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Taxation Of The Set Of 4-Tuples With Separate Characteristics

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Abstract:
In This Article, The Collection Of 3-Tuples (X_1, X_2, X_3) Where X_1, X_2, X_3 Are Three Distinct Non-Zero Integers In Which The Product Of Any Two Values Increased By $2s, s \in Z$ Is Evaluated.
It Is Proved That Such 3-Tuples Can Be Prolonged To The Set Of 4-Tuples (X_1, X_2, X_3, X_4) Sustaining The Same Condition. Arithmetic Illustrations Are Also Offered And The Conditions Are Confirmed By Python Program.
***Key Word**: Integer Sequence, Diophantine 4-Tuples.*

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I. Introduction

A set of positive integers $\{x_1, x_2, x_3, \dots, x_m\}$ is said to have the characteristic $D(s)$ where $s \in Z - \{0\}$ if $x_i x_j + s$ is a square. Fermat discovered the first $D\ (1)$- quadruple $\{1, 3, 8, 120\}$. Many authors are interested to investigate the quadruple of special numbers with various properties in [1-10]. In this communication, variety of 4-tuples (X_1, X_2, X_3, X_4) where X_1, X_2, X_3, X_4 are distinct non-zero integers in such a way that the multiplication of any two quantities increased by $2s$ is a square of an integer is exhibited.

II. Taxation of the set of 4-tuples

Let X_1, X_2 be two distinct non-zero integers united with the peculiar statement that their product enlarged by $2s$ is a square.
In Mathematical notation, it is taken as

$$X_1X_2 + 2s = {\alpha_1}^2 \tag{1}$$

Let X_3 be a newly selected integer with additional statements that

$$X_1X_3 + 2s = {\beta_1}^2 \tag{2}$$

$$X_2X_3 + 2s = {\gamma_1}^2 \tag{3}$$

From (2) and (3), it is eradiated by

$$2s(X_2 - X_1) = {\beta_1}^2X_2 - {\gamma_1}^2X_1 \tag{4}$$

To evaluate the possibility of the third element X_3 in the 3-tuples (X_1, X_2, X_3), let us initiated with the succeeding transformations

$$\beta_1 = B_1 + X_1 \text{ and } \gamma_1 = B_1 + X_2$$

Execution of these two alternations in (4) reveals the chance of B_1 by

$$B_1 = \alpha_1$$

Consequently, $\beta_1 = X_1 + \alpha_1, \gamma_1 = X_2 + \alpha_1$
Replacement of the above value of $\beta_1$in (2) produces the choice of X_3 by

$$X_3 = X_1 + X_2 + 2\alpha_1 \tag{5}$$

Note that,(X_1, X_2, X_3) is an integer 3-tuples sustaining the characteristics $D(2s)$.
Different sets of quadruples obtained from elements in the triple(X_1, X_2, X_3) are explained through the following procedure.

Starting with the pair (X_2, X_3) where $X_3 = X_1 + X_2 + 2\alpha_1$ such that $X_2X_3 + 2s = (X_2 + \alpha_1)^2$ and then following the similar procedure as enlightened previously, it can be prolonged into the triple (X_2, X_3, X_4) with the same characteristics where

$$X_4 = X_2 + X_3 + 2(X_2 + \alpha_1) \quad (6)$$
$$\Rightarrow X_4 = X_1 + 4X_2 + 4\alpha_1$$

Suppose the quadruple (X_1, X_2, X_3, X_4) satisfies the characteristics $D(2s)$.
One of the necessary conditions for the above statement is

$$X_1X_4 + 2s = {\mu_1}^2 \quad (7)$$

From (7), it is perceived that

$$(X_1 + 2\alpha_1)^2 - {\mu_1}^2 = 6s$$
$$\Rightarrow (X_1 + 2\alpha_1 - \mu_1)(X_1 + 2\alpha_1 + \mu_1) = 6s$$

Let $(X_1 + 2\alpha_1 - \mu_1) = 6$ (8)

$$(X_1 + 2\alpha_1 + \mu_1) = s \quad (9)$$

Afterwards, application of some simple mathematical calculations in (8) and (9) provides that

$$2s = 4X_1 + 8\alpha_1 - 12 \quad (10)$$

Retaining (10) in (1), it is estimated that

$$X_1(X_2 + 4) = (\alpha_1 - 2)(\alpha_1 - 6) \quad (11)$$

Case 1: Elect $\alpha_1 = X_1k + 2$ where $k \in Z$
Presentation of this choice of α in (10) and (11) gives that

$$2s = 4X_1(2k + 1) + 4$$
$$X_2 = X_1k^2 - 4k - 4$$

Correspondingly,

$$X_3 = X_1(k + 1)^2 - 4k$$
$$X_4 = X_1(2k + 1)^2 - 16k - 8$$

Hence, the quadruple $\{X_1, X_1k^2 - 4k - 4, X_1(k + 1)^2 - 4k, X_1(2k + 1)^2 - 16k - 8\}$ has he characteristics $D(4X_1(2k + 1) + 4)$ where X_1 is any arbitrary integer.

Table no 1. Authentication of the conditions of the 4-tuples are demonstrated in the ensuing table.

k	$2s$	X_1	X_2	X_3	X_4	$X_1X_2 + 2s$	$X_1X_3 + 2s$	$X_1X_4 + 2s$	$X_2X_3 + 2s$	$X_2X_4 + 2s$	$X_3X_4 + 2s$
1	-20	-2	-10	-12	-42	0^2	2^2	8^2	10^2	20^2	22^2
2	-16	-1	-16	-17	-65	0^2	1^2	7^2	16^2	32^2	33^2
3	4	0	-16	-12	-56	2^2	2^2	2^2	14^2	30^2	26^2
4	40	1	-4	9	9	6^2	7^2	7^2	2^2	2^2	11^2
5	92	2	26	52	154	12^2	14^2	20^2	38^2	64^2	90^2

Case 2: Put $\alpha = X_1k + 6$, $k \in Z$
As in case (1), it is discovered by

$$2s = 4X_1(2k + 1) + 36$$
$$X_2 = X_1k^2 + 4k - 4$$
$$X_3 = X_1(k + 1)^2 + 4k + 8$$
$$X_4 = X_1(2k + 1)^2 + 16k + 8$$

where X_1 is any arbitrary integer.
Identify that $\{X_1, X_1k^2 + 4k - 4, X_1(k + 1)^2 + 4k + 8, X_1(2k + 1)^2 + 16k + 8\}$ is the quadruple with the property $D(4X_1(2k + 1) + 36)$.

Table no 2. Mathematical calculations for few chances of X_1 and k are listed in the table below.

k	$2s$	X_1	X_2	X_3	X_4	$X_1X_2 + 2s$	$X_1X_3 + 2s$	$X_1X_4 + 2s$	$X_2X_3 + 2s$	$X_2X_4 + 2s$	$X_3X_4 + 2s$
1	12	-2	-2	4	6	4^2	2^2	0^2	2^2	0^2	6^2
2	16	-1	0	7	15	4^2	3^2	1^2	4^2	4^2	11^2
3	36	0	8	20	56	6^2	6^2	6^2	14^2	22^2	34^2
4	72	1	28	49	153	10^2	11^2	15^2	38^2	66^2	87^2
5	124	2	66	100	330	16^2	18^2	28^2	82^2	148^2	182^2

Python Program for the confirmation of the needed quadruples with numerical values are displayed below.

```
import math
Section = int(input('ENTER THE VALUE OF SECTION'))
if Section == 1:
```

```
k = int(input('ENTER THE VALUE OF k = '))
X1 = int(input('ENTER THE VALUE OF X1 = '))
X2 = X1 * k * k - 4 * k - 4
X3 = X1 * (k + 1) * (k + 1) - 4 * k
X4 = X1 * (2 * k + 1) * (2 * k + 1) - 16 * k - 8
S = 4 * X1 * (2 * k + 1) + 4
print('X1 = ', X1, 'X2 = ', X2, 'X3 = ', X3, 'X4 = ', X4, 'S = ', S)
 A = X1 * X2 + S
 root = math.sqrt(A)
 if int(root + 0.5) ** 2 == A:
        print('X1 * X2 + S = ', A, "product of X1 and X2 is incresased by 2S")
 else:
        print('X1 * X2 + S = ', A, "product of X1 and X2 is not incresased by 2S")
B = X1 * X3 + S
oot = math.sqrt(B)
if int(root + 0.5) ** 2 == B:
        print('X1 * X3 + S = ', B, "product of X1 and X3 is incresased by 2S")
else:
        print('X1 * X2 + S = ', B, "product of X1 and X3 is not incresased by 2S")
C = X1 * X4 + S
root = math.sqrt(C)
if int(root + 0.5) ** 2 == C:
        print('X1 * X4 + S = ', C, "product of X1 and X4 is incresased by 2S")
else:
                print('X1 * X4 + S = ', C, "product of X1 and X4 is not incresased by 2S")
D = X2 * X3 + S
 root = math.sqrt(D)
 if int(root + 0.5) ** 2 == D:
        print('X2 * X3 + S = ', D, "product of X2 and X3 is incresased by 2S")
else:
                print('X2 * X3 + S = ', D, "product of X2 and X3 is not incresased by 2S")
E = X2 * X4 + S
root = math.sqrt(E)
if int(root + 0.5) ** 2 == E:
        print('X2 * X4 + S = ', E, "product of X2 and X4 is incresased by 2S")
else:
        print('X2 * X4 + S = ', E, "product of X2 and X4 is not incresased by 2S")
F = X3 * X4 + S
root = math.sqrt(F)
if int(root + 0.5) ** 2 == F:
        print('X3 * X4 + S = ', F, "product of X3 and X4 is incresased by 2S")
else:
        print('X3 * X4 + S = ', F, "product of X3 and X4 is not incresased by 2S")
elif Section == 2:
k = int(input('ENTER THE VALUE OF k = '))
X1 = int(input('ENTER THE VALUE OF X1 = '))
X2 = X1 * k * k + 4 * k - 4
X3 = X1 * (k + 1) * (k + 1) + 4 * k + 8
X4 = X1 * (2 * k + 1) * (2 * k + 1) + 16 * k + 8
S = 4 * X1 * (2 * k + 1) + 36
print('X1 = ', X1, 'X2 = ', X2, 'X3 = ', X3, 'X4 = ', X4, 'S = ', S)
G = X1 * X2 + S
root = math.sqrt(G)
if int(root + 0.5) ** 2 == G:
        print('X1 * X2 + S = ', G, "product of X1 and X2 is incresased by 2S")
else:
        print('X1 * X2 + S = ', G, "product of X1 and X2 is not incresased by 2S")
H = X1 * X3 + S
```

```
root = math.sqrt(H)
if int(root + 0.5) ** 2 == H:
        print('X1 * X3 + S = ', H, "product of X1 and X3 is incresased by 2S")
else:
        print('X1 * X2 + S = ', H, "product of X1 and X3 is not incresased by 2S")
I = X1 * X4 + S
root = math.sqrt(I)
if int(root + 0.5) ** 2 == I:
        print('X1 * X4 + S = ', I, "product of X1 and X4 is incresased by 2S")
else:
        print('X1 * X4 + S = ', I, "product of X1 and X4 is not incresased by 2S")
J = X2 * X3 + S
root = math.sqrt(J)
if int(root + 0.5) ** 2 == J:
        print('X2 * X3 + S = ', J, "product of X2 and X3 is incresased by 2S")
else:
        print('X2 * X3 + S = ', J, "product of X2 and X3 is not incresased by 2S")
K = X2 * X4 + S
root = math.sqrt(K)
if int(root + 0.5) ** 2 == K:
        print('X2 * X4 + S = ', K, "product of X2 and X4 is incresased by 2S")
else:
        print('X2 * X4 + S = ', K, "product of X2 and X4 is not incresased by 2S")
L = X3 * X4 + S
root = math.sqrt(L)
if int(root + 0.5) ** 2 == L:
        print('X3 * X4 + S = ', L, "product of X3 and X4 is incresased by 2S")
else:
        print('X3 * X4 + S = ', L, "product of X3 and X4 is not incresased by 2S")
```

Output of Some values of k:

```
ENTER THE VALUE OF SECTION 1
ENTER THE VALUE OF k = 2
ENTER THE VALUE OF X1 = −1
X1 = −1 X2 = −16 X3 = −17 X4 = −65 S = −16
X1 * X2 + S = 0 product of X1 and X2 is incresased by 2S
X1 * X3 + S = 1 product of X1 and X3 is incresased by 2S
X1 * X4 + S = 49 product of X1 and X4 is incresased by 2S
X2 * X3 + S = 256 product of X2 and X3 is incresased by 2S
X2 * X4 + S = 1024 product of X2 and X4 is incresased by 2S
X3 * X4 + S = 1089 product of X3 and X4 is incresased by 2S
ENTER THE VALUE OF SECTION 2
ENTER THE VALUE OF k = 5
ENTER THE VALUE OF X1 = 2
X1 = 2 X2 = 66 X3 = 100 X4 = 330 S = 124
X1 * X2 + S = 256 product of X1 and X2 is incresased by 2S
X1 * X3 + S = 324 product of X1 and X3 is incresased by 2S
X1 * X4 + S = 784 product of X1 and X4 is incresased by 2S
X2 * X3 + 2 * S = 6724 product of X2 and X3 is incresased by 2S
X2 * X4 + S = 21904 product of X2 and X4 is incresased by 2S
X3 * X4 + S = 33124 product of X3 and X4 is incresased by 2S
```

III. Conclusion

In this manuscript, variety of 4-tuples with members are integers together with the characteristic that the multiplication of any pair of elements among them added with $2s$ is a square number is evaluated. In such a way that, one can search quadruples, quintuples etc sustaining some other characteristics.

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