = \frac{L(P_i)}{\lambda_{P_i}} = \frac{1}{\mu - \lambda} \quad (4)$$

where $L(P_i)$ is the average queue length at P_i and λ_{P_i} is the arrival rate at P_i

The expected queue length $L(P_i)$ for each place P_i can be modeled using the relationship

$L(P_i) = \lambda_{P_i} \cdot W(P_i)$ as in equation (4) above.

The throughput $X(T_i)$ of a transition T_i is defined as the number of voters processed per unit time by the relation

$$X(T_i) = \lambda_{P_i} \cdot P(T_i) \quad (5)$$

where $P(T_i)$ is the probability of transition T_i

The utilization rate $U(P_i)$ of a place P_i measures the proportion of time it is actively processing tokens (voters) and it is given by

$$U(P_i) = \frac{\lambda_{P_i}}{\mu_{T_i}} \quad (6)$$

This is the ratio of the arrival rate at P_i to the service rate of its associated transition T_i

The Optimization problem is given below with the objective to minimize waiting times and queue lengths while maximizing throughput and ensuring fair resource allocation.

Objective Function:

Minimize the total waiting time across all places $P_1, P_2 \dots P_5$

$$\text{Minimize } Z = \sum_{i=1}^5 W(P_i) \quad (7)$$

Constraints:

(a) Conservation of tokens:

$$\sum_{i=1}^5 M(P_i) = M_{\text{total}} \quad (8)$$

where $M(P_i)$ is the marking (number of tokens) at P_i and M_{total} is the total number of voters.

(b) Service Capacity:

$$U(P_i) \leq 1 \quad \forall i \quad (9)$$

(c) Non-negative queue lengths:

$$L(P_i) \geq 0 \quad \forall i \quad (10)$$

(d) Probabilistic Transitions:

$$\sum_{j=1}^n P(T_i) = 1 \quad (11)$$

Solution of the Stochastic Petri-Net Simulation Model

The model was implemented using a simulation software capable of handling SPNs namely the Colour Petri Net (CPN) also called the CPN tools. Data collected during the study serves as input for the parameters, ensuring the model reflects real-world conditions.

Modelling of the Voting System using Stochastic Coloured Petri-Net Tools

Colour Petri Net (CPN) tools is a powerful modelling and simulation environment used to design, simulate, and analyze systems using a graphical language for modelling concurrent, discrete-event and stochastic systems. It is especially useful for complex system like election voting processes, where multiple voters interact with multiple polling activities such as accreditation, ballot issuance and voting booth access.

Key features of CPN tools include the following:

a). Graphical Interface

This include drag-and-drop interface for places, transitions and arcs

b). Color Sets

Token can carry complex data such as voter ID, time etc

c). Timing and Stochastic Support

CPN tools supports timed tokens and exponential distributions for simulation

d). State Space Analysis

CPN tools can check for deadlocks, bottlenecks, throughput and queue length.

e). Simulation Engine

CPN tool can simulate real-time behavior and dynamics of modeled systems.

Below is the Step-by-Step Process of how the Election process in FCT Abuja can be modeled using CPN tools:

Step 1: Define the Core Components of the Voting System

Here the following component (Voters, Accreditation, Voting Queue, Ballot Issuance, Voting Booth and Exit) are model (Tokens, Transitions & Places, Place, Transition, Place + Transistion and Place) respectively.

Step 2: Define Time and Probability Dynamics

Here we use exponential delays and Poisson arrivals

fun arrivalTime() = exponential (1.0 / l); // Inter-arrival time

fun serviceTime() = exponential (1.0 / m); // Serice time at a station

Step 3: Define Color Sets

Here Token now represent voters with attributes such as ID, status, and timestamps.

colset Voter = record id: INT

hasVoted: BOOL * timeStamp: INT;

Step 4: Define Sequence of CPN Flow

Here we define the sequence of the CPN flow in seven order as follows:

1). **Place:** Voters_Arrival

Voters arrival are given as initial marking and subsequently dynamically populated.

2). **Transition:** Start_Accreditation

Verification by API and APII INEC Officers.

3). **Place:** Accredited_Voters

Accredited voters now wait for ballot issuance.

4). **Transition:** Issue_Ballot

Ballot handed over.

5). **Place:** Voting_Booth

Voters makes selection

6). **Transition:** Casts_Vote

Ballot casted.

7). **Place:** Voters_Arrival

Voters arrival are given as initial marking and subsequently dynamically populated.

Step 5: Add Timing and Delays

Here each service transition should include delays (exponential or constant) and Resources (numbers of INEC staff).

@+serviceTime(); // adds stochastic delay to the transition

III. Results**Data Collection Instrument for Election Monitoring and Evaluation**

Data collection is very important since the results and findings of this study in the best case possible are as good as the input information. In this study, different methods were used to collect data, one is designed to collect relevant demographic information (see Table 1) and the other is the election process experiences (see Table 2) of voters in one polling unit each in the six area councils as per the field study carried out by INEC.

Table 1 Demographic Information of voters in one polling unit each in the six area councils in the study area

S/ N	Area Council	Age Group					Gender		Educational Level				Voter Status		Total
		[18-25]	[26-35]	[36-45]	[46-60]	[60+]	M	F	NF	Pri	Sec	Ter	Reg	UReg	
1	AAC	55	36	59	19	2	90	81	18	34	52	67	143	28	171
2	AMAC	96	83	90	57	2	155	173	31	76	56	165	314	14	328
3	BAC	24	30	43	29	0	73	53	33	12	57	24	121	05	126
4	GAC	63	30	25	43	4	88	77	15	51	34	65	141	24	165
5	KAC	58	54	75	33	1	113	108	29	38	56	98	215	6	221
6	KWAC	51	67	59	12	3	97	95	36	71	40	45	181	11	192
Total		347	300	351	193	12	616	587	162	282	295	464	1115	88	1203
Distri (%)		29	25	29	16	01	51	49	13	23	25	39	93	07	

Source: Field Survey (INEC, 2023)

Key: NF: Non Formal, Pri: Primary, Sec: Secondary, Ter: Tertiary, Reg: Registered, UReg: Unregistered
AAC: Abaji Area Council, AMAC: Abuja Municipal Area Council, BAC: Bwari Area Council, GAC: Gwagwalada Area Council, KAC: Kuje Area Council, KWAC: Kwali Area Council

Using the Colour Petri-Net (CPN) model, we present the simulated arrival pattern (see Table 3 and Table 4) for both the accreditation and voting process in one polling unit each in the six area council in the study area. CPN Tools (version 4.0.1) was used in constructing a Stochastic CPN model. The developed CPN model consist of socket places and transition drawn with rectangles. Each transition is a module which has completely defined operations such as the voting station module that models arrival and departure of voters and voting booth module that model queuing of voters, the operation the presiding officers and voters behavioral pattern. The CPN model was further broken it into different sub modules such as arrival module, voting process module, queue module, departure module and voter behavioral module (that modelled reneging and balking of disenfranchised voters.)

The major places, transitions, color sets, variables, initial parameters, and functions used in developing the CPN model of the voting system are described and depicted in section 2.5, though details are not shown here.

Table 2 Election Process Experience Evaluation in one polling unit each in the six area councils.

Area Council		AA C	AMAC	BA C	GAC	KAC	KWAC	Total	Per (%)
Did you vote in the last election?	Yes	113	284	116	131	198	169	1011	91%
	No	30	30	5	10	17	12	104	9%
When did you arrive at the voting venue for Accreditation?	[Before 8am]	24	63	12	14	25	14	152	15%
	[8-9am]	28	72	36	34	17	32	219	22%
	[9-10am]	18	43	27	35	38	54	215	21%
	[10-11am]	20	34	21	18	41	37	171	17%
	[11-12noon]	13	56	17	9	56	15	166	16%
	[12-1pm]	10	16	3	21	21	17	88	9%
When did you arrive for the voting?	[B4 12noon]	12	67	34	23	43	34	213	21%
	[12-1pm]	14	45	25	16	22	23	145	14%

	[1-2pm]	21	45	31	45	56	41	239	24%
	[2-3pm]	23	67	21	32	21	28	192	19%
	[3-4pm]	24	23	3	6	34	12	102	10%
	[4-5pm]	8	23	2	5	13	12	63	6%
	[5-6pm]	11	14	0	4	9	19	57	6%
How long did Accreditation process take? (Service Time)	[<3 min.]	44	98	73	61	87	70	433	43%
	[3-5 min.]	23	80	17	37	54	33	244	24%
	[5-8 min.]	12	87	26	22	37	56	240	24%
	[>8 min.]	34	19	0	11	20	10	94	9%
How long did Voting process take (Service Time)	[<3 min.]	23	73	33	45	65	61	300	30%
	[3-5 min.]	21	92	18	23	76	50	280	28%
	[5-8 min.]	45	65	31	42	34	43	260	26%
	[>8 min.]	24	54	34	21	23	15	171	17%
Were there cases of voter disenfranchisement in your polling unit?	Yes	2	2	5	6	5	0	20	2%
	No	111	282	111	125	193	169	991	98%
Was there adequate security at the polling unit?	Yes	56	168	65	2	110	89	490	48%
	No	57	116	51	129	88	80	521	52%
Were election materials delivered on time?	Yes	113	284	116	131	198	169	1011	100%
	No	0	0	0	0	0	0	0	0%
How do you rate the efficiency of the electronic voting system (if applicable)?	[Poor]	5	25	7	14	34	22	107	10.6%
	[Average]	20	102	72	35	89	69	387	38.3%
	[Good]	34	92	33	27	45	54	285	28.2%
	[Excellent]	54	65	4	55	30	24	232	22.9%

Source: Field Survey (INEC, 2023)

Key: AAC:Abaji Area Council, AMAC:Abuja Municipal Area Council, BAC:Bwari Area Council, GAC: Gwagwalada Area Council, KAC: Kuje Area Council, KWAC:Kwali Area Council

Table 3 Arrival Pattern Data for Accreditation Process in one polling unit each in the Study Areas

	Abaji Area Council (AAC)					Abuja Municipal Area Council (AMAC)					Bwari Area Council (BAC)				
Time Interval	% Voters Turn out	Observed Population	Simulated Population	Arrival Rate (λ) (minutes)	Av. Inter-Arrival Time ($\frac{1}{\lambda}$) (minutes)	% Voters Turn out	Observed Population	Simulated Population	Arrival Rate (λ) (minutes)	Av. Inter-Arrival Time ($\frac{1}{\lambda}$) (minutes)	% Voters Turn out	Observed Population	Simulated Population	Arrival Rate (λ) (minutes)	Av. Inter-Arrival Time ($\frac{1}{\lambda}$) (minutes)
[Before 8am]	21.2	24	30.4	Prelod	Prelod	22.2	63	69.7	Prelod	Prelod	10.3	12	12.5	Prelod	Prelod
[8-9am]	24.8	28	35.4	0.5	2.1	25.4	72	79.6	1.2	0.8	31.0	36	37.6	0.6	1.7
[9-10am]	15.9	18	22.8	0.3	3.3	15.1	43	47.5	0.7	1.4	23.3	27	28.2	0.5	2.2
[10-11am]	17.7	20	25.3	0.3	3.0	12.0	34	37.6	0.6	1.8	18.1	21	21.9	0.4	2.9
[11-12noon]	11.5	13	16.5	0.2	4.6	19.7	56	61.9	0.9	1.1	14.7	17	17.7	0.3	3.5
[12-1pm]	8.8	10	12.7	0.2	6.0	5.6	16	17.7	0.3	3.8	2.6	3	3.1	0.1	20.0
Grand Total	100	113	143	1.9	0.5	100	284	314	4.7	0.2	100	116	221	1.9	0.5
TRV		143					314					121			

TRV: Total Registered Voters

	Gwagwalada Area Council (GAC)					Kuje Area Council (KAC)					Kwali Area Council (KWAC)				
Time Interval	% Voters Turn out	Observed Pop	Simulated Pop	Arrival Rate	Av. Inter-Arrival Time	% Voters Turn out	Observed Pop	Simulated Pop	Arrival Rate	Av. Inter-Arrival Time	% Voters Turn out	Observed Pop	Simulated Pop	Arrival Rate	Av. Inter-Arrival Time

	n out	ulat ion	ulati on	e (λ) (mi nut es)	Arri val Tim e ($\frac{1}{\lambda}$) (mi nute s)	rn out	ulat ion	ulati on	e (λ) (mi nut es)	val Tim e ($\frac{1}{\lambda}$) (mi nute s)	n out	ulat ion	ulati on	(λ) (mi nut es)	val Time ($\frac{1}{\lambda}$) (min utes)
[Before 8am]	10.7	14	15.1	Prel oad	Prel oad	12. 6	25	27.1	Prel oad	Prel oad	8.3	14	15.0	Prel oad	Prel oad
[8- 9am]	26.0	34	36.6	0.6	1.8	8.6	17	18.5	0.3	3.5	18. 9	32	34.3	0.5	1.9
[9- 10am]	26.7	35	37.7	0.6	1.7	19. 2	38	41.3	0.6	1.6	32. 0	54	57.8	0.9	1.1
[10- 11am]	13.7	18	19.4	0.3	3.3	20. 7	41	44.5	0.7	1.5	21. 9	37	39.6	0.6	1.6
[11- 12noon]	6.9	9	9.7	0.2	6.7	28. 3	56	60.8	0.9	1.1	8.9	15	16.1	0.3	4.0
[12- 1pm]	16.0	21	22.6	0.4	2.9	10. 6	21	22.8	0.4	2.9	10. 1	17	18.2	0.3	3.5
Grand Total	100	131	141	2.2	0.5	100	198	215	3.3	0.3	100	169	181	2.8	0.4
TRV		141					215					181			

TRV: Total Registered Voters

Table 4 Arrival Pattern Data for the Voting Process in the Study Areas in one polling unit each in the Study Areas

	Abaji Area Council (AAC)					Abuja Municipal Area Council (AMAC)					Bwari Area Council (BAC)				
Time Inter val	% Vot ers Tur n out	Obs erv ed Pop ulat ion	Sim ulat ed Pop ulat ion	Arr ival Rat e (λ) (mi nut es)	Av. Inter- Arri val Tim e ($\frac{1}{\lambda}$) (mi nute s)	% Vot ers Tur n out	Obs erve d Pop ulat ion	Sim ulat ed Pop ulat ion	Arr ival Rat e (λ) (mi nut es)	Av. Inter- Arri val Tim e ($\frac{1}{\lambda}$) (mi nute s)	% Vot ers Tur n out	Obs erv ed Pop ulat ion	Sim ulat ed Pop ulat ion	Arr ival Rat e (λ) (mi nute s)	Av. Inter- Arri val Time ($\frac{1}{\lambda}$) (min utes)
[B4 12noon]	10.6	12	15.2	Prel oad	Prel oad	23. 6	67	74.1	Prel oad	Prel oad	29. 3	34	35.5	Prel oad	Prel oad
[12- 1pm]	12.4	14	17.7	0.2	4.3	15. 8	45	49.8	0.8	1.3	21. 6	25	26.1	0.6	1.7
[1- 2pm]	18.6	21	26.6	0.4	2.9	15. 8	45	49.8	0.8	1.3	26. 7	31	32.3	0.5	2.2
[2- 3pm]	20.4	23	29.1	0.4	2.6	23. 6	67	74.1	1.1	0.9	18. 1	21	21.9	0.4	2.9
[3- 4pm]	21.2	24	30.4	0.4	2.5	8.1	23	25.4	0.4	2.6	2.6	3	3.1	0.3	3.5
[4- 5pm]	7.1	8	10.1	0.1	7.5	8.1	23	25.4	0.4	2.6	1.7	2	2.1	0.1	20.0
[5- 6pm]	9.7	11	13.9	0.2	5.5	4.9	14	15.5	0.2	4.3		0	0.0	0.0	
Grand Total	100. 0	113	143.			100 .0	284	314.			100 .0	116	121.		
TRV		143					314					121			

TRV: Total Registered Voters

	Gwagwalada Area Council (GAC)					Kuje Area Council (KAC)					Kwali Area Council (KWAC)				
Time Inter val	% Vot ers Tur n out	Obs erv ed Pop ulat ion	Sim ulat ed Pop ulat ion	Arr ival Rat e (λ)	Av. Inter- Arri val Tim e ($\frac{1}{\lambda}$)	% Vot ers Tur n out	Obs erve d Pop ulat ion	Sim ulat ed Pop ulat ion	Arr ival Rat e (λ)	Av. Inter- Arri val Tim e	% Vot ers Tur n out	Obs erv ed Pop ulat ion	Sim ulat ed Pop ulat ion	Arr ival Rat e (λ)	Av. Inter- Arri val Time ($\frac{1}{\lambda}$) (minut es)

				(mi nut es)	(mi nute s)				(mi nut es)	($\frac{1}{2}$) (mi nute s)				(mi nut es)	
[B4 12noon]	17.6	23	24.8	Prel oad	Prel oad	21. 7	43	46.7	Prel oad	Prel oad	20. 1	34	36.4	Prel oad	Preloa d
[12- 1pm]	12.2	16	17.2	0.3	3.8	11. 1	22	23.9	0.4	2.7	13. 6	23	24.6	0.4	2.6
[1- 2pm]	34.4	45	48.4	0.8	1.3	28. 3	56	60.8	0.9	1.1	24. 3	41	43.9	0.7	1.5
[2- 3pm]	24.4	32	34.4	0.5	1.9	10. 6	21	22.8	0.4	2.9	16. 6	28	30.0	0.5	2.1
[3- 4pm]	4.6	6	6.5	0.1	10.0	17. 2	34	36.9	0.6	1.8	7.1	12	12.9	0.2	5.0
[4- 5pm]	3.8	5	5.4	0.1	12.0	6.6	13	14.1	0.2	4.6	7.1	12	12.9	0.2	5.0
[5- 6pm]	3.1	4	4.3	0.1	15.0	4.5	9	9.8	0.2	6.7	11. 2	19	20.3	0.3	3.2
Gran d Total	100. 0	131	141. 0			100 .0	198	215. 0			100 .0	169	181. 0		
TRV		141					215					181			

TRV: Total Registered Voters

The Election process data collected for the CPN model validation is shown in Table 5. The polling officers (APOI and APOII) were assumed to maintain an average of 101 and 93 seconds for accreditation and voting respectively. These were choosing according to data released during field trials carried out by INEC for the card readers for the 2023 general elections. Five hours were dedicated to the arrival of intending voters during accreditation exercise (8.00am - 1.00pm) while seven hours were dedicated to the arrival of voters during the voting process (1.30pm - 7.30pm).

Table 5 Election Process Data collected for the CPN model validation in one polling unit each in the study area

S/N	Area Councils	Data Code	Election Processes Type	Voter Population	Service Time (Sec)		Balloting Time (Sec)
					APO I	APOII	
1	Abaji	ACC2514	Accreditation	143	107	105	78
		VOT1151	Voting	113	110	95	
2	Abuja	ACC1214	Accreditation	314	101	97	65
		VOT1012	Voting	284	86	85	
3	Bwari	ACC6171	Accreditation	121	97	105	67
		VOT4121	Voting	116	86	90	
4	Gwagwalada	ACC7131	Accreditation	141	95	105	58
		VOT4010	Voting	131	80	95	
5	Kuje	ACC9410	Accreditation	215	105	90	75
		VOT1256	Voting	198	104	107	
6	Kwali	ACC2119	Accreditation	181	112	106	61
		VOT4120	Voting	169	94	85	

The voters' arrival pattern for accreditation and voting processes being assumed followed the observed pattern monitored and reported in (Olabisi and Chukwunoso, 2012). Each hourly period is assumed to follow a Poisson pattern of voter arrival for the voter population within the hour. The modelled voters' arrival pattern is consistent with the observation made in (Olabisi and Chukwunoso, 2012). Monitors facilities of Coloured Peti Net Tools (CPN Tools) were used to obtain the required data from the developed CPN model during the simulation runs. Ten replications of the simulation run were executed each for the accreditation and voting processes in each of the six (6) study area. The simulations were coded to allow easy remembrance of the data as shown in Table 5 above.

Validation of the developed CPNs model

To ensure the accuracy and reliability of the proposed CPN model, a validation process was undertaken to confirm that the model accurately represents the real-world system. This was achieved by comparing the simulation results; specifically, the number of voter arrivals over varying time intervals during both the accreditation and voting phases. Service time, waiting time and other queuing metric with actual data derived from queuing theory. Table 6 and Table 7 below shows the solution of equation (1) to equation (7). This present

computation of some queuing metric (stochastic parameter) as it concern one polling unit each in the six study area.

Table 6 Optimization Results of Queuing Parameters from the study area

Study Area	Accredited Voters	Actual Voters	Arrival Rate (λ)	Service Rate (μ)	Waiting Time (W) - mins	Queue Length (L) - voters	Utilization Rate (ρ)
AAC	143	113	0.3973	0.59 (0.58)	5.08 (5.31)	2.02 (2.11)	0.67 (0.68)
AMAC	314	284	0.8722	0.60 (0.70)	-3.76 (-5.89)	-3.28 (-5.12)	1.44 (1.24)
BAC	121	116	0.3361	0.59 (0.68)	3.88 (2.89)	1.30 (0.97)	0.57 (0.49)
GAC	141	131	0.3917	0.60 (0.69)	4.80 (3.40)	1.88 (1.33)	0.65 (0.57)
KAC	215	198	0.5972	0.62 (0.57)	55.06 (-35.09)	32.88 (-20.95)	0.97 (1.05)
KWAC	181	169	0.5028	0.55 (0.67)	20.97 (5.97)	10.54 (3.00)	0.91 (0.75)

Key: AAC:Abaji Area Council, AMAC:Abuja Municipal Area Council, BAC:Bwari Area Council, GAC: Gwagwalada Area Council, KAC: Kuje Area Council, KWAC:Kwali Area Council. Number in parenthesis are information for the voting process.

Table 7 Optimization Results of Queuing Parameters from the study area (Continue...)

Study Area	Accreditation (mins)		Voting (mins)		Balloting Time (mins)	Throug h-put Rate	Optimized Data		
	APO I	APO II	APO I	APO II			Accreditation (Voting) (μ)	Waiting Time (W)	Optimiz ed Officer
AAC	1.783	1.583	1.833	1.583	1.300	0.397	0.59 (0.58)	5.08 (5.32)	1
AMAC	1.683	1.617	1.433	1.417	1.083	0.872	1.21 (1.40)	2.94 (1.88)	2
BAC	1.617	1.750	1.433	1.500	1.117	0.336	0.59 (0.68)	3.88 (2.89)	1
GAC	1.583	1.750	1.333	1.583	0.967	0.391	0.60 (0.68)	4.80 (3.40)	1
KAC	1.750	1.500	1.733	1.783	1.250	0.597	1.23 (1.14)	1.58 (1.85)	2
KWAC	1.867	1.767	1.566	1.417	1.017	0.503	0.55 (0.67)	20.97 (5.97)	1

Key: AAC:Abaji Area Council, AMAC:Abuja Municipal Area Council, BAC:Bwari Area Council, GAC: Gwagwalada Area Council, KAC: Kuje Area Council, KWAC:Kwali Area Council. Number in parenthesis are information for the voting process.

We now plot the graph of Table 3 and Table 4, so as to compare the arrival rate and service rate as shown in Figure 1 and Figure 3 below.

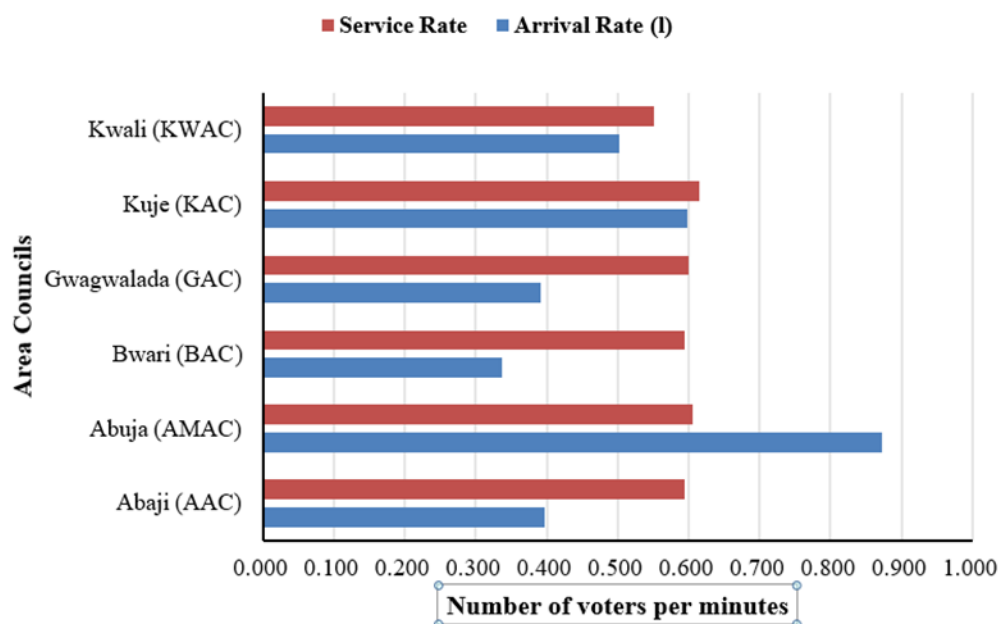


Figure 1 Relationship between Arrival Rate (λ) and Service Rate (μ) in the study areas

In order to study the relationship between waiting time (W) and queue length (L), we plot these metrics from Table 6 as shown in Figure 2 below.

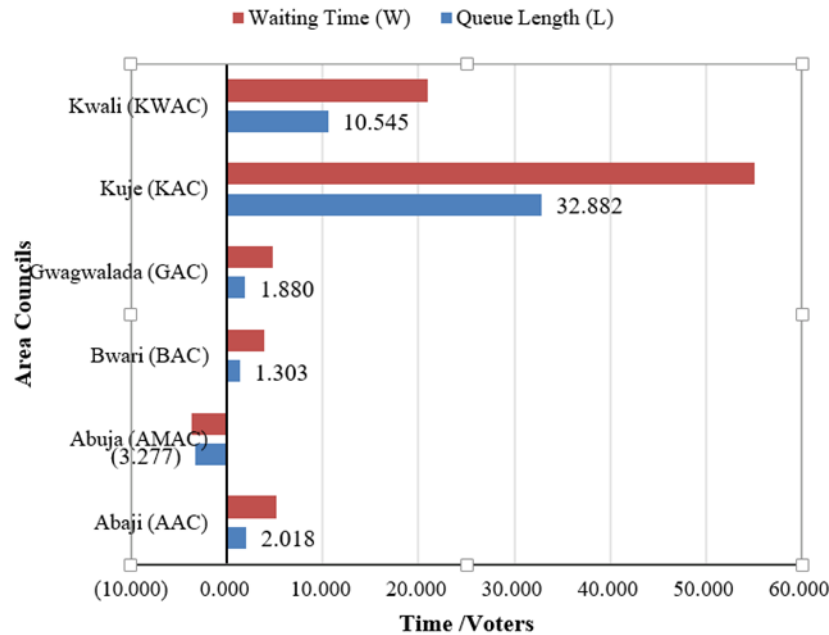


Figure 2 Relationship between Waiting Time (W) and Queue Length (L) in the study areas

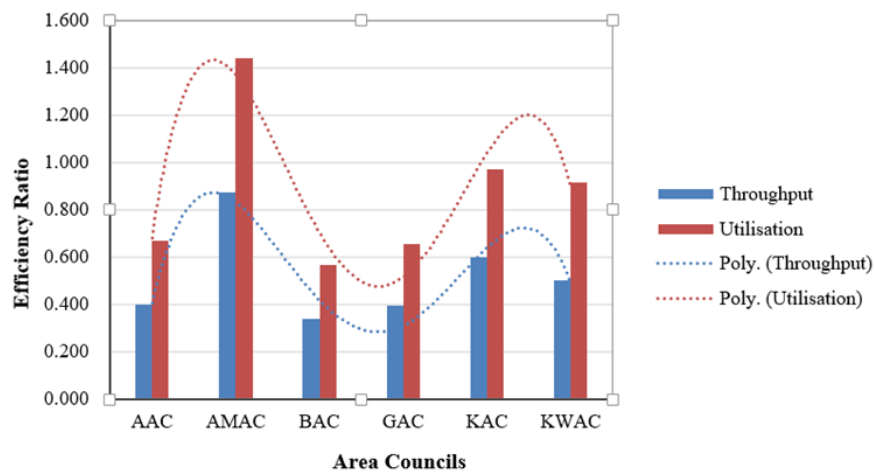


Figure 3 Relationship between Arrival Rate (λ) and Service Rate (μ) in the study areas

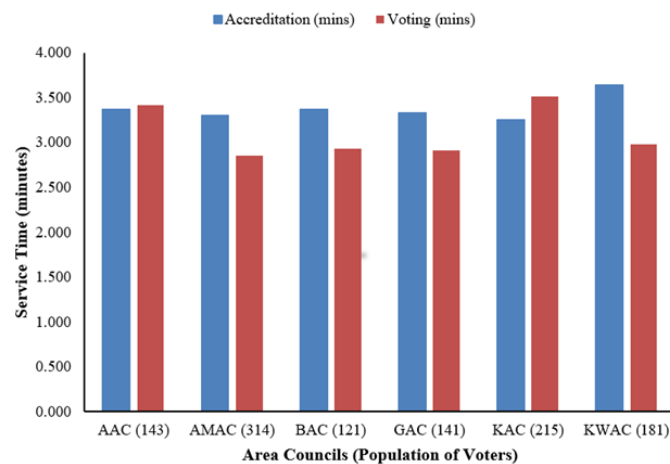


Figure 4 Implication of Service Time in the study areas

IV. Discussion Of Results

In this section, we presents an in-depth analysis and interpretation of simulation results based on a Stochastic Petri Net (SPN) framework for evaluation electoral voting processes in six Areas Councils of the Federal Capital Territory (FCT), Abuja. The simulation used a Queuing-based Colored Petri Net (CPN) model to analyze voter flow, assess delays and optimize resource deployment at polling stations. Key performance indicators derived from the model include arrival rate (λ), service rate (μ), waiting time (W) and utilization rate (ρ). From equations (1) through to equation (7) provided the mathematical foundation for computing system performance metrics listed above. Real time data collected include arrival time of voters, service times for the Assistant Polling Officers (APOs), balloting times, and actual counts of accredited and participating voters. We now presents optimization results addressing polling staff requirements and evaluates performance improvements with corresponding visual illustrations.

Relationship between Arrival Rate (λ) and Service Rate (μ)

The arrival rate reflects the number of voters arriving at the polling unit per minute, calculated by dividing the total number of accredited voters by the polling duration (360 minutes). The highest arrival rate was recorded in Abuja Municipal Area Council (AMAC) with $\lambda = 0.8722$ voters/min, followed by Kuje ($\lambda = 0.5972$), Kwali ($\lambda = 0.5028$), and Abaji ($\lambda = 0.3972$) as shown in Table 4.6 and Figure 4.1. These values indicate the voter influx to the polling stations and directly affect queue buildup. According to [3], arrival rate plays a critical role in determining the stability of election systems. When λ approaches or exceeds μ (service rate), the system tends toward saturation, leading to excessive queues and potential voter disenfranchisement. This scenarios are similarly found in restaurant business as seen in the work of [8] and [9].

Service rate (μ) represents the rate at which voters are processed at accreditation or voting stations. It is influenced by the efficiency of polling officers (APO I and II), whose performance times were collected and averaged per area council. A higher μ mean faster processing leading to shorter queues and waiting time. If μ is too low, it can lead to congestion since voter arrival rate (λ) exceeds service capacity. Most service rates hovered between 0.5747 and 0.6154 voters/min, except in AMAC where the rate was significantly lower compared to λ ($0.6061 < 0.8722$), creating instability as can be seen in Figure 1. This aligns with [10], who argue that bottlenecks are inevitable when voter arrival overwhelms processing capabilities, especially in urbanized zones.

The analysis also revealed that even though balloting time remained relatively consistent across study areas (ranging from 58 to 78 seconds), the total accreditation and voting service times varied due to differences in APO efficiency. Figure 3 plotted service time against voter population, showing that higher populations tend to inflate service delays when APO response times aren't proportionally scaled. In particular, Kuje recorded a total accreditation time of 3.25 minutes per voter, which coupled with a voter population of 215, led to long queue delays and utilization beyond 0.97 before optimization. This supports findings by [3], who highlighted the importance of synchronized APO performance to avoid systemic bottlenecks.

Relationship between Voter Population, Queue Length (L) and Waiting Time (W).

According to Little's Law (Equation 4), queue length (L) is a product of arrival rate (λ) and waiting time (W). The voters population determines the value of λ , while service characteristics is govern by the parameter μ . Figure 4.1 shows that AMAC, which had the highest voter count (314), also recorded the longest queue for both accreditation and voting processes (See Table 6). By contrast, BAC with only 121 accredited voters showed minimal queue formation. In Kuje, the waiting time of 55.06b minutes indicates voters waited nearly an hour before accreditation and voting respectively. This is highly impractical and can trigger drop-off rate leading to renege or disenfranchisement. AMAC with a negative waiting time (-3.7s minutes) reflect system instability in the specified polling unit, thus confirming that the polling unit was overwhelm since $\lambda > \mu$. In Gwagwalada and Abaji, as can be seen in Table 6 and Figure 2, we notice a waiting time of 4.80 minutes and 5.08 minutes respectively, which indicate manageable queues in the selected polling units due to the fact that $\lambda < \mu$. It should be observed that Kuje showed a queue length of 32.88 voters while AMAC produced an illogical negative value indicating system instability and overload due to high queue length since $\lambda > \mu$ underscoring the need for optimization.. This trend supports the hypothesis that longer queues are directly proportional to higher arrival rates, consistent with earlier findings by [10], who demonstrated that increased voter turnout in limited-resource environments leads to queue saturation and increased disenfranchisement. Also these findings corroborate [11], who simulated voter queue dynamics and find out that queue length exceeding 10 voters can deter voter participation in the voting process. The queue length peaked where the service rate (μ) was below or marginally above λ .

Waiting time in a queuing system is highly sensitive to λ and μ . Figure 2 illustrates the waiting time profile before optimization, where some study areas such as AMAC and Kuje experienced unstable queue behavior, as indicated by negative or exaggerated waiting times, due to service rates being lower than arrival rates. Empirical literature corroborates this pattern as seen in [12], who opined that election queues often become unmanageable when λ approaches or surpasses μ , particularly in urban or high-density polling stations. Their

analysis recommends dynamic resource reallocation, which our optimization model achieves through increased staffing.

Effect of Throughput rate, Utilization rate and Process Efficiency

Throughput, as shown in Table 7 and consistent with Equation (5), is directly equal to λ under stable conditions. Throughput reflects how well the system converts voters' arrivals into completed accreditations or votes. After optimization, all polling units achieved throughput levels matching arrival rates, thus eliminating bottlenecks. From our results, it should be noted that throughput is throttled in the selected polling unit is AMAC due to excessive arrival rate, hence voters accumulate in long queue as compared to area councils like Bwari (BAC) and Abaji (AAC), where voters arrivals are with minimal delay. The model demonstrates that throughput targets are achievable with optimized staffing, even within resource-limited scenarios.

Utilization rate (ρ) is an indicator of system load and saturation, reflecting the proportion of polling staff workload. This is a critical performance metric that represents the ratio of voter arrival rate (λ) to service rate (μ) and directly informs us about how heavily the accreditation and voting facilities were being used. According to [13], an ideal system should operate below 0.8, but anything above 0.8 suggests inefficiency or insufficient staffing. From Table 6, Table 7 and Figure 3, we observed that Abaji (AAC), Bwari (BAC) and Gwagwalada (GAC) all have utilization rates of 0.67, 0.57 and 0.65 respectively showing a well-balanced system, while Kuje (KAC), Abuja (AMAC) and Kwali (KWAC) have utilization rates of 0.97, 1.44 and 0.91 respectively showing a very close to saturation state vulnerable to collapse under slight increase in arrival rate for Kuje area council (KAC), a severely overloaded state guaranteeing excessive waiting and probable disenfranchisement for Abuja Municipal Area Council (AMAC) and a very close to saturation for Kwali area council (KWAC).

Both the throughput and utilization rate help to provide a balance between service capacity and demand, which provides a basis for measuring election voting process efficiency. Process efficiency is a combination of stable utilization rate ($\rho < 1.0$), minimal waiting time (W) and manageable queue length (L). Efficiency is optimal when λ is well below μ , queues remain short, and voters are processed quickly. Based on our results in Table 6 and Table 7, efficient polling areas are Bwari (BAC), Abaji (AAC) and Gwagwalada (GAC) area councils respectively, while inefficient and failing polling areas are Kuje (KAC) with $\rho = 0.97$, $W = 55.06$ minutes, Abuja (AMAC) with $\rho = 1.44$, $W = -3.76$ minutes and Kwali (KWAC) with $\rho = 0.91$, $W = 20.97$ minutes respectively. These findings match the conclusions in [14], where overloading due to high voters population led to inefficient electoral logistics. Moreover, the Colored Petri Net model confirms the non-linear impact of arrival spikes, that is a small increase in λ beyond μ drastically reduces efficiency and voter experience.

Effects of Optimization of Officer Allocation

Equation (7) was utilized to compute the optimal number of Assistant Polling Officers (APOs) required per polling unit, balancing between resource constraints and service quality. The model recommends up to four officers, a limit set by INEC guidelines. As seen in Table 7, optimization improved waiting times drastically in AMAC (from unstable to 2.94 minutes) and in Kuje (from 55 minutes to 1.58 minutes). Table 7 shows that officer allocation aligns proportionally with voter load. Notably, while Abaji and BAC required only one officer due to their low voter inflow and efficient APO service times, AMAC and Kuje required additional two officers for system stability. Table 6 and Table 7 further validates this strategy by comparing utilization rates before and after optimization, clearly indicating improvements and system stabilization ($\rho < 1$) across all units. This approach agrees with [3], who emphasized staffing flexibility and modular APO deployment as key factors in mitigating queue surges in high-volume voting environments.

Effects of Disenfranchisement and Voter Drop-off

By comparing number of voters accredited versus number of actual voters who cast their ballots, disenfranchisement rates were computed. The results showed that Abuja (AMAC) and Abaji (AAC) had the highest drop-off (21%), which is also where service inefficiencies and longer queue lengths were most evident prior to optimization. This again supports the assumption that long queues and waiting times discourage voter retention, a finding well-supported in the work of [15], who modeled voter fatigue as a function of queue delay.

In summary, major findings in this work include the following:

- Arrival Rate (λ) varied considerably across councils, with AMAC having the highest voter influx ($\lambda = 0.8722$) and Bwari the lowest ($\lambda = 0.3361$), reflecting demographic and logistical variations across the FCT.
- Utilisation Rate (ρ) served as a strong predictor of system performance. AMAC recorded $\rho = 1.44$, indicating a severely overloaded system, while Bwari ($\rho = 0.57$) and Abaji ($\rho = 0.67$) operated within optimal ranges.
- Waiting Time (W) was longest in Kuje (55.06 minutes) and unrealistic (negative) in AMAC due to over saturation, pointing to voter delays and potential disenfranchisement.
- Queue Lengths (L) were longest in Kuje (32.88 voters) and stable in Gwagwalada (1.88) and Bwari (1.30).

(e). Throughput was throttled in overloaded systems, where arrival exceeded service capacity (e.g., AMAC), resulting in voter congestion and service breakdown.

(f). The CPN model was effective in visualizing the discrete and stochastic nature of voter flow and bottlenecks. It provided a predictive framework to test "what-if" scenarios and optimize staff deployment.

These findings are consistent with earlier simulation-based studies on voter queues and polling logistics [3], [8].

V. Conclusion

This paper underscores the complexity of electoral queue dynamics and the importance of modeling tools in forecasting and resolving logistical challenges. The CPN model, grounded in stochastic process theory, proved invaluable for simulating real-world voter behavior and quantifying stress points within the system. Findings reveal that certain polling units in the FCT operate dangerously close to or beyond saturation. Without proactive interventions, this leads to extended voter queues, service delays, and in certain cases disenfranchisement of voters which can undermine the democratic process. Through simulation, this research provides actionable intelligence on how to structure polling stations, allocate staff, and manage voter flow for more transparent, timely, and equitable elections.

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