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# An Approximate Solution Of Fingero Phenomenon With Inclination By Using The Finite Difference Schmidt Method

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***Abstract:***
*This paper represents an approximate solution of the fingero phenomenon with inclination. The fingering phenomenon is a well-known instability phenomenon that occurs when a fluid contained in a porous medium is displaced by another of lower viscosity, leading to the formation of perturbations or "fingers" that advance rapidly through the medium. This phenomenon is particularly significant in petroleum engineering, where water injection is commonly used for oil recovery. The fingering phenomenon in two-phase flow through homogeneous porous media, specifically considering oil and water with a mean capillary pressure. The mathematical formulation leads to a second-order linear partial differential equation governing the displacement process. The problem is modelled as water being injected into a dipping oil-saturated porous medium, resulting in well-developed finger structures. An approximate solution is obtained by using the finite difference Schmidt method and appropriate initial and boundary conditions. The solution of the problem is shown in tables with graphs by using MATLAB.*

***Key Words**: Fingero phenomenon; Homogeneous porous medium; Inclined porous medium; Double phase; Schmidt method; Immiscible fluids; Capillary pressure; Phase saturation*

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## I. Introduction

It is a well-known physical fact that when a fluid contained in a porous medium is displaced by another of lesser viscosity instead of regular displacement of whole front, perturbations (fingers) occur which shoot through the porous medium at relatively great speeds. This phenomenon of occurrence of instabilities is called fingering [1] In fig. 1 fingering process shows oil-water flow into a porous medium. In petroleum engineering, the fingering process is well known phenomenon occurring in displacement of oil by water injection which is common in oil by water injection which is common in oil recovery process. This problem has great importance for oil recovery processes petroleum technology.

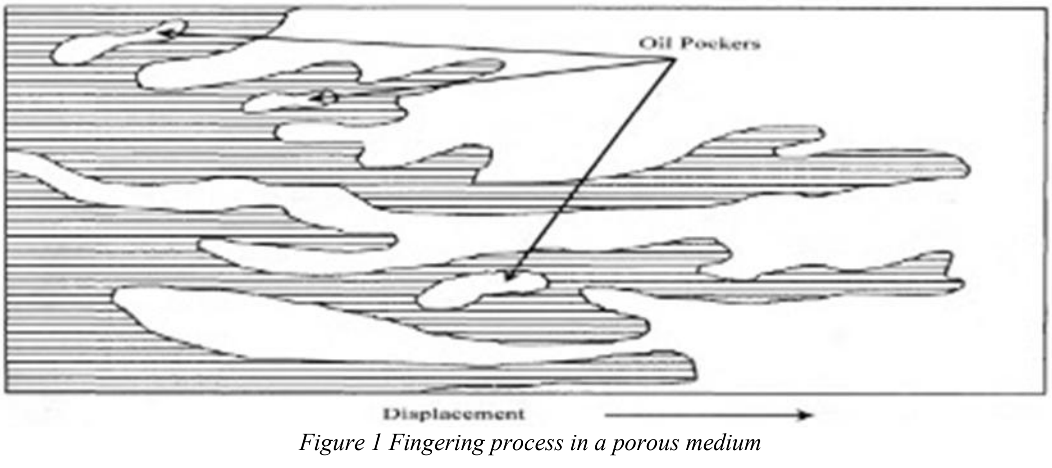


*Figure 1 Fingering process in a porous medium*

Many authors have discussed such an important problem from different point of view, for example Scheidegger and Johnson [2] discussed only the average cross-sectional area occupied by the fingers and it was taken into account that the size and shape of individual fingers are disregarded. Also introduced the idea of discussing the statistical behaviour of instabilities in homogeneous porous media and considered the phenomenon without the effect of capillary pressure. Verma [3] examined the behaviour of fingering phenomenon in a displacement process through heterogeneous medium from statistical point of view. Scheidegger [4], Chouke [5], Jecquard [6], Marle [7], Verma [8] have investigated this phenomenon with different aspects. Mishra [9], Mehta [10], Patel [11], Patel [12], Tailor [13] Mukhserjee [14] and Shome [15] also have discussed this phenomenon in different point of view with means pressure by employing different mathematical technique to get more accurate result. Patel [16] has obtained an approximate solution of instability phenomenon in heterogeneous porous media with mean pressure in terms of ascending power series. This phenomenon is also discussed by K.K.Patel [17] with inclination and applied Homotopy analysis method to get approximate solution and Shah [18] has obtained approximate solution by homotopy perturbation new integral transform method. Shah [19] has obtained a mathematical solution of fingering phenomenon in vertical downward direction through heterogeneous porous medium by using variational iteration method. More [20] applied reduced differential transform method to solve this phenomenon in double phase flow through heterogeneous porous media for vertical downward direction. The phenomenon of instability occurring in inclined porous media is solved using optimal homotopy analysis method [21]

In this paper, we have discussed the phenomenon of fingering (instabilities) in double phase flow through homogenous porous media (namely oil and water) with mean capillary pressure. The mathematical formulation yield to linear partial differential equation.

Instabilities or fingering in porous media are the topic of this section. In the secondary recovery process of petroleum technology, it is very significant.

It is a well-known physical fact that when a fluid contained in a porous medium is displaced by another fluid with lower viscosity, protuberances may form instead of the regular displacement of the entire fluid and shoot through the porous medium at a relatively high speed. Because the protuberances resemble the shape of fingers, this phenomenon of instabilities is known as fingering. Fig. 2. The concept of examining the statistical behavior of this phenomenon in homogeneous porous media without taking into account capillary pressure was first proposed by Scheidegger and Johnson [2] . Only the cross-sectional area of the fingers is considered in the statistical analysis; the size and shape of each individual finger are ignored. Fig. 2.

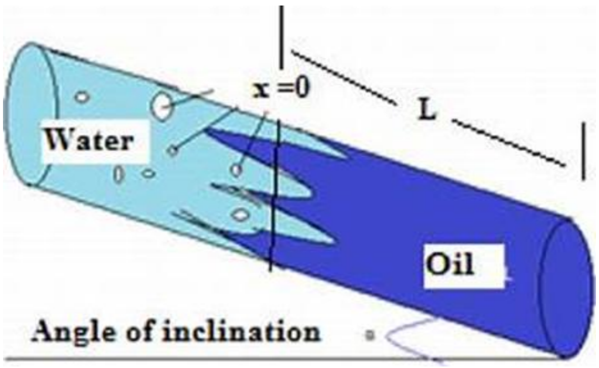


*Figure 2 Representation of the fingering phenomenon in a cylindrical piece of inclined porous media*

For a cartesian value of X and t, fingers are present in the porous medium if our displacement process is parallel to the X direction and advances over time. The average relative area occupied by the i[th] fluid at level 'x' is the definition of the saturation of the i[th] fluids (Si), which is a function of 'x' and 't. i.e. $S_i = S_i(x,t)$.

As a result, the fingers' relative cross-sectional area represents the saturation of the displacing fluid in the porous medium. The description of the displacement of immiscible fluids, in which the seepage velocity of phases is determined by Darcy's law, becomes formally identical with the treatment of fingers with the idea of fictitious relative permeability.

$$K_w = S_w^2 \tag{1.1}$$

$$K_0 = S_0 = 1 - S_w^2 \tag{1.2}$$

Where $K_w$ and $K_0$ denote the fictitious relative permeability of water and oil, respectively. and $S_w$, $S_0$ stand for their saturations. The stabilization of fingers under particular circumstances has since been discussed in a number of studies by Verma [22, 23] and Chouke [24]

## II. Statement Of Problem

In the investigated problem we consider that water is injected with given velocity into a dipping oil-saturated porous media of homogeneous physical characteristics, such that the injecting water shoot through the oil formation and gives rise to a well-developed finger flow. The schematics fingers are given by fig. 2. The displacement of an entire oil at the initial boundary $X = 0$ (X being measured in the direction of displacement) through a small distance due to the water injection, which is assumed that the initial saturation $S_{w_0}$ exists at common interface.

In the statistical treatment of finger [4] only the average behaviour of the two fluids involved is taken into consideration in this problem. The saturation of water $(S_w)$ is defined as the average cross-sectional area occupied by the water at level $X$ for $t > 0$ is $S_w(X, t)$. Thus the saturation of displacing fluid in porous medium represents the average cross-sectional area occupied by fingers [25].

Assuming that the flow of two immiscible liquids is governed by Darcy's law, we may write the seepage velocities of water $(V_w)$ and oil $(V_o)$ can be expressed as

$$V_w = -\frac{K_w}{\delta_w} K \left[ \left( \frac{\partial P_w}{\partial X} + \rho_w \cdot g \cdot \sin\alpha \right) \right] \quad (3.1)$$

$$V_o = -\frac{K_o}{\delta_o} K \left[ \left( \frac{\partial P_o}{\partial X} + \rho_o \cdot g \cdot \sin\alpha \right) \right] \quad (3.2)$$

Where,
K= permeability of the homogeneous medium,
$K_w$ & $K_o$ = relative permeability of water and oil respectively which are the functions of $S_w$ and $S_o$
$S_w$ & $S_o$ = saturation of water and oil respectively
$P_w$ & $P_o$ = pressure of water and oil respectively
$\delta_w$ and $\delta_o$ are the constant kinematics viscosities
$\alpha =$ the inclination of the bed
g = gravitational constant
$\rho_w$ and $\rho_o$ are constant densities of water and oil respectively.
The equation of continuity of the two phases (phase densities are regarded as constant) are given by

$$P\left(\frac{\partial S_w}{\partial t}\right) + \left(\frac{\partial V_w}{\partial X}\right) = 0 \quad (3.3)$$

$$P\left(\frac{\partial S_o}{\partial t}\right) + \left(\frac{\partial V_o}{\partial X}\right) = 0 \quad (3.4)$$

Where,
P = Porosity of the medium,
The definiteness of phase saturation is that

$$S_w + S_o = 1 \quad (3.5)$$

As we have discussed in the previous problem, capillary pressure $P_c$ defined as the pressure discontinuity between the phases across their common interface, is a function of the phase saturation, we may write as

$$P_c = \beta(S_w^{-1} - C) \quad (3.6)$$

$$P_c = P_o - P_w \quad (3.7)$$

Where $\beta$ and $C$ is the constant

For definiteness, the following important relationships between the saturation $S_w$ and $S_o$ and the fictitious relative permeability $K_w$ and $K_o$ have been assumed

$$K_w = S_w^2 \quad (3.8)$$

$$K_o = S_0 = 1 - S_w^2 \quad (3.9)$$

## III. Mathematical Formulation Of The Problem

The equation of motion for saturation can be obtained by substituting the values of $V_w$ and $V_o$ form equations (3.1) and (3.2) into equations (3.3) and (3.4), respectively, we get

$$P\left(\frac{\partial S_w}{\partial t}\right) = \frac{K_w}{\delta_w} K \left(\frac{\partial^2 P_w}{\partial X^2}\right) + \left(\frac{K}{\delta_w}\right)\left(\frac{\partial K_w}{\partial X}\right)\left(\frac{\partial P_w}{\partial X}\right) + \left(\frac{K_w}{\delta_w}\right)\left(\frac{\partial K}{\partial X}\right)\left(\frac{\partial P_w}{\partial X}\right) + \rho_w \cdot g.\sin\alpha \frac{\partial}{\partial X}\left(\frac{K_w}{\delta_w} K\right) \quad (4.1)$$

$$P\left(\frac{\partial S_o}{\partial t}\right) = \frac{K_o}{\delta_o} K \left(\frac{\partial^2 P_o}{\partial X^2}\right) + \left(\frac{K}{\delta_o}\right)\left(\frac{\partial K_o}{\partial X}\right)\left(\frac{\partial P_o}{\partial X}\right) + \left(\frac{K_o}{\delta_o}\right)\left(\frac{\partial K}{\partial X}\right)\left(\frac{\partial P_o}{\partial X}\right) + \rho_o \cdot g \cdot \sin\alpha \frac{\partial}{\partial X}\left(\frac{K_o}{\delta_o} K\right) \quad (4.2)$$

When effects due to pressure discontinuity and gravity term in an inclined porous medium, the general flow equations of the phases in the homogeneous medium are considered here.

## IV. Instability With Capillary Pressure

In this part, we examine the behaviour of fingering in a homogeneous porous medium with capillary pressure. The equation of motion for saturation obtained by substituting the values of $V_w$ and $V_o$ from the equation (3.3) and (3.4), respectively, we get

$$P\left(\frac{\partial S_w}{\partial t}\right) = \frac{\partial}{\partial X}\left[\frac{K_w}{\delta_w} K \left(\frac{\partial P_w}{\partial X} + \rho_w \cdot g \cdot \sin\alpha\right)\right] \quad (5.1)$$

$$P\left(\frac{\partial S_o}{\partial t}\right) = \frac{\partial}{\partial X}\left[\frac{K_o}{\delta_o} K \left(\frac{\partial P_o}{\partial X} + \rho_o \cdot g \cdot \sin\alpha\right)\right] \quad (5.2)$$

Substituting the value of $\frac{\partial P_w}{\partial X}$ from (3.7) into (5.1), we obtain

$$P\left(\frac{\partial S_w}{\partial t}\right)=\frac{\partial}{\partial X}\left[\frac{K_w}{\delta_w}\,K\,\left(\frac{\partial P_o}{\partial X}-\frac{\partial P_c}{\partial X}+\rho_w\cdot g\cdot\sin\alpha\right)\right] \quad (5.3)$$

From the combination of equations (5.2) and (5.3) and using (3.5), we get

$$\frac{\partial}{\partial X}\left[\begin{array}{c}\left(\frac{K_w}{\delta_w}\,K+\frac{K_o}{\delta_o}\,K\right)\frac{\partial P_o}{\partial X}-\ \frac{K_w}{\delta_w}\,K\left(\frac{\partial P_c}{\partial X}\right)+\\ \left(\frac{K_w}{\delta_w}\,K\rho_w+\frac{K_o}{\delta_o}\,K\rho_o\right)g\cdot\sin\alpha\end{array}\right]=0 \quad (5.4)$$

Integrating (5.4) with respect to $X$, we obtain

$$\left[\begin{array}{c}\left(\frac{K_w}{\delta_w}\,K+\frac{K_o}{\delta_o}\,K\right)\frac{\partial P_o}{\partial X}-\ \frac{K_w}{\delta_w}\,K\left(\frac{\partial P_c}{\partial X}\right)+\\ \left(\frac{K_w}{\delta_w}\,K\rho_w+\frac{K_o}{\delta_o}\,K\rho_o\right)g\cdot\sin\alpha\end{array}\right]=-q \quad (5.5)$$

Where $q$ is the integrating constant.
Simplification of (5.5)

$$\frac{\partial P_o}{\partial X}=\frac{1}{\left(1+m\frac{K_o}{K_w}\right)}\frac{\partial P_c}{\partial X}-\frac{q}{\left(\frac{K_w}{\delta_w}K+\frac{K_o}{\delta_o}K\right)}-\frac{\left(\rho_w+\frac{K_o}{K_w}m\,\rho_o\right)}{\left(1+m\frac{K_o}{K_w}\right)}\cdot g\cdot\sin\alpha \quad (5.6)$$

Where $m=\frac{\delta_w}{\delta_o}$

Using (5.6) and (5.3), we have

$$P\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial X}\left[\frac{K\frac{K_o}{\delta_o}}{\left(1+m\frac{K_o}{K_w}\right)}\left(\frac{\partial P_c}{\partial X}-(\rho_w-\rho_o)\cdot g\cdot\sin\alpha\right)+\frac{q}{\left(1+m\frac{K_o}{K_w}\right)}\right]=0 \quad (5.7)$$

The value of the pressure of oil ($P_o$) can be written as in Oroveanu [26] in the form

$$P_o=\frac{P_o+P_w}{2}+\frac{P_o-P_w}{2}=\bar{P}+\frac{1}{2}P_c \quad (5.8)$$

Where $\bar{P}$ is the mean pressure which is constant, equation (5.8) written as

$$\frac{\partial P_o}{\partial X}=\frac{1}{2}\frac{\partial P_c}{\partial X} \quad (5.9)$$

Using (5.6) and (5.2), we get

$$q=\frac{1}{2}\left(\frac{K_w}{\delta_w}\,K-\frac{K_o}{\delta_o}\,K\right)\frac{\partial P_c}{\partial X}-\left(\frac{K_w}{\delta_w}\,K\rho_w+\frac{K_o}{\delta_o}\,K\rho_o\right)g\cdot\sin\alpha \quad (5.10)$$

Substituting the value of q from equation (5.10) in (5.7), we get

$$P\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial X}\left[\frac{K_w}{\delta_w}\,K\left(\frac{1}{2}\frac{\partial P_c}{\partial S_w}\frac{\partial S_w}{\partial x}-\rho_w\cdot g\cdot\sin\alpha\right)\right]=0 \quad (5.11)$$

Substituting the values of $K_w$ and $P_c$ from (3.7) and (4.1), we get

$$P\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial X}\left[\frac{S_w^2}{\delta_w}\,K\left(-\frac{\beta}{2}S_w^{-2}\frac{\partial S_w}{\partial x}-\rho_w\cdot g\cdot\sin\alpha\right)\right]=0 \quad (5.12)$$

$$P\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial X}\left[\frac{S_w^2}{\delta_w}\,K\left(-\frac{\beta}{2}\right)S_w^{-2}\frac{\partial S_w}{\partial x}-\frac{S_w^2}{\delta_w}\,K\rho_w\cdot g\cdot\sin\alpha\right]=0 \quad (5.13)$$

$$P\left(\frac{\partial S_w}{\partial t}\right)-\frac{1}{2}\frac{\beta}{\delta_w}K\frac{\partial^2 S_w}{\partial X^2}-2K\frac{\rho_w}{\delta_w}\cdot g\cdot\sin\alpha\,S_w\frac{\partial S_w}{\partial x}=0 \quad (5.14)$$

Rewriting equation (1.6.14)

$$P\left(\frac{\partial S_w}{\partial t}\right)-\gamma\frac{\partial^2 S_w}{\partial X^2}-\eta S_w\frac{\partial S_w}{\partial x}=0 \quad (5.15)$$

Where, $\gamma=\frac{1}{2}\frac{\beta}{\delta_w}K$

$$\eta=2K\frac{\rho_w}{\delta_w}\cdot g\cdot\sin\alpha$$

With boundary condition

$$S_w(0,t)=0.1;\ \ t\geq 0$$
$$S_w(1,t)=1;\ t\geq 0$$
$$S_w(x,0)=0.05;\ 0<x<1$$

This is the second-order partial differential equation which is the governing equation.

## V. Solution Of The Problem

We get,

$$0.5\,\frac{u_{i,j+1}-u_{i,j}}{k}-0.5\,\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{h^2}-0.5u_{i,j}\,\frac{u_{i,j}-u_{i-1,j}}{h}=0 \quad (6.4)$$

$$0.5\,\frac{u_{i,j+1}-u_{i,j}}{k}=0.5\,\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{h^2}+0.5u_{i,j}\,\frac{u_{i,j}-u_{i-1,j}}{h} \quad (6.5)$$

$$\frac{u_{i,j+1}-u_{i,j}}{k}=\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{h^2}+u_{i,j}\,\frac{u_{i,j}-u_{i-1,j}}{h} \quad (6.6)$$

$$u_{i,j+1}-u_{i,j}=\frac{k}{h^2}\left[u_{i-1,j}-2u_{i,j}+u_{i+1,j}\right]+\frac{k}{h}u_{i,j}\left[u_{i,j}-u_{i-1,j}\right] \quad (6.7)$$

$$u_{i,j+1} = u_{i,j} + \frac{k}{h^2}\left[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right] + \frac{k}{h}u_{i,j}\left[u_{i,j} - u_{i-1,j}\right] \quad (6.8)$$

This expression represents the explicit scheme for the governing partial differential equation. The solution of the expression in the form of tables and graphs is obtained using MATLAB.

## VI. Numerical And Graphical Representation Of Solution

| t | x=0 | x=0.25 | x=0.5 | x=0.75 | x=1 |
|---|---|---|---|---|---|
| 0 | 0.1 | 0.05 | 0.05 | 0.05 | 1 |
| 0.025 | 0.1 | 0.06975 | 0.05 | 0.43 | 1 |
| 0.05 | 0.1 | 0.07373901 | 0.20980125 | 0.52234 | 1 |
| 0.075 | 0.1 | 0.13847466 | 0.28324646 | 0.60471365 | 1 |
| 0.1 | 0.1 | 0.18152629 | 0.35802522 | 0.65368087 | 1 |
| 0.125 | 0.1 | 0.22099526 | 0.41200702 | 0.69327271 | 1 |
| 0.15 | 0.1 | 0.2516758 | 0.45597841 | 0.72295673 | 1 |
| 0.175 | 0.1 | 0.27654384 | 0.49036445 | 0.74628409 | 1 |
| 0.2 | 0.1 | 0.29633676 | 0.51768906 | 0.76450147 | 1 |
| 0.225 | 0.1 | 0.31216116 | 0.53933227 | 0.77884476 | 1 |
| 0.25 | 0.1 | 0.32478799 | 0.55652089 | 0.79015617 | 1 |
| 0.275 | 0.1 | 0.3348668 | 0.57017826 | 0.79910043 | 1 |
| 0.3 | 0.1 | 0.34290957 | 0.58103949 | 0.80618457 | 1 |
| 0.325 | 0.1 | 0.34932731 | 0.58968184 | 0.81180356 | 1 |
| 0.35 | 0.1 | 0.35444788 | 0.59656199 | 0.81626537 | 1 |
| 0.375 | 0.1 | 0.35853322 | 0.60204131 | 0.8198115 | 1 |
| 0.4 | 0.1 | 0.36179244 | 0.60640634 | 0.82263187 | 1 |

Table 1 The numerical value of saturation of water $S_w$

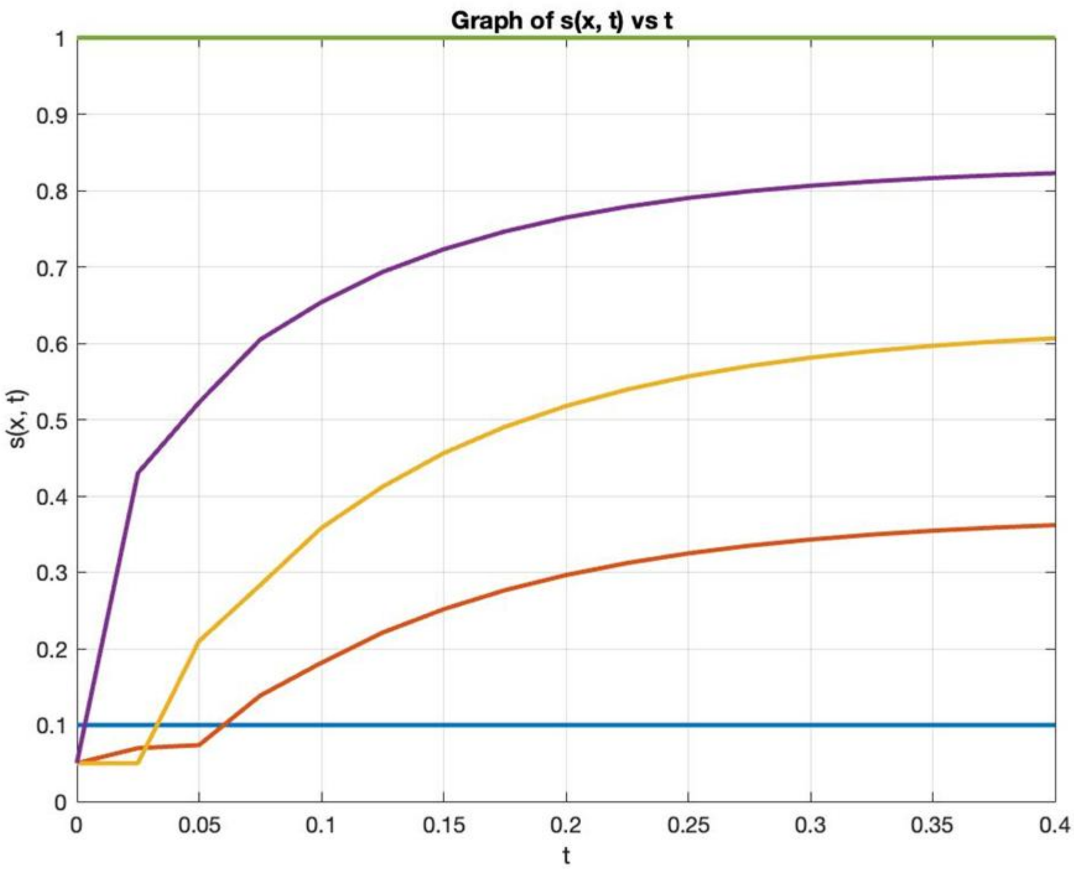


*Figure 3 Graph of* $S_w(x,t) \rightarrow t$

| x | t=0 | t=0.025 | t=0.05 | t=0.075 | t=0.1 | t=0.125 |
|---|---|---|---|---|---|---|
| 0 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 0.25 | 0.05 | 0.06975 | 0.07373901 | 0.13847466 | 0.18152629 | 0.22099526 |
| 0.5 | 0.05 | 0.05 | 0.20980125 | 0.28324646 | 0.35802522 | 0.41200702 |
| 0.75 | 0.05 | 0.43 | 0.52234 | 0.60471365 | 0.65368087 | 0.69327271 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| x | t=0.15 | t=0.175 | t=0.2 | t=0.225 | t=0.25 | t=0.275 |
|---|---|---|---|---|---|---|
| 0 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 0.25 | 0.2516758 | 0.27654384 | 0.29633676 | 0.31216116 | 0.32478799 | 0.3348668 |
| 0.5 | 0.45597841 | 0.49036445 | 0.51768906 | 0.53933227 | 0.55652089 | 0.57017826 |
| 0.75 | 0.72295673 | 0.74628409 | 0.76450147 | 0.77884476 | 0.79015617 | 0.79910043 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| x | t=0.3 | t=0.325 | t=0.35 | t=0.375 | t=0.4 |
|---|---|---|---|---|---|
| 0 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 0.25 | 0.34290957 | 0.34932731 | 0.35444788 | 0.35853322 | 0.36179244 |
| 0.5 | 0.58103949 | 0.58968184 | 0.59656199 | 0.60204131 | 0.60640634 |
| 0.75 | 0.80618457 | 0.81180356 | 0.81626537 | 0.8198115 | 0.82263187 |
| 1 | 1 | 1 | 1 | 1 | 1 |

*Table 2 The numerical value of saturation of water $S_w$*

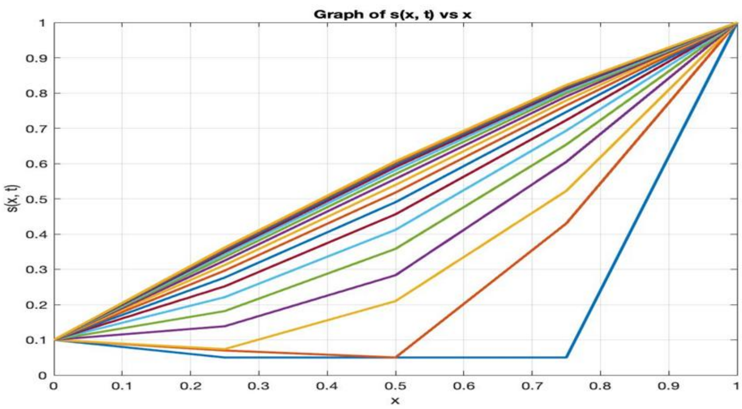


*Figure 4 Graph of $S_w(x,t) \rightarrow x$*

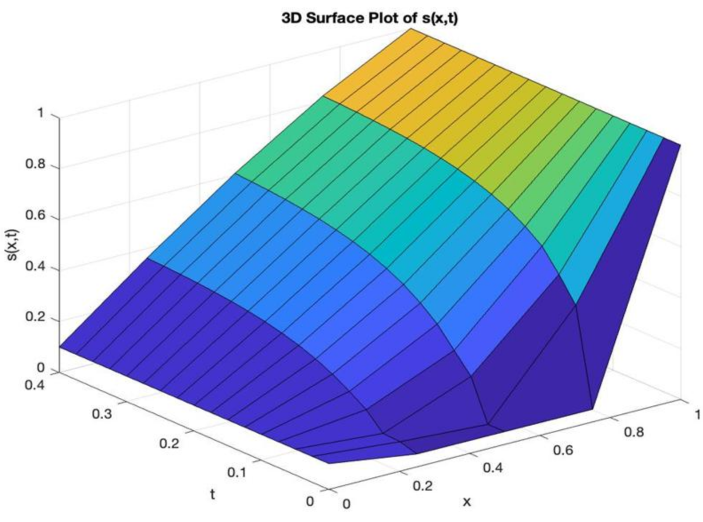


*Figure 5 3D surface plot of $S_w(x,t)$*

## VII. Conclusion

Value of saturation of water $S_w$ at different points of $x$ and $t$ are given in the above table 1. The figure 3 represent the graph of $S_w(x,t)$ vs $t$. The graph of $S_w(x,t)$ vs $x$ is defined in the figure 4. Now we can generalize the problem with further iterations and the solution becomes clearer. For that we used MATLAB programming and get the nearest solution of the problem. The 3D graph of value of saturation of water $S_w$ at different points of $x$ and $t$ shows in figure 5. This states that saturation of water $S_w$ increasing when $x$ and $t$ increasing.

## References


[1]. Scheidegger, The Physics Of Flow Through Porous Media (3rd Edition), Toronto: University Of Toronto Press, 1960, Pp. 229-230.
[2]. E. Scheidegger And E. F. Johnson, "The Statistical Behaviour Of Instabiliies In Displacement Processes In Porous Media," Canadian Journal Of Physics , Vol. 39, No. 2, Pp. 326-334, 1961.
[3]. Verma, "Motion Of Immiscible Liquids In A Cracked Heterogeneous Porous Medium With Capillary Pressure," Rev. Roun. Science And Techlogy Ser. Mecan. Appl., Vol. 13, No. 2, Pp. 277-292, 1968.
[4]. E. Scheidegger, "Growth Of Instabilities On Displacement Fronts In Porous Media," Physics Of Fluids, Vol. 3, No. 1, Pp. 94-104, 1960.
[5]. R. Chuoke, P. V. Meurs And C. V. D. Poel, "The Instability Of Slow, Immiscible, Viscous Liquid-Liquid Displacements In Permeable Media," Petroleum Transactions, Vol. 216, No. 1, Pp. 188-194, 1959.
[6]. P. Jecquard And P. Sequier, "Movement Dedeux Fluids On Contact Unmilien Porex," J. Mech., Vol. 1, No. 4, 1940.
[7]. Marle, "Cours De Production ( In Inless Ecoulemen's Polyphasique)," Inst. Fr. Petrol, 1965.
[8]. Verma, "Fingero-Imbibiton In Slightly Dipping Porous Medium," Journal Of Applied Physics, Vol. 41, No. 9, Pp. 3638-3642, 1970.
[9]. S. Mishra And A.P.Verma, "Similarity Solution For Instabilites In Double Phase Flow Through Porous Media," Journal Of Applied Physics, Vol. 44, No. 4, Pp. 1622-1624, 1973.
[10]. M.N.Mehta, "An Approximate Soluiton Of Double Phase Flow Due To Differential Wettablity," Journal Of Indain Academic Maths, Vol. 16, No. 2, Pp. 157-162, 1966.
[11]. S. Patel, An Approximate Solution Of Displacement Processes In Porous Media, Ph.D. Thesis, South Gujarat University, 1997.
[12]. R. Patel, An Approximate Of Laplace Transformation In Phenomeno Of Fingering Through Homogeneous Porous Media, Ph.D Thesis, South Gujarat University, 1999.
[13]. M. R. Tailor, An Approximate Solution Of Polyphase Fluid Flow Through Porous Media, Ph.D Thesis, South Gujarat University, 2002.
[14]. Mukherjee And S. P., "An Analytic Solution Of Fingering Phenomenon Arising In Fluid Flow Through Porous Media By Using Techniques Of Calculus Of Variation And Similarity Theory," Journal Of Mathematical Research, Vol. 1, No. 2, Pp. 64-72, 2009.
[15]. M. Prajapati, Numerical Technique In Fluid Flow Through Porous Media, Ph.D Thesis, Pacific University, 2017.
[16]. K. R. Patel, M. N. Mehta And T. R. Patel, "An Approximate Solution Of Instability Phenomenon In Heterogeneous Porous Media With Mean Pressure," General Mathematics Notes, Vol. 10, No. 2, Pp. 9-21, 2012.
[17]. K. K. Patel, M. N. Mehta And T. R. Singh., "Application Of Homotopy Analysis Method In One-Dimensional Instability Phenomenon Arising In Inclined Porous Media," American Journal Of Applied Mathematics And Statistics, Vol. 2, No. 3, Pp. 106-114, 2014.
[18]. K. Shah And T. Singh, "The Combined Approach To Obtain Approximate Analytical Solution Of Instability Phenomenon Arising In Secondary Oil Recovery Process," Computational And Applied Mathematics , Vol. 37, Pp. 3593-3607, 2018.
[19]. A. Shah And A. K. Parikh, "Athematical Solution Of Fingering Phenomenon In Vertical Downward Direction Through Heterogeneous Porous Medium," Advances In Mathematics: Scientific Journal, Vol. 10, No. 1, Pp. 483-496, 2021.
[20]. P. A. More And P. V. Tandel, "Solution Of Fingering Phenomenon In Double Phase Flow Through Heterogeneous Porous Media For Vertically Downward Direction," International Journal Of Mathematics Trends And Technology, Vol. 67, No. 9, Pp. 118-129, 2021.
[21]. Prajapati And N. B. Desai, "Analytic Analysis For Oil Recovery During Cocurrent Imbibition In Inclined Homogeneous Porous Medium," International Journal On Recent And Innovation Trends In Computing And Communication, Vol. 5, No. 7, Pp. 189-194, 2017.
[22]. Verma, "On Stabilization Of Fingers In A Slightly Cracked Heterogeneous Porous Medium," In Developments In Soil Science, Elsevier, 1972, Pp. 221-228.
[23]. Verma, "Instabilities In Two Phases Flow Through Porous Media With Magnetic Field," In Multi-Phase Transport: Fundamentals, Reactor Safety, Applications, Volume 2, Hemisphere Publishing Corporation, 1980, Pp. 1323-1335.
[24]. E. Chouke, A. Scheidegger And C. Chow, "Petrol. Trans. Aime," Journal Of Hydrology, Vol. 15, No. 1, P. 216, 1952.
[25]. Verma, "Statistical Behavior Of Fingering In A Displacement Process In Heterogeneous Porous Medium With Capillary Pressure," Canadian Journal Of Physics , Vol. 47, No. 3, Pp. 319-324, 1969.
[26]. T. Oroveanu, "Scurgerea Fluidelor Multifazice Prin Meddi Poroase Eriture Academic," Repulic, Socialistic Remania, Pp. 268-271,329,325, 1966.
[27]. C.G.Mingham And D.M.Causon, Introductory Finite Difference Methods For Pdes, Ventus Publishing Aps, 2010.