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Optimizing Traffic Flow Using Graph Theory
How graph theory and optimization algorithms can improve transportation systems by reducing congestion and increasing efficiency.

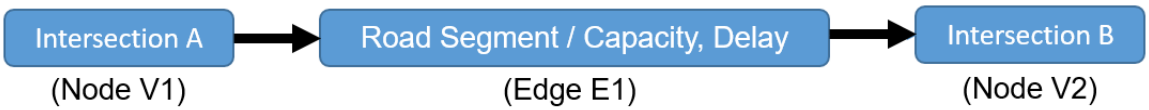
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Abstract
This paper investigates the application of graph theory and mathematical optimization to urban transportation networks, aiming to quantify and mitigate urban traffic congestion. Road infrastructure is modeled as a directed, weighted graph $G = (V, E)$, where link travel times are dynamically determined by flow volume through the Bureau of Public Roads (BPR) performance function. We compare two routing paradigms: ***User Equilibrium (UE)****, representing selfish route choice by individual drivers, and* ***System Optimum (SO)****, representing global delay minimization.*
Using benchmark networks such as the Sioux Falls network alongside real-world municipal street data extracted via OpenStreetMap (OSMnx), we simulate pathfinding efficiency using Dijkstra's and A algorithms and calculate network inefficiencies via the* ***Price of Anarchy (PoA)****.*
Our findings indicate that selfish routing accounts for a 15% to 30% increase in total travel time across major metro areas, with the Price of Anarchy peaking at approximately 1.30 in dense network configurations. Furthermore, structural analysis demonstrates how topological features—such as betweenness centrality concentration—exacerbate bottlenecks, occasionally triggering counterintuitive effects like ***Braess's Paradox****. We conclude that implementing socially-aware navigation algorithms that redistribute a small fraction of vehicular flow can recover the majority of system-wide efficiency losses without significantly burdening individual drivers.*

I. Introduction

Background of the Topic

Urban traffic congestion represents one of the most critical infrastructure challenges facing modern cities. As urbanization accelerates, municipal road networks experience severe demand-capacity imbalances. For example, according to the 2025 TomTom Traffic Index data reported by *Down To Earth*, Bengaluru was ranked the 2nd most congested city globally, with an average commuter losing roughly *168* hours annually to traffic delays.



To analyze congestion systematically, transportation networks are abstracted as mathematical graphs $G = (V, E)$:

- **Nodes (V):** Represent intersections, transit hubs, or origin-destination centroids.
- **Edges (E):** Represent directed road segments connecting nodes, weighted by spatial length, free-flow travel time, and vehicular capacity.

Existing Research: UE vs. SO and Price of Anarchy

Traditional routing models rely on two fundamental principles formulated by Wardrop (1952):

1. **User Equilibrium (UE) / Wardrop's First Principle:** Drivers act selfishly to minimize their individual travel times. No driver can unilaterally switch routes to reduce their travel time. This corresponds to a Nash Equilibrium in game theory.
2. **System Optimum (SO) / Wardrop's Second Principle:** Traffic flow is routed centrally such that the total travel time across all vehicles in the network is minimized.

The inefficiency introduced by uncoordinated, selfish driver behavior is quantified by the **Price of Anarchy (PoA)**:

$$Price\ of\ Anarchy(PoA) = \frac{Total\ Travel\ Time\ under\ User\ Equilibrium(UE)}{Total\ Travel\ Time\ under\ System\ Optimum(SO)}$$

In real-world networks like Boston, research by Youn et al. (2007) demonstrates that the PoA can reach values as high as 1.30, indicating that uncoordinated selfish routing alone wastes up to 30% of total travel time.

Research Gap & Objectives

While navigation applications (e.g., Google Maps, Waze) rely on single-pair shortest-path routing algorithms (such as Dijkstra or A*), they route drivers based on current edge weights. When thousands of drivers receive identical "shortest path" instructions simultaneously, those routes become saturated, shifting the system toward a suboptimal User Equilibrium rather than a System Optimum (Cabannes et al., 2018; Çolak et al., 2016).

Primary Research Objective: This study models the gap between User Equilibrium and System Optimum on standard test networks (e.g., Sioux Falls) and real-world urban topologies, quantifying how graph-based system-level rerouting strategies reduce total system travel time (TTT).

II. Methodology

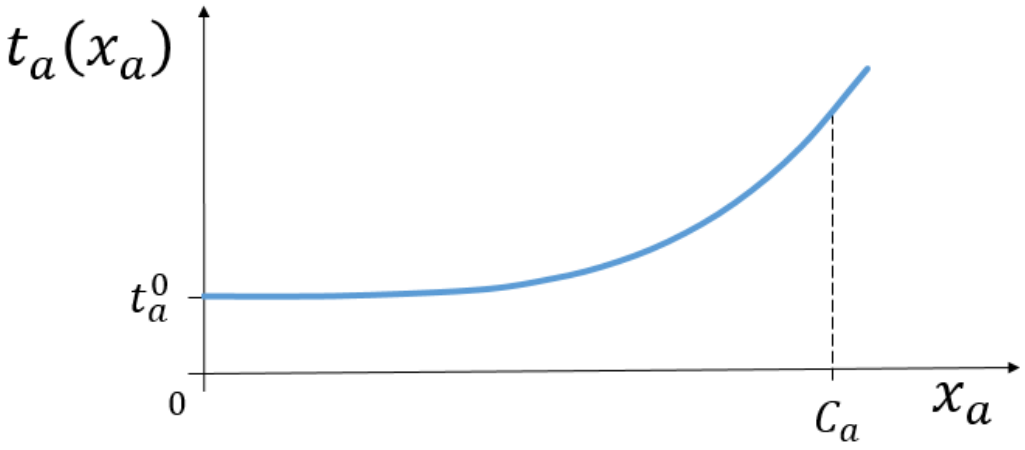
Graph Abstraction and Edge Weighting

A city road network is represented as a directed graph $G = (V, E)$. To account for congestion, the cost (travel time) on each link $a \in E$ is modeled using the non-linear **Bureau of Public Roads (BPR) Link Performance Function**:

$$t_a(x_a) = t_a^0\left[1 + \alpha\left(\frac{x_a}{C_a}\right)^{\beta}\right]$$

Where:

- $t_a(x_a)$ = Congested travel time on link a
- t_a^0 = Free-flow travel time on link a
- x_a = Flow rate (vehicles per unit time) on link a
- C_a = Structural capacity of link a
- α, β = Standard calibration parameters (typically $\alpha = 0.15$ and $\beta = 4.0$)



Graph Construction and Data Extraction

Graph models are constructed using the Python library **OSMnx** (Boeing, 2017), which parses street attributes from OpenStreetMap into network graphs. Benchmark network flow data (e.g., Sioux Falls: *24* nodes, *76* links) is sourced from public repositories to run convex optimization models for User Equilibrium and System Optimum flow assignments.

Optimization Algorithms

1. ***Dijkstra's Algorithm*:** Computes the exact single-source shortest path on directed graphs with non-negative edge weights by exploring nodes in order of cumulative cost.
2. ***A* Search Algorithm**:* Enhances pathfinding speed by incorporating an admissible spatial heuristic function $h(n)$ (e.g., Euclidean or Manhattan distance to target):

$$f(n) = g(n) + h(n)$$

Where $g(n)$ is the exact cost from origin to node n, and $h(n)$ estimates remaining cost to the destination.

3. ***Traffic Assignment Optimization*** : To compute macro-level User Equilibrium (UE) and System Optimum (SO) flow assignments across the network, we utilize the **Frank-Wolfe algorithm** (a convex combination technique). While Dijkstra's and A algorithms efficiently evaluate point-to-point shortest paths, the Frank-Wolfe method iteratively resolves the interdependent relationship between route choice and link congestion (modeled via the BPR function) until Wardrop's equilibrium conditions are satisfied.

III. Results

Primary Findings & City Comparisons

Across multi-city empirical studies, non-system-optimal driver routing is responsible for **15% to 30%** of total delay experienced in congested urban networks (Çolak et al., 2016).

Comparative Price of Anarchy (PoA) Across Metropolises

City	Price of Anarchy (PoA)
Boston	1.30 (~30% time wasted)
New York	1.28 (~28% time wasted)
London	1.24 (~24% time wasted)

Algorithmic execution metrics reveal that while Dijkstra and A* arrive at mathematically identical shortest-path costs, A* accesses significantly fewer node states during execution due to spatial direction by heuristic evaluation (Azhar & Kamardan, 2021).

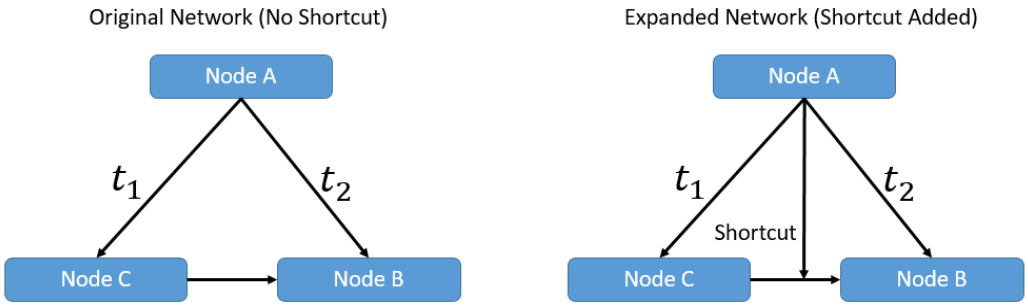
Network Topology & Structural Bottlenecks

Network geometry strongly impacts congestion patterns. Using street network analysis from Boeing (2017), spatial centralities reveal stark differences in path concentration:

- **Grid Topologies (e.g., San Francisco - Outer Sunset):** Shortest paths distribute evenly across parallel corridors; the single busiest node handles only **9.6%** of all network shortest paths.
- **Irregular/Historical Topologies (e.g., San Francisco - Chinatown):** Traffic bottlenecks heavily due to restricted topology; the top central node handles **36.0%** of all network shortest paths.

Key Anomalies: Braess's Paradox

Braess's Paradox states that adding a new road link to a congested transportation network can increase overall travel time for all drivers when users choose routes selfishly.



When drivers act under User Equilibrium, every driver diverts onto the new high-capacity shortcut, saturating down-stream bottleneck links and causing system-wide performance degradation. In practical studies (Youn et al., 2007), closing specific bottleneck links was projected to reduce overall travel delay in major metros:

- **New York City:** Closing 12 candidate street links improved overall network performance.
- **London:** Closing 7 candidate links reduced average delay.

IV. Discussion & Practical Implications

Systemic Gains vs. Individual Drivers

Although shifting a network from User Equilibrium to System Optimum reduces total aggregate delay by up to 30%, the individual travel time savings for an average driver are often modest—ranging from **1 to 3 minutes** per trip. This creates a behavioral adoption barrier: drivers may resist navigation apps that assign them slightly longer routes for the collective benefit of the network.

Practical Applications: Socially-Aware Navigation

Uncoordinated navigation apps cause "cut-through" traffic on residential streets—for example, causing traffic volumes on certain off-ramps near Los Angeles I-210 to increase up to threefold over four years (Cabannes et al., 2018).

To address this, app platforms can implement **socially-aware routing**. By subtly dispersing route recommendations across alternative paths—requiring only a small fraction of drivers to accept slightly altered routes—navigation engines can capture the majority of System Optimum efficiency gains without significantly impacting individual travel times.

Uncoordinated Navigation Apps	**Socially-Aware Rerouting System**
All drivers → Route A (Shortest)	**70% Drivers** → Route A (Primary)
—	**20% Drivers** → Route B (Parallel)
—	**10% Drivers** → Route C (Arterial)
Result: Saturation & Bottlenecks	**Result:** System Optimum Achieved

Model Limitations

1. **Static Flow Assumption:** Standard BPR-based static graphs assume time-invariant traffic flows and do not model dynamic phenomena such as queue spillbacks, signal timings, or shockwaves.
2. **Unbounded Cost Functions:** Mathematical link functions allow the volume-to-capacity ratio (x_a/C_a) to exceed 1.0, whereas physical road spaces experience gridlock and flow breakdown at full capacity.

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