IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-0661,p-ISSN: 2278-8727, Volume 20, Issue 1, Ser. 1 (Jan. – Feb. 2024), PP 09-17
www.iosrjournals.org

"Stability Of Charged Black Holes In String Theory"

Prabir Saha
Dr. Farook Rahaman
Jadavpur University
188, Raja S.C. Mallick Rd, Kolkata 700032.

Abstract
The charged black hole in string theory is firstly found by Gibbons and Maeda and independently found by Garfinkle,Horowitz, and Strminger. Reissner-Nordström Anti-de Sitter (RNAdS) black holes are unstable against the charged scalar field perturbations due to the well-known superradiance phenomenon.

Date of Submission: 25-12-2023 Date of Acceptance: 05-01-2024

I. Introduction

String Theory,in 1984,received from two physicists, Gary Horowitz and Andy Strominger.They were excited about a model for describing the vacuum state of the universe, based on a new theory called string theory. In a paper written in 1939, Albert Einstein attempted to reject the notion of black holes that his theory of general relativity and gravity, published more than two decades earlier, seemed to predict. "The essential result of this investigation," claimed Einstein, who at the time was six years into his appointment as a Professor at the Institute, "is a clear understanding as to why the 'Schwarzschild singularities' do not exist in physical reality."

Schwarzschild singularities, later coined "black holes" by John Wheeler, for- mer Member in the School of Mathematics, describe objects that are so massive and compact that time disappears and space becomes infinite. The same year that Einstein sought to discount the existence of black holes, J. Robert Oppenheimer, who would become Director of the Institute in 1947, and his student Hartland S. Snyder used Einstein's theory of general relativity to show how black holes could form.

Below, Juan Maldacena, Professor in the School of Natural Sciences, explains the development of a string theoretic interpretation of black holes where quan- tum mechanics and general relativity, theories previously considered incompati- ble, are united. Work by Maldacena and others has given a precise description of a black hole, which is described holographically in terms of a theory living on the horizon. According to this theory, black holes behave like ordinary quantum mechanical objects—information about them is not lost, as previously thought, but retained on their horizons—leading physicists to look at black holes as lab- oratories for describing the quantum mechanics of spacetime and for modeling strongly interacting quantum systems.

The ancients thought that space and time were preexisting entities on which motion happens.According to Einstein's theory of general relativity, we know that this is not true. Space and time are dynamical objects whose shape is modified by the bodies that move in it. The ordinary force of gravity is due to this deformation of spacetime.Spacetime is a physical entity that affects the motion of particles and, in turn, is affected by the motion of the same particles.For example,the Earth deforms spacetime in such a way that clocks at different altitudes run at different rates. For the Earth, this is a very small (but measurable) effect. For a very massive and very compact object the deformation (or warping) of spacetime can have a big effect. For example, on the surface of a neutron star a clock runs slower, at 70 percent of the speed of a clock far away.

In fact,you can have an object that is so massive that time comes to a complete standstill.These are black holes.General relativity predicts that an object that is very massive and sufficiently compact will collapse into a black hole.A black hole is such a surprising prediction of general relativity that it took many years to be properly recognized as a prediction. Einstein himself thought it was not a true prediction, but a mathematical oversimplification.

Black holes are big holes in spacetime. They have a surface that is called a "horizon".It is a surface that marks a point of no return.A person who crosses it will never be able to come back out. However,he will not feel anything spe- cial when he crosses the horizon.Only a while later will he feel very uncomfort- able when he is crushed into a "singularity," a region with very high gravitational fields. The horizon is what makes black holes "black";nothing can escape from the horizon,not even light.Fortunately,if you stay outside the horizon, nothing bad happens to you. The singularity remains hidden behind the horizon.

Something surprising happens when we take into account the effects of quan- tum mechanics.Due to quantum mechanical fluctuations near the horizon,black holes should emit radiation,called Hawking radiation.This is a famous theoreti- cal prediction that Stephen Hawking made in the seventies.This means that black holes are not completely black.A black hole can glow, like an ember,or,if it is hot enough,you can even have the oxymoronic possibility of a white black hole.A black hole gets hotter the smaller it is.A white black hole should have the size of a bacterium,and the mass of a continent.Such black holes,while theoretically possi- ble,are not known to be naturally produced anywhere in the universe.Black holes that are naturally produced have masses bigger than the Sun and sizes bigger than a few miles.Such black holes are also supposed to emit Hawking radiation,but it is swamped by other matter falling into the black hole. For this reason,Hawking radiation has not been directly measured.The existence of this radiation has im- portant consequences.The first is that black holes have a temperature.We know that temperature is due to the motion of the elementary constituents of the ob- ject.For example,air is hotter or colder depending on whether air molecules are moving faster or slower. In the case of black holes, what is moving? Black holes only involve gravity, so what is moving is spacetime itself.Since the nineteenth century,we have understood that when we have thermal systems we can compute a quantity called the "entropy," which tells us about the number of microscopic configurations that the system has. From Hawking's formula for the temperature of a black hole one can also find this entropy. It turns out to be proportional to the area of the horizon,or the square of the mass of the black hole. This is also a bit strange. The entropy of almost every substance grows in proportion to the amount of substance that we have. Here it grows like the square.

A second consequence of Hawking radiation is that black holes lose mass,since they are radiating energy away.Thus,a black hole left alone in an otherwise empty universe would eventually completely disappear.We call this process "black hole evaporation," since the black hole appears to evaporate as a droplet of water.

Hawking radiation from black holes has given rise to very profound and in- teresting theoretical puzzles.Einstein has taught us that spacetime is a physical object.We also know that all other physical objects, such as those made with mat- ter or radiation,obey the laws of quantum mechanics.Thus, spacetime should be no different and should also obey the laws of quantum mechanics. Any quantum mechanical theory of spacetime should be able to describe precisely how black holes form and evaporate.

Here one finds an interesting paradox.The black hole can form in many dif- ferent ways but it appears to evaporate always in the same way. This is a contra- diction with standard quantum mechanics. In quantum mechanics (as in classical mechanics) the information about a system is not lost. Different initial condi- tions lead to different outcomes.Hawking suggested that black holes imply that this basic principle of quantum mechanics would not hold in the presence of grav- ity.Namely,the radiation coming out of black holes would be completely thermal and devoid of the information of what fell into black holes. Thus, black holes ap- pear to be sinks of information, perverse monsters that threaten the fundamental laws of quantum mechanics.

String theory is a theory being constructed to describe the quantum mechanics of spacetime.As such,the theory should explain whether black holes are consis- tent with quantum mechanics or not.In fact,since string theory obeys the usual principles of quantum mechanics, we expect that information should not be lost in black holes. For this reason the problem of information loss was actively stud- ied during the nineties. The problem was difficult in the original formulation of string theory because the quantum spacetime was described by starting with a flat spacetime and then considering small quantum fluctuations, or ripples, that prop- agate in it.As long as these ripples interact weakly with each other,the theory is relatively simple.However,in order to form a black hole you need a strong devi- ation from a flat spacetime.You need to put a lot of these ripples together,and by the time the black hole forms,the simplest formulation of string theory becomes unmanageable.

In the mid-nineties, Joseph Polchinski (at the University of California, Santa Barbara) made a breakthrough by discovering that string theory contains other objects, called D-branes.These are particle-like objects that are heavier than the spacetime ripples we discussed above.Nevertheless, one can give a very precise description for them within the rules of string theory.It soon became clear that they were ideally suited for studying black holes.

The description of a single D-brane is fairly simple.A single D-brane is very similar to a particle;it is characterized by its position in space.However,a single D- brane is not heavy enough to curve spacetime in a significant way.So,we need to bring many D-branes together.When we bring them together,there is a surprising new symmetry that emerges.In ordinary quantum mechanics,elementary particles are identical,in the sense that there is no way to distinguish them.The full descrip- tion is completely invariant under the interchange of any two identical elementary particles, such as two electrons.D-branes are invariant under a bigger symme- try group:a full continuous symmetry,called a gauge symmetry.When N D-branes come together,the positions of the branes become N x N matrices.We would have expected that N branes are described by N positions,the positions of each of the objects individually.However,we find that they are described by N^2 numbers.The dynamics of these N^2 variables is governed by a gauge theory.Now, if we want to separate the D-branes by a big amount, we find that there is a force that does not allow them to be separated unless the matrices are diagonal,reducing then to the usual description in terms of N identical particles.When all these D-branes are close together,the number of possible ways to arrange them grows very fast with its number.It grows like N^2,rather than the N expected for a usual extensive system.

This has become a bit abstract, so let us make an analogy. Say D-branes are

people. Imagine that we have a group of N people (say N is a big number, e.g., a thousand). Now imagine that each person can be happy or sad. The entropy is just the information that you need to completely specify the emotional state of everybody. In this case you need to specify N bits of information: whether each of the N persons is happy or sad. If N is a thousand, you need a kilobit of information. On the other hand, imagine that each person can like or dislike every other person. Now to capture the complete set of likes or dislikes of everybody you need to give N^2 bits of information. If N is a thousand, you need a megabit of information. The case of black holes is similar to this latter situation, where one has to keep track of variables that involve pairs of D-branes,rather than single D- branes.In this analogy,you can only separate the D-branes when they dislike (and are disliked by) all the other D-branes,so the number of configurations becomes much smaller.

A large number of D-branes is heavy enough to warp the spacetime around them and to produce a black hole.In order to produce a black hole with some temperature it is necessary to excite these N^2 degrees of freedom.This leads to a precise microscopic accounting for the entropy of the black holes, as shown by Andrew Strominger and Cumrun Vafa.These N^2 degrees of freedom produce a highly entangled state that cannot be described in terms of the motion of the individual particles.However,it can be described very precisely in terms of the gauge theory of the N x N matrices.However,in some very important respects it is the same.First,it obeys the usual rules of quantum mechanics. Second,it lives on a fixed spacetime—in this case,the point in spacetime where the branes are sitting.

We said that we can describe the branes in terms of a gauge theory living at a spatial point.On the other hand,we said that the branes form a black hole,which has a non-zero horizon size.

In string theory,these two descriptions are viewed as equivalent.The gauge theory is describing the whole region around the black hole.If we view the black hole from very far away,it looks like a point—that is why the matrices live at a point.On the other hand,the matrices also give rise to the whole spacetime region around the horizon of the black hole.This is the gauge/gravity correspondence proposed by Edward Witten and Steven Gubser, Igor Klebanov, and Alexander Polyakov.

The gauge theory gives a precise description of the black hole and its surround- ing geometry.It is described in terms of a perfectly ordinary quantum mechanical system.This explains its entropy.It also gives a completely quantum mechanical description of the black hole and the spacetime around the black hole. This de- scription is sometimes called "holographic" because the whole spacetime emerges dynamically from a quantum mechanical description that lives in a smaller num- ber of dimensions.

Going back to the analogy of the group of people and the pattern of likes and dislikes, the idea is that the whole spacetime is encoded in the pattern of likes and dislikes among the various people. A spacetime ripple is a change in that pattern.The "gauge theory" is a simple dynamical law that says how this pattern changes.

This description has been explored actively here at the Institute and elsewhere. It is best understood in very special configurations in string theory.However, simi- lar descriptions are expected to be valid for black holes in general.These theoreti- cal developments were made with the goal of showing that black holes behave like ordinary quantum mechanical objects. More recently, the same relation is being explored in order to model strongly interacting quantum systems via black holes.

Scalar clouds in charged stringy black hole-mirror system

It was reported that massive scalar fields can form bound states around Kerr black holes . These bound states are called scalar clouds, which have a real frequency $\omega = m\Omega_H$, where m is the azimuthal index and Ω_H is the horizon angular velocity of Kerr black hole. These bound states satisfy the superradiant critical frequency condition $\omega = q\Phi_H$ for charged scalar field, where q is charge of scalar field, and Φ_H is horizon electrostatic potential. It was firstly proposed by S. Hod that the scalar field can have real bound states in the near-extremal Kerr black hole . Soon later, it was reported that massive scalar fields can form bound states around Kerr black holes by using the numerical method to solve the scalar field equation in the background. This bound states are the stationary scalar configurations in the black hole backgrounds, which are regular at the horizon and outside. They are named as scalar clouds. It is suggested that whenever clouds of a given matter field can be found around a black hole, in a linear analysis, there exists a fully non- linear solution of new hairy black hole correspondingly.However, it requires that the field originating clouds yields a time independent energy momentum tensor. Generally, the field should be complex, and have a factor $e^{i\omega ct}$, where ω_c is the superradiance critical frequency. For instance, real scalar fields can give rise to clouds but not hairy black holes.

Generally speaking, the existence of stationary bound states of matter fields in the black hole backgrounds requires two necessary conditions. The first is that the matter fields should undergo the classical superradiant phenomenon in the black hole background. This condition can be satisfied by the bosonic fields in the ro- tating black holes or the charged scalar fields in the charged black holes.When the frequencies of these Corresponding matter fields ω are smaller than the super- radiant critical frequency ω_c, there are time growing quasi-bound states. When $\omega > \omega_c$, the fields are time decaying. So, the scalar clouds exists at the bound- ary between these two regimes, i.e. the frequencies of the fields are taken as the superradiant critical frequency ω_c.

It is proved that the the massive charged scalar field is stable in this background and there is no superradiant instability. Correspondingly, the scalar clouds are only possible with the mirror-like boundary condition.

DESCRIPTION OF THE SYSTEM

The parameters M and Q are the mass and the electric charge of the charged stringy black hole, respectively. The event horizon of black hole is located at r

= 2M. The area of the sphere approaches to zero when $r = Q^2/M$. Therefore, the sphere surface of the radius $r = Q^2/M$ is singular. When $Q^2 \le 2M^2$, this singular surface is surrounded by the event horizon. In this paper, we will always assume the cosmic censorship hypothesis, i.e. we will only consider the black hole with the parameters satisfying the condition $Q^2 \le 2M^2$.

The dynamics of the charged scalar field is then governed by the Klein-Gordon equation

$$(\nabla_\nu - iqA_\nu)(\nabla^\nu - iqA^\nu)\Psi = 0, \quad (4)$$

where q denotes the charge of the scalar field. By taking the ansatz of the scalar field

$\Psi = e^{i\omega t}R(r)Y_{lm}(\theta, \phi)$,

NUMERICAL PROCEDURE AND RESULTS

The numerical methods employed in this problem are based on the shooting method, which is also called the direct integration (DI) method.

Firstly, near the event horizon r = 2M, we require the radial function is regular and expand the radial function R(r) as a generalized power series in terms of $(r\ r_+)$ as have done in the first line of Eq.(9). Because the radial equation is linear, we can take $R_0 = 1$ without loss of generality. Substituting expansion of the radial wave function into the radial equation (5), we can solve the coefficient R_k order by order in terms of $(r \quad r_+)$. have only considered six terms in the expansion. The $R_k s$ can be expressed in terms of the parameters (M, Q, q, l),which are not exhibited here.

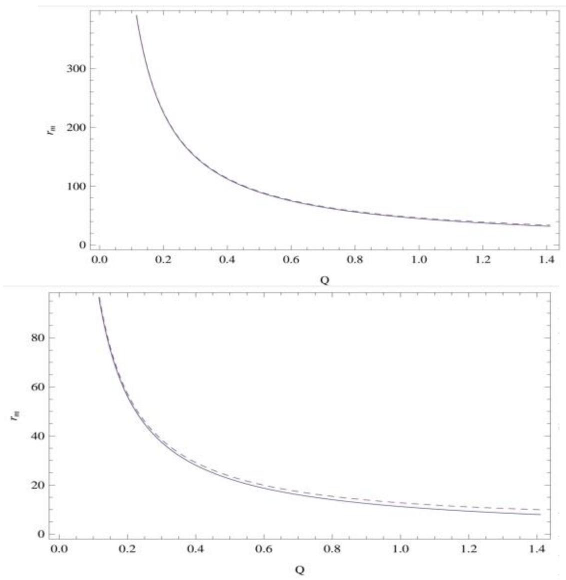

FIG. 1: Mirror location rm plotted versus the black hole charge Q for M = 1, l

= 1, n = 0 and for various scalar charge q. For the first panel, q = 0.2, while for the second panel, q = 0.8. The solid line and the dashed line represent the analytical and the numerical results respectively.

Then, we can integrate the radial equation (6) from $r = r_+(1 + \epsilon)$ and stop the integration at the radius of the mirror. In this procedure, we have taken the small ϵ as 10^{-6} . The procedure can be repeated by varying the input parameters (M, Q, q, l) until the mirror-like boundary condition $R(r_m) = 0$ is reached with the desired precision. For the given input parameters (M, Q, q, l), scalar clouds exist for a discrete set of r_m, which is labeled by the quantum number n of nodes of the radial function R(r).

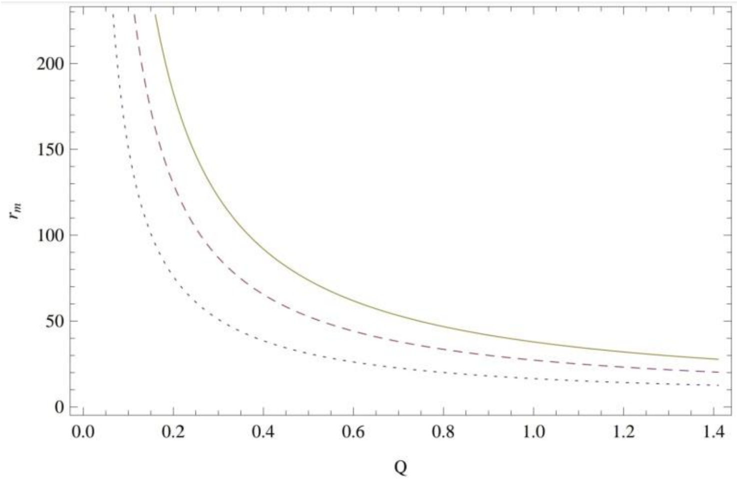

FIG. 2: Mirror location rm plotted versus the black hole charge Q for M = 1, l = 1, q = 0.6 and for various node number n. The dotted, dashed, and solid lines represent n = 0, 1, and 2 respectively.

When q = 0.2, the analytical approximation is always precise in all range of Q. When q = 0.8, the analytical results have obvious difference with the numerical results only for large Q.

In Fig.(2), we have drawn the mirror location r_m that support the scalar cloud as a function of the black hole charge Q for various values of node number n of the radial function. It is observed that, when the black hole charge Q increases, we need to place the reflecting mirror more closer to the horizon in order to have a scalar cloud. When the node number n of radial function increases, the plotted lines become away from the axis.

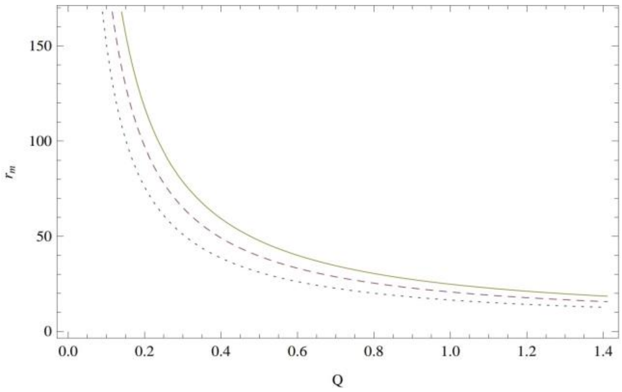

FIG. 3: Mirror location r_m plotted versus the black hole charge Q for M = 1, n = 0, q = 0.6 and for various l. The dotted, dashed, and solid lines represent l = 1, 2, and 3 respectively.

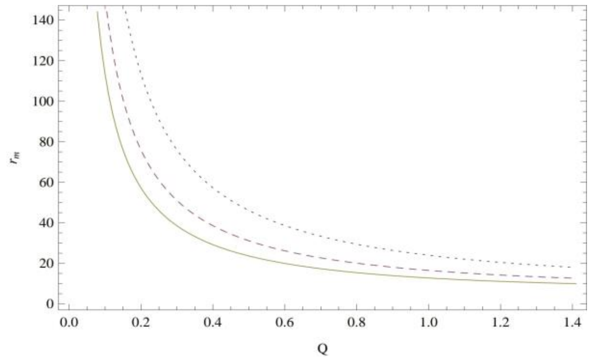

FIG. 4: Mirror location r_m plotted versus the black hole charge Q for M = 1, l = 1, n = 0 and for various scalar charge q. The dotted, dashed, and solid lines represent q = 0.4, 0.6, and 0.8 respectively.

In Fig.(3) and (4), we display the mirror location rm as a function of the black hole charge Q for various different l and q. We can observe that, the lines become far away from the axis when increasing l, while the lines become more closer to the axis when increasing the scalar charge q.In addition, Fig.(3) and (4) together with Fig.(2) show that, when $Q \to 0$, $r_m \to \infty$. This indicates that there is no massless scalar cloud for Schwarzschild black hole with the mirror-like boundary condition, even thought it is possible for massive scalar fields in Schwarzschild black hole to have arbitrarily long-lived quasi-bound states.In Fig.(5) and (6), we have fixed the mirror radius as $r_m = 40$.

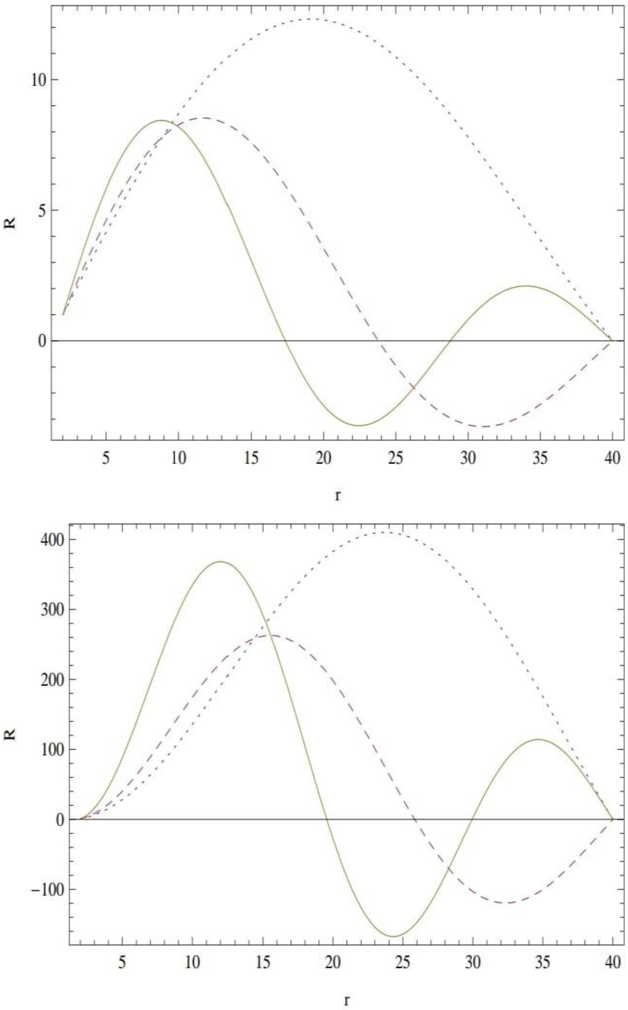

FIG. 5: Radial functions R(r) of scalar clouds for M = 1, q = 0.6, rm = 40 with different harmonic index l and node number n. The first and the second panels correspond l = 1 and 2 respectively. The dotted, dashed, and solid lines represent n = 1, 2, and 3 respectively.

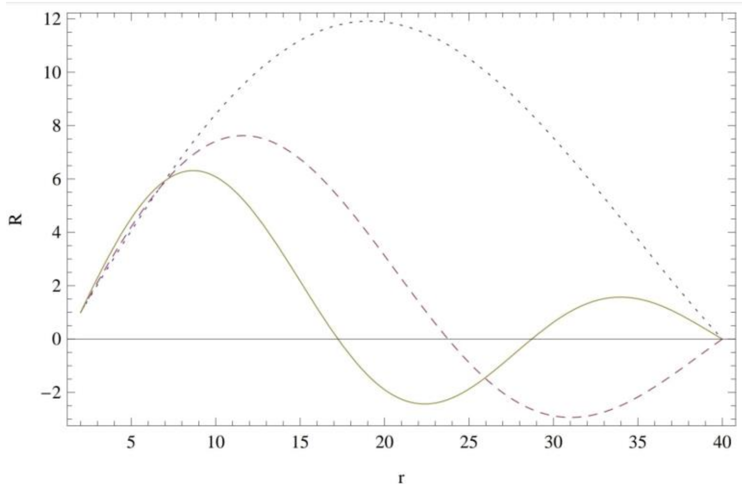

FIG. 6: Radial functions R(r) of scalar clouds for M = 1, q = 0.8, l = 1, rm = 40 with different node number n. The dotted, dashed, and solid lines represent n = 1, 2, and 3, respectively, and the corresponding black hole charge Q are 0.219882, 0.583819, and 0.956562.

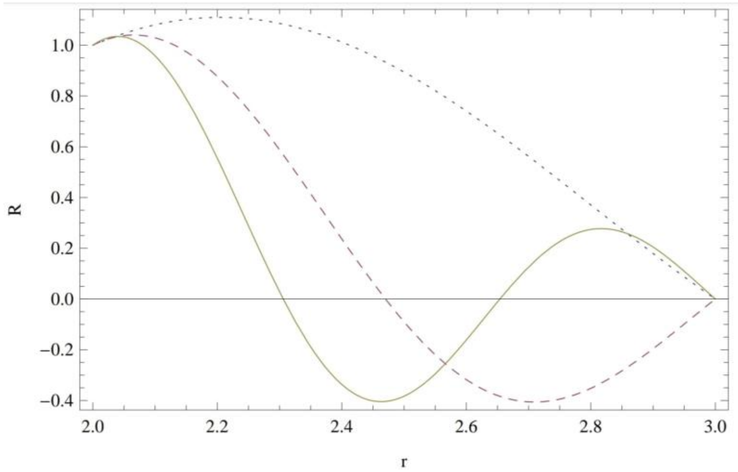

FIG. 7: Radial functions R(r) of scalar clouds for the small mirror radius rm =

The parameters of black hole and scalar field are taken as M = 1, q = 20, and l = 1. The dotted, dashed, and solid lines represent n = 1, 2, and 3, respectively, and the corresponding black hole charge Q are 0.306384, 0.600699, 0.913741.

In Fig.(7), we consider the case that the mirror location is very close to the horizon. We take the mirror radius as $r_m = 3$. From our previous analytical and numerical work on the superradiant instability of scalar field in the background of the charged stringy black hole plus mirror system, we need a large scalar field charge q. Here, we set q = 20. We can see that, the scalar field can be bounded by the reflecting mirror very near the horizon to form the clouds.

II. Stability of charged black holes

This implies that there exists at least one maximum point in the region r > 2M and at least one maximum point in the region $Q^2/M < r < 2M$. We denote this two maximum points as z_1 and z_2 respectively. Then we have $z_1 > z_2 > 0$. For the nonextremal black hole, $Q^2 < 2M^2$ .Then, we have c > 0 , d < 0 . This implies $z_3 < 0$ .This means that in the superradiant regime the effective potential V (r) has only one maximum outside the event horizon. This implies that there is no trapping potential well outside of the event horizon which is separated from the horizon by a potential barrier.

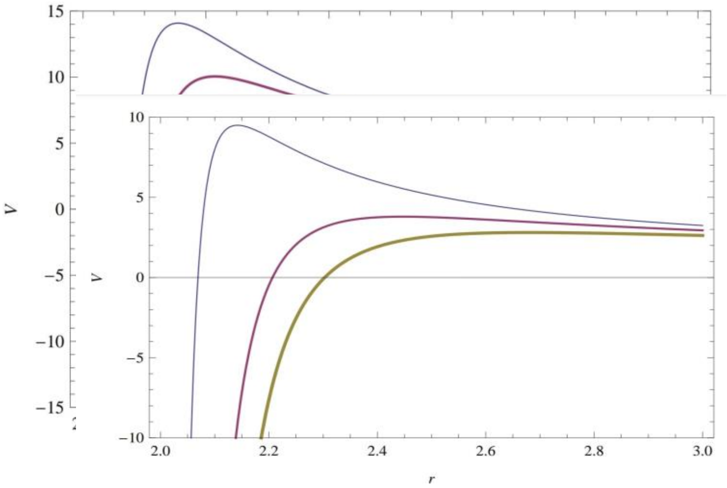

FIG. 8: Qualitative shape of the effective potential V for different ω. The parameters are given by M = 1, Q = 1, q = 1, μ = 1 and l = 1. From top to down, the three curves correspond to ω = 1/3, 1/4 and 1/5 respectively.

In FIG. 8, we have plotted the shape of the effective potential V given in Eq.(20) for different ω . The analytical conclusion for the nonextremal black hole case is explicitly shown in this figure. The mass of the scalar field is never able to generate a potential well outside of the horizon to trap the superradiant modes. Thus, there are no meta-stable bound states of the charged massive scalar field in the superradiant regime. In other words, the superradiance in the nonextremal charged stringy black hole can not trigger the instability.

But $z_0 < 0$, i.e. the root of V’(z) = 0 locates at the non-physical regime r < 2M. This implies that there is neither an maximum point nor an minimum point outside the horizon. In the black-hole exterior, the effective potential will gradually bring down to a finite value.

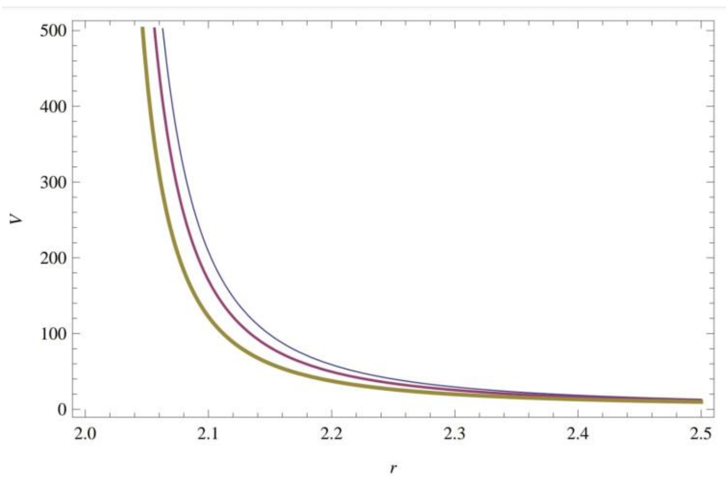

FIG. 9: Qualitative shape of the effective potential V for different ω. The parameters are given by M = 1, Q = $\sqrt{2}$, q = 1, μ = 1 and l = 1. From right to left, the three curves correspond to ω = 1/2, 1/3 and 1/5 respectively.

In FIG. 9, we have plotted the shape of the effective potential given in Eq.(34) for different ω . The parameters are selected to satisfying the extremal condition and b < 0 simultaneously. One can see that, outside the horizon, there exists neither a potential barrier nor a potential well. In this case, the superradiance can not lead to the instability.

But $z_0 > 0$, i.e. there is a root of V’(z) = 0 in the region of r > 2M. This implies that the effective potential have only one maximum point outside the horizon, i.e. there is only a potential barrier outside of the horizon.

FIG. 10: Qualitative shape of the effective potential V for different ω. The parameters are given by M = 1, Q $\sqrt{2}$, q = 1, μ = 1 and l = 0. From top to down, the three curves correspond to ω = 1/2, 1/3 and 1/4 respectively.

In FIG. 10, we have plotted the effective potential for different ω which satisfy the condition $b > 0$. The shape of the effective potential in this case is very similar to that of the nonextremal black hole. So for the same reason, in the present case, the superradiant modes can not be trapped as well and the black hole is also stable.

III. Conclusion

We have shown that the classical superradiance phenomenon presents in the charged stringy black holes for the charged scalar field perturbations.The superradiant con- dition is also obtained by analyzing the asymptotic solutions near the horizon and at the spatial infinity, which is similar to that of RN black hole.Then we investigate the possibility of instability triggered by the superradiance. It is shown by ana- lyzing the behavior of the effective potential that for both the nonextremal black holes and the extremal black holes there is no potential well which is separated from the horizon by a potential barrier.Thus, the superradiant modes of charged massive scalar field can not be trapped and lead to the instabilities of the black holes. This indicates that the extremal and the nonextremal charged black holes in string theory are stable against the massive charged scalar field perturbations.

At last, we should note that although the mass of the scalar field can not pro- vide an effective potential well outside the black hole, one can still make the black hole unstable by placing a reflecting mirror around the black hole.

IV. Acknowledgement

I would like to express my special thanks to my project supervisor Dr. Farook Rahaman, Associate Professor of Department of Mathematics,Jadavpur University,Kolkata, who gave me a golden opportunity to do this wonderful project on the topic "STABILITY OF CHARGED BLACK HOLES IN STRING THEORY",which also helped me in doing a lot of research and i came to know about so many new things which will help me for my future studies.I am really thankful to him.

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