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A Fuzzy Goal Programming Technique For Solving Multi-Objective Chance Constrained Programming Problems With Gumbel Distributed Random Variables

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Abstract
In this work, we developed a multi-objective chance constrained programming model involving Gumbel distributed random variables. It also applies the fuzzy goal program- ming technique in solving multi-objective chance constrained problems. We employed the quantitative research method following a descriptive approach to solve a multi-objective chance constrained problem where the right hand side parameter of the constraints are considered as triangular fuzzy numbers. The deterministic equivalence of the probabilistic problem was obtained and solved by the fuzzy programming approach to obtain the best and worst values of the objective functions. Finally, the minsum weighted goal program- ming technique was employed to find the best compromise solution to the problem. Results indicate that a multi-objective chance constrained problem can be sufficiently modelled by the Gumbel distribution. Moreover, the fuzzy goal programming technique is shown to be an efficient tool for satisficing a multi-objective chance constrained problem.

Keywords: *Fuzzy programming, triangular fuzzy numbers, chance constrained programming, Gumbel random variables, membership function.*

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I. Introduction

Goal programming is an analytical procedure that tackles real- life decision-making problems where targets are assigned to all objective functions which may sometimes be conflicting and non-commensurate. It seeks to find a compromise solution to each objective. Goal programming was first introduced by Charnes and Cooper in 1961 for a linear model. Several advances in goal programming were by (Ijiri, 1965), (Lee, 1972), (Ignizo, 1976), and many other researchers. Goal programming as a multi objective mathematical model attempts to proffer solution to conflicting objectives. The solution may not be optimal with respect to all the conflicting objectives but it provides a fair compromise solution by increasing the satisfaction in the system. A minus for the goal programming technique is that it does not make room for randomness of the at- tributes in the system. To curb this limitation, the chance constrained programming technique was adopted into goal programming. This approach, introduced by (Charnes and Cooper, 1959) involves converting the parameters of the probabilistic problem into an equivalent deterministic problem according to some specified confidence levels. More recently, some researchers have studied several aspects of chanced constrained programming. The work (Qahtani et al., 2019) studied a multi choice multi objective transportation problem using a goal programming ap- proach where the parameters of supply and demand are random variables. A multi-objective chance constrained problem involving three parameters lognormal distribution was considered in (Archarya et al., 2021). A new methodology for solving multi objective chance constrained problems was elucidated in (Atalay et al., 2021), they developed an algorithm which uses the global criterion method to solve multi objective deterministic equivalent of chance constrained problems. The work (Nasseri and Chitgar, 2020) studied multi objective stochastic optimization models with chance constraints. The paper (Abou-El-Enien, 2000), proposed a mixed algorithm for multi objective stochastic problem using MATLAB program and obtained non dominated solutions for stochastic multiple objective programming problems using the chance constrainedprogramming technique.

The work (Biswas and Modak, 2011) presented a fuzzy goal programming technique to solve multi-objective decision making problems in probabilistic decision- making environment where the righthand side parameters associated with the system constraints are exponentially dis- tributed fuzzy random variables. A multi-objective fuzzy chance constrained fuzzy goal pro- gramming for capacitated transportation problem was studied by (Bhargava et al., 2014) in the solution process, the fuzzy parameters were defuzzified by applying graded mean integration method which provided a satisfactory result. (Barik, 2015) investigated a linearly constrained probabilistic fuzzy goal programming where the right hand parameters in some constraints follow Pareto distribution with known mean. (Li and Murata, 2017) proposed a chance constrained multi-objective goal programming model for supplier selection problem with uncertain factors. The proposed model provides chance constraints leading to order allocation decision - making result. The work also compared deterministic and chance constraints showing the superiority of the later. (Mohanty et al., 2019), presented multi objective chance constrained programming problems involving some continuous random parameters. They considered all the righthand side parameters of the constraints as random variables following some continuous distributions. Numerical illustrations proved the efficiency of the solution procedures of the model.

In real life situations, decision makers have difficulty specifying precisely the goal target not just because of the uncertain behavior of the parameters of the system but also due to the imprecise nature of the parameters.

To curb this difficulty fuzzy set theory which was first proposed by (Zadeh, 1965), and first applied by (Narasimham, 1980) in the field of goal programming was also introduced by (Ludhandjula, 1983) to the field of stochastic programming. This paper presents a chance constrained approach to solving multi-objective fuzzy goal pro- gramming problem where the right side parameter of the constraints are considered as Gumbel distributed random variables. It is assumed that the sample data represents the distribution of the maximum values of an exponential random variable. Also, the coefficients of the system constraints are considered as triangular fuzzy numbers. The general chance constrained pro- gramming (CCP) techniques are used to remove the probabilistic nature of the problem and convert it into an equivalent deterministic fuzzy programming problem (FPP) in the model for- mulation process (Biswas and Modak, 2012). Given the imprecise nature of the parameters of the system constraints , it is assumed that the mode of the right hand side parameters are Gumbel random variables associated with some constants of admissible violations. The optimal decision is obtained by applying a linear membership function to measure the degree of achievability of the objectives. Then, a fuzzy goal programming model is used to obtain a compromise solution that increases the degree of satisfaction of the defined fuzzy goals. Finally, a numerical example is solved by fuzzy programming technique to show the efficiency of our approach.

II. Mathematical Model for a Fuzzy Chance ConstrainedMulti-Objective Programming Problem (FCCMOP))

The FCCMOP model is given as

$$\text{Opt } Z_k = \sum_{j=1}^{q} C_{kj}x_j, \qquad k = 1, 2, \cdots, K; \tag{2.1}$$

Subject to

$$P\left[\sum_{j=1}^{q} \tilde{a}_{ij}x_j \leq \tilde{b}_i\right] \geq 1 - \gamma_i \qquad i = 1, 2, \cdots, p;$$

$$x_j \geq 0, \qquad j = 1, 2, \cdots, q.$$

Here, b_i are gumbel random variables and a_{ij} are triangular fuzzy numbers, $\gamma_i (0 \leq \gamma_i \leq 1)$ $i = 1, 2, \cdots, p$ are real numbers representing the tolerance level of the parameters, C_{kj} are the cost coefficients and x_j are the decision variables.

III. Equivalent Deterministic Fuzzy Programming Model

Considering the probabilistic nature of the model in subsection 3 above, an equivalent determin-istic fuzzy programming model is given as

$$\text{Opt } Z_k = \sum_{j=1}^{q} C_{kj}x_j \qquad k = 1, 2, \cdots, K \tag{4.1}$$

Subject to

$$y_i \leq \alpha_i - \beta_i[ln(-ln(\gamma_i))]$$

$$x_j \geq 0, \qquad i = 1, 2, \cdots, p, \; j = 1, 2, \cdots, q,$$

where α_i represents the mode of the distribution, β_i represents the standard deviation, γ_i are real numbers representing the tolerance level of the parameters, a_{ij}, $i = 1, 2, \cdots, p$; $j = 1, 2, \cdots, q$ are the constraint coefficients and C_{kj}, $j = 1, 2, \ldots, q$; $k = 1, 2, \ldots, K$ are the coefficients associated with the objective function.

IV. Representation of membership functions

Considering the right sided fuzzy goals b_i of the *ith* ($i = 1, 2, \cdots, p$) system constraints, and assigning ρ_i as the right admissible violations for the ith fuzzy goals. The linear membership function of $\mu(b_i)$is represented as

V. Triangular Fuzzy numbers

Let A be the triangular fuzzy number represented by the triple of three real numbers $\tilde{A} = (a^L, a, a^R)$ which is defined by its continuous membership function $\mu_A : \mathrm{R} \rightarrow [0, 1]$ defined by

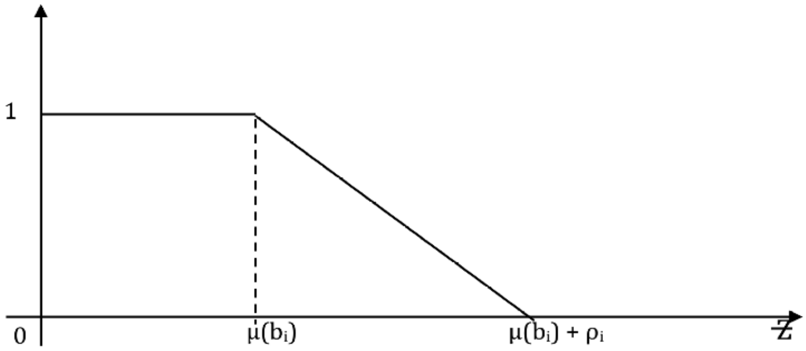

Figure 1: Right sided fuzzy number

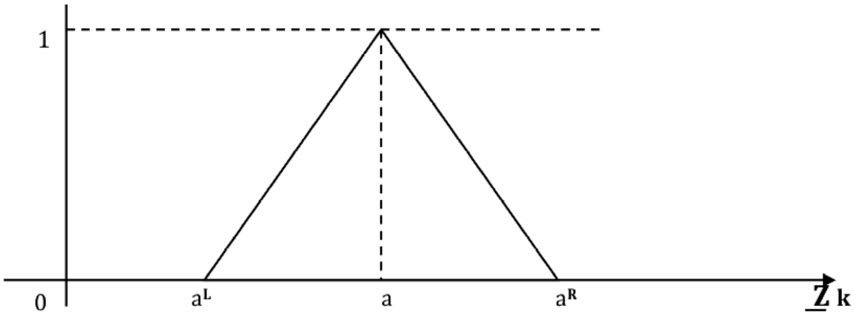

Figure 2: Triangular fuzzy number

VI. Conclusions

In this work, we considered a chance constrained multi-objective programming problem involv-ing continuous random variables. The righthand side parameters are considered as random variables. By considering that the random variables follow a Gumbel distribution, we establish the equivalent deterministic form of the chance constraints. Fuzzy goal programming technique is used to solve the multi-objective deterministic model. The methodology used in this work can be extended to solve FMOCCP problems which follow other types of distribution. It can be employed in a fully fuzzy environment. The study can be extended for non-linear chance constrained problems. However, it is concluded that the described methodology will add a new dimension into solving MOCCP in a fuzzy defined probabilistic decision-making environment.

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