

Slip and Pulsatile Mhd Blood Flow Through An Inclined Stenosed Artery With Body Acceleration And Heat Source Effects.

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Abstract

We shall be studying theoretically the effects of body acceleration, heat source and slip on a pulsatory blood flow where the blood is assumed to be an unsteady, non-Newtonian blood flow flowing through an artery with stenosis present at the porous artery walls which is permeable with the results from the study discussed. The application of the magnetic field is in the region perpendicular to the inclined artery with a permeable wall and stenosis at the wall with the inclination angle varying where the fluid flowing through the artery is anelastic-viscous and electrically conducting fluid. The dimensional momentum equation was transformed to a dimensionless form and the Frobenius series solution was gotten for the symmetric axial differential momentum equation with the applied boundary conditions. For clarity of the applicability of the study, results were shown graphically with the behavior of the blood flow through the artery with stenosis shown for the velocity in the axial direction, blood acceleration, wall shear stress and volumetric flow rate.

Keywords: Magneto-hydrodynamic (MHD), Body acceleration, pulsatile pressure, Slip velocity, Permeability of the porous medium.

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Nomenclature

P_0	Pressure gradient in steady state
P_l	Pressure gradient in pulsatile state
w_p	Amplitude of Pulse rate frequency
w_b	Amplitude of the body frequency
f_p	Heart pulse frequency
f_b	Body acceleration frequency
G_0	Body acceleration
$G(t)$	Body acceleration dependent on time
θ	Angle of inclined artery
Φ	Angle of phase difference
z	Axial blood flow direction
t	Time
C	Blood concentration
θ	Blood energy
B_0	Magnetic field
g	Acceleration due to gravity
D	Mass Diffusivity coefficient
K	Porosity coefficient
Re	Reynolds number
C_p	Specific heat capacity
Pe	Peclet number
S_c	Schmidt number
r	Coordinates of the radial flow
z	Coordinate of the Axial flow
μ	Coefficient Blood viscosity
ρ	Blood density
K_r	Chemical reaction term

σ	Blood flow electrical conductivity
Sc	Schmidt number
C_p	Specific heat capacity
M	Magnetic parameter
θ	Temperature
G_r	Grashof energy number
G_c	Grashof concentration number
β_T	Volume expansion Coefficient due to temperature
β_C	Volume expansion Coefficient due to concentration.
C_w	Artery wall concentration
T_w	Artery wall temperature
H	Heat source term
δ	Stenosis height
l_0	Stenosis length,
$R(z)$	Radius of the stenotic vessel
R_0	Radius of the normal artery
s	Stenosis shape term
ξ	Tapering term
d	Stenosis position.

I. Introduction.

This theoretical study analysis the flow of blood through an inclined artery having stenosis at the artery walls with the effect of heat source and body acceleration which has immense significance and importance in the growth of tumor and cardiovascular disease. Blood flow can be both steady and unsteady where it doesn't depend on time and on the other hand depends on time. Certain factors can affect the viscosity of the blood where the shear stress could reduce its viscosity because of certain factors such as heat source and agitation of the blood due to body acceleration effect. The pulsatile blood flow past an artery has caught the attention from researchers because of the relevant applicability in the biotechnological, biomedical and medical sciences. Blood circulation takes place when blood is pumped from the heart to different muscles of the body through the arteries which transports the blood to the body muscles where the pressure gradient creates a pulsatile flow of the blood. The application of heat source to the areas where the stenosis is present due to plaques of cholesterol could help to increase blood flow and arrest diseases resulting to hypotension for cardiac failure. Furthermore the application of heat source and induced slip at the wall can help treat ailments such as cancer and tumor growth.

Stenosis in the artery affects blood flowing from the heart through the artery Rabbi et al. [1] and Ellahi et al. [2]. Pralhad et al. [3] studied the blood flow through the artery with stenosis with the wall shear stress and the resistance at the wall while Ellahi et al. [4] did a mathematical model explaining the blood flow through an artery with a composite stenosis. Magnetic field applied on bio fluids have effect on the dynamics of the fluid hence this fluids are bio magnetic fluids with rich application in medical sciences and biomedical engineering. Haik et al. [5] gave a clear distinction between bio magnetic fluid (BFD) and hydro magnetic fluid (MHD). Abdullah et al. [6] studied the effect of magneto-hydrodynamic on the flow of blood through a stenosis that is irregular with the results obtained from the study showing that, the rate of flow of blood reduced as a result of magnetic field applied to the arterial segment. Prakash et al. [7] did a study on the Magneto-hydrodynamic blood flow through an artery that is bifurcated with the effect of heat source where the blood flowing past the artery is considered to be unsteady and a non-Newtonian fluid. The results showed that the magnetic field and the heat source modify the pattern of the blood flow while the temperature of the blood is increased with heat source increasing the velocity.

Srivastava [8] did an analysis of the blood flow motion that is steady through an artery inclined with applied magnetic field with the conclusion that velocity of the blood flow decreases as a result of the increase in the magnetic field. The study showed that MHD fluids had electrically conducting properties due to the magnetic field applied to the fluid. Eldesoky [9] proposed a mathematical modelling of blood flow that is parallel with applied magnetic field in the transverse direction with effect of heat source. Results showed that increase in the heat source increases the temperature and axial velocity while increase in the decay reduces the temperature and axial velocity. Sudden change in the velocity could disturb the flow of blood through the artery hence having an effect on the blood flow. The prolonged sudden velocity change with the inclined body artery caused by body movement during aircraft flight, car driving, etc., could have a dangerous effect on the human body. Saddiqui, et al. [10] studied the effect of slip and body acceleration on the pulsatile flow of blood on a Casson fluid flowing through a stenotic artery. Sinha et al. [11] did a study on the slip and periodic body acceleration effect on pulsatile flow of blood passing through a segmented stenosed artery. Sinha et al. [12] did a

study on the effect of the transfer of heat on unsteady Magneto-hydrodynamic blood flow in a vessel that is permeable with non-uniform heat source present. The stretching velocity that is time-dependent and surface temperature of the vessel causes unsteady coupled flow and temperature fields with the heat source/sink effect on the blood flow which non-uniform was considered. The study showed its clinical application in treating cardiovascular disorders with accelerated circulation. Tripathi and Sharma [13] did a study on heat and mass transfer effects of a blood flow two phases which is pulsatile past a stenosed artery that is narrow with chemical reaction and radiation. Karthikeyan and Jeevitha [14] did a study analysis on the effect of heat and mass transfer on a model in two phases for unsteady blood flow that is pulsatile past an artery with stenosis having a wall that is permeable with radiation and chemical reaction effects. Abubakar and Adeoye [15] did a study analyzing the effect of heat radiation and magnetic field on blood flowing in a tapered and inclined porous artery with stenosis. Amos and Ogulu [16] studied the magnetic field effect on Pulsatile Blood Flow through an axis-symmetric channel that is constricted. Bunonyo and Amos [17] studied the effect of lipid concentration on the blood that flows through an artery channel inclined with magnetic field present.

This study has theoretically analyzed, showing the effects of slip, heat source, body acceleration, inclined artery angle and pulsatile pressure on the non-Newtonian unsteady blood flow through a stenotic artery. The artery walls are porous and permeable with the analysis done by solving the problem of the governing equation using the Frobenius power series method to obtain the velocity of the blood flow, blood acceleration, wall shear stress and volumetric flow rate solutions with a graph illustrating the behavior of the effect of slip, heat source, body acceleration, magnetic field, pulsatile pressure gradient and inclined artery on the blood flow velocity, blood acceleration, shear stress and volumetric flow rate.

II. Formulation of the problem

The motion of the flow of blood is axisymmetric with the coordinate's (r', θ', z') flowing horizontally in the axis z' . The one dimensional blood flow is transported past a rigid and cylindrical stenotic artery whose walls are porous with permeability where the fluid considered is an incompressible, non-Newtonian, viscous, electrically conducting blood fluid under the influence of a magnetic field applied normal to the tangential artery. The height and the position of the stenosis is dependent on the height of the constricted artery wall.

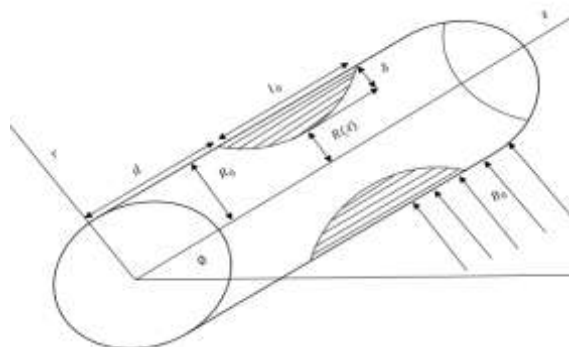


Figure 1.1 Flow geometry of the stenotic artery

The geometry of the one dimensional blood flow through the segmented stenotic artery, symmetrical in shape was proposed by Eldesoky [18] and Kumar et al. [19] as,

$$R(z') = \left\{ d'(z) - \frac{\delta'}{2} \left[1 + \cos \frac{2\pi}{l_0} \left\{ z' - d' - \frac{l_0'}{2} \right\} \right] \right\}, d' \leq z' \leq d' + l_0' \quad (1)$$

The greatest stenosis height (R) happens at the center, Nadeem et al. [20].

$$z = d + \frac{L_0}{s} \left(\frac{1}{s-1} \right) \quad \text{For } s \geq 2 \text{ and } l_1 = \frac{d'}{R_0}$$

III. Governing equation

The blood temperature and blood flow velocity which is steady and unsteady flowing past an inclined artery with the applied magnetic field in the radial direction perpendicular to the axial direction is considered

$$\rho \frac{\partial u'}{\partial t'} = - \frac{\partial p'}{\partial z} + \rho G(t) + \mu \frac{\partial}{\partial r'} \left(r' \frac{\partial u'}{\partial r'} \right) - \sigma_c B_0^2 u' + g \sin \phi - \frac{\mu}{k_p} u' \quad (2)$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + N^2 \theta \quad (3)$$

The non-dimensional form of pressure gradient is expressed as

$$- \frac{\partial p'}{\partial z} = P_0' + P_1' \cos(w_p t'); t \geq 0 \quad (4)$$

Where $w_p = 2\pi f_p$ and $w_b = 2\pi f_b$

$$G(t) = G_0 \cos(w_b t + \varphi); \quad t \geq 0 \tag{5}$$

The dimensionless flow geometry with stenosis, Eldesoky[18] and Kumar et al. [19].

$$R(z) = \left\{ \begin{array}{l} (1 + \xi z) - \frac{\delta}{2} \left[1 + \cos \frac{2\pi}{l_0} \left\{ z - l_1 - \frac{l_0}{2} \right\} \right] \\ (1 + \xi z) \end{array} \right\}, \quad l_1 \leq z \leq l_1 + l_0 \tag{6}$$

The non-dimensional body acceleration and Pressure gradient

$$G(t) = G_0 \cos(bt + \varphi); \quad t \geq 0 \tag{7a}$$

$$-\frac{\partial p}{\partial z} = P_0 + P_l \cos(w_p t); \quad t \geq 0 \tag{7b}$$

The momentum equation in dimensionless form is written as for first consideration:

$$Re \frac{\partial u}{\partial t} = P_0 + P_l \cos t + G_0 \cos(bt + \varphi) + \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \left(M^2 + \frac{1}{K} \right) u + \frac{\sin \theta}{Fr} \tag{8}$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + N^2 \theta \tag{9}$$

The dimensional initial and boundary slip conditions are

$$\left\{ \begin{array}{l} \frac{\partial u'}{\partial r'} = -h' u', T' = T'_a \text{ at } r' = R'(z) \\ \frac{\partial u'}{\partial r'} = 0, \frac{\partial \theta'}{\partial r'} = 0 \text{ at } r' = 0 \end{array} \right\} \tag{10a}$$

The dimensionless initial and boundary slip conditions are

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial r} = -hu, \theta = \theta_a \text{ at } r = R(z) \\ \frac{\partial u}{\partial r} = 0, \frac{\partial \theta}{\partial r} = 0 \text{ at } r = 0 \end{array} \right\} \tag{10b}$$

η depends on the property of the porous material, K is the permeability parameter.

$$a = \frac{R(z)}{R_0 + \xi z}; \quad \xi = \tan \phi, \quad h = -\frac{\eta}{R_0 \sqrt{K}}$$

Non dimensional parameters in the governing equations and boundary conditions are transformed to dimensionless form.

$$\begin{aligned} u &= \frac{u'}{u_0}; \delta = \frac{\delta'}{R_0}; d(z') = R_0 + \xi z'; r = \frac{r'}{R_0}; z = \frac{z'}{R_0}; b = \frac{w_b}{w_p}; t = w_p t'; R(z) = \frac{R'(z)}{R_0}; P = \frac{R_0 \rho'}{u_0 \mu}; Re = \frac{\rho \omega R_0'^2}{\mu}; \theta = \\ & \frac{T' - T_0}{T_w - T_0}; \theta_a = \frac{T'_a - T_0}{T_w - T_0}; C = \frac{C' - C_\infty}{C_w - C_\infty}; C_a = \frac{C'_a - C_\infty}{C_w - C_\infty} \delta = \frac{\delta'}{R_0}; N^2 = \frac{R_0'^2 Q_0}{\rho c_p k_p}; M^2 = \frac{\sigma R_0'^2 B_0^2}{\mu}; P_e = \frac{\rho R_0'^2 c_p}{k_p}; S_c = \frac{\theta}{D'}; Kr = \\ & \frac{E' R_0'^2}{\theta D'}; G_r = \frac{g \rho R_0'^2 \beta_T \theta (T_w - T_0)}{u_0 \mu}; G_C = \frac{g \rho R_0'^2 \beta_C (C_w - C_0)}{u_0 \mu}; P_l = \frac{P'_l R_0'^2}{u_0 \mu}; P_0 = \frac{P'_0 R_0'^2}{u_0 \mu}; G_0 = \frac{\rho G'_0 R_0'^2}{u_0 \mu}; f_r = \frac{u_0 \mu}{g R_0'^2}; D = \\ & \frac{D'}{D_0}; k = \frac{k'_p}{R_0'^2}; \end{aligned} \tag{11}$$

IV. Method of Solution

The differential equation analytically solved using the Frobenius method. The solutions of the governing non-linear partial differential equation for the steady and pulsatile blood flow velocity and blood temperature is expressed as a function of time

$$u(r, t) = u_0(r) + u_p(r) \epsilon e^{i\omega t} \tag{12}$$

$$\theta(r, t) = \theta_0(r) + \theta_p(r) \epsilon e^{i\omega t} \tag{13}$$

V. Solution to the Governing equation

The temperature for the steady and pulsatile state is expressed below as

$$\frac{\partial^2 \theta_0}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_0}{\partial r} + N^2 \theta_0 = 0 \tag{14}$$

$$\frac{\partial^2 \theta_p}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_p}{\partial r} + \alpha_1 \theta_p = 0 \tag{15}$$

Where $\alpha_1 = N^2 - i\omega Pe$

The blood flow velocity for the steady and pulsatile state is expressed below as

$$\frac{\partial^2 u_0}{\partial r^2} + \frac{1}{r} \frac{\partial u_0}{\partial r} - \beta_1 u_0 = -G - G_r \theta_0 \tag{16}$$

$$\frac{\partial^2 u_p}{\partial r^2} + \frac{1}{r} \frac{\partial u_p}{\partial r} - \beta_2 u_p = -F - G_r \theta_p \tag{17}$$

$$\beta_1 = M^2 + \frac{1}{K}; \quad G = P_0 + \frac{\sin \theta}{Fr}; \quad \beta_2 = M^2 + \frac{1}{K} + Re i \omega \text{ and } F = P_l \cos t + G_0 \cos(bt + \varphi)$$

The power series solution for the steady state and the pulsatile state for both the temperature and blood flow velocity using the Funch's theorem, called the Frobenius series expressed as

$$\theta_0(r) = \sum_{n=0}^{\infty} a_n r^{n+k} \text{ Where } a_n, k \in C_1 \tag{18}$$

$$\theta_p(r) = \sum_{n=0}^{\infty} b_n r^{m+k} \text{ Where } b_n, k \in C_2 \tag{19}$$

$$u_0 = \sum_{n=0}^{\infty} c_n r^{n+k} \text{ Where } c_n, k \in C_3 \tag{20}$$

$$u_p = \sum_{m=0}^{\infty} d_m r^{m+k} \text{ Where } d_m, k \in C_4 \tag{21}$$

The temperature in the steady state is expressed as

$$\theta_0 = C_1 \left[1 - \frac{N^2 r^2}{2^2} + \frac{N^4 r^4}{2^2 4^2} - \frac{N^6 r^6}{2^2 4^2 6^2} + \frac{N^8 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + D_1 \left[\ln r \left(1 - \frac{N^2 r^2}{2^2} + \frac{N^4 r^4}{2^2 4^2} - \frac{N^6 r^6}{2^2 4^2 6^2} + \frac{N^8 r^8}{2^2 4^2 6^2 8^2} + \dots \right) + \left(\frac{N^2 r^2}{2^2} - \frac{3N^4 r^4}{2^3 4^2} + \frac{N^6 r^6}{4^3 6^3} + \frac{N^8 r^8}{2^5 6^3 8^3} - \dots \right) \right] \tag{22}$$

Apply the boundary condition in equation (10) to equation (22) with $D_1 = 0$, then

$$\theta_0 = C_1 \left[1 - \frac{N^2 r^2}{2^2} + \frac{N^4 r^4}{2^2 4^2} - \frac{N^6 r^6}{2^2 4^2 6^2} + \frac{N^8 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \tag{23}$$

$$\text{Where } C_1 = \frac{\theta_R}{\left[1 - \frac{N^2 R^2}{2^2} + \frac{N^4 R^4}{2^2 4^2} - \frac{N^6 R^6}{2^2 4^2 6^2} + \frac{N^8 R^8}{2^2 4^2 6^2 8^2} + \dots \right]}$$

The temperature in the pulsatile state is expressed as

$$\theta_p = C_2 \left[1 - \frac{\alpha_1 r^2}{2^2} + \frac{\alpha_1^2 r^4}{2^2 4^2} - \frac{\alpha_1^3 r^6}{2^2 4^2 6^2} + \frac{\alpha_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + D_2 \left[\ln r \left(1 - \frac{\alpha_1 r^2}{2^2} + \frac{\alpha_1^2 r^4}{2^2 4^2} - \frac{\alpha_1^3 r^6}{2^2 4^2 6^2} + \frac{\alpha_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right) + \left(\frac{\alpha_1 r^2}{2^2} - \frac{3\alpha_1^2 r^4}{2^3 4^2} + \frac{\alpha_1^3 r^6}{4^3 6^3} + \frac{\alpha_1^4 r^8}{2^5 6^3 8^3} - \dots \right) \right] \tag{24}$$

Apply the boundary condition in equation (10) to equation (24) with $D_2 = 0$, then

$$\theta_p = C_2 \left[1 - \frac{\alpha_1 r^2}{2^2} + \frac{\alpha_1^2 r^4}{2^2 4^2} - \frac{\alpha_1^3 r^6}{2^2 4^2 6^2} + \frac{\alpha_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \tag{25}$$

$$\text{Where } C_2 = \frac{\theta_R}{\left[1 - \frac{\alpha_1 R^2}{2^2} + \frac{\alpha_1^2 R^4}{2^2 4^2} - \frac{\alpha_1^3 R^6}{2^2 4^2 6^2} + \frac{\alpha_1^4 R^8}{2^2 4^2 6^2 8^2} + \dots \right]}$$

The solution for the temperature in equation (13) is gotten by combining both equation (23) and (25) which is expressed as

$$\theta(r, t) = C_1 \left[1 - \frac{N^2 r^2}{2^2} + \frac{N^4 r^4}{2^2 4^2} - \frac{N^6 r^6}{2^2 4^2 6^2} + \frac{N^8 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + \left(C_2 \left[1 - \frac{\alpha_1 r^2}{2^2} + \frac{\alpha_1^2 r^4}{2^2 4^2} - \frac{\alpha_1^3 r^6}{2^2 4^2 6^2} + \frac{\alpha_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \right) e^{i\omega t} \tag{26}$$

The complementary solution for the blood flow velocity in the steady state in equation (16) is expressed as

$$u_{0c} = C_3 \left[1 + \frac{\beta_1 r^2}{2^2} + \frac{\beta_1^2 r^4}{2^2 4^2} + \frac{\beta_1^3 r^6}{2^2 4^2 6^2} + \frac{\beta_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + D_3 \left[\ln r \left(1 + \frac{\beta_1 r^2}{2^2} + \frac{\beta_1^2 r^4}{2^2 4^2} + \frac{\beta_1^3 r^6}{2^2 4^2 6^2} + \frac{\beta_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right) + \left(-\frac{\beta_1 r^2}{2^2} - \frac{3\beta_1^2 r^4}{2^3 4^2} - \frac{\beta_1^3 r^6}{4^3 6^3} + \frac{\beta_1^4 r^8}{2^5 6^3 8^3} - \dots \right) \right] \tag{27}$$

Apply the boundary condition in equation (10) to equation (27) with $D_3 = 0$, then

$$u_{0c} = C_3 \left[1 + \frac{\beta_1 r^2}{2^2} + \frac{\beta_1^2 r^4}{2^2 4^2} + \frac{\beta_1^3 r^6}{2^2 4^2 6^2} + \frac{\beta_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \tag{28}$$

The particular solution for the steady state blood flow velocity in equation (16) is expressed as

$$u_{0p} = M_0 + M_1r^2 + M_2r^4 + M_3r^6 + M_4r^8 \quad (29)$$

The solution for the blood flow velocity in equation (16) is the combination of equation (28) and (29).

$$u_0 = C_3 \left[1 + \frac{\beta_1 r^2}{2^2} + \frac{\beta_1^2 r^4}{2^2 4^2} + \frac{\beta_1^3 r^6}{2^2 4^2 6^2} + \frac{\beta_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + M_0 + M_1r^2 + M_2r^4 + M_3r^6 + M_4r^8 \quad (30)$$

Where $C_3 = -$

$$\frac{[h(M_0 + M_1R^2 + M_2R^4 + M_3R^6 + M_4R^8) + 2M_1R + 4M_2R^3 + 6M_3R^5 + 8M_4R^7]}{\left[\frac{\beta_1 R^2}{2} + \frac{\beta_1^2 R^3}{2^2 4} + \frac{\beta_1^3 R^5}{2^2 4^2 6} + \frac{\beta_1^4 R^7}{2^2 4^2 6^2 8} + \dots \right] + h \left[1 + \frac{\beta_1 R^2}{2^2} + \frac{\beta_1^2 R^4}{2^2 4^2} + \frac{\beta_1^3 R^6}{2^2 4^2 6^2} + \frac{\beta_1^4 R^8}{2^2 4^2 6^2 8^2} + \dots \right]}$$

The complementary solution for the blood flow velocity in the pulsatile state in equation (17) is expressed as

$$u_{pc} = C_4 \left[1 + \frac{\beta_2 r^2}{2^2} + \frac{\beta_2^2 r^4}{2^2 4^2} + \frac{\beta_2^3 r^6}{2^2 4^2 6^2} + \frac{\beta_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + D_4 \left[\ln r \left(1 + \frac{\beta_2 r^2}{2^2} + \frac{\beta_2^2 r^4}{2^2 4^2} + \frac{\beta_2^3 r^6}{2^2 4^2 6^2} + \frac{\beta_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right) - \beta_2 r^2 - 3\beta_2^2 r^4 - 4\beta_2^3 r^6 - 5\beta_2^4 r^8 - \dots \right] \quad (31)$$

Apply the boundary condition in equation (10) to equation (31) with $D_4 = 0$, then

$$u_{pc} = C_4 \left[1 + \frac{\beta_2 r^2}{2^2} + \frac{\beta_2^2 r^4}{2^2 4^2} + \frac{\beta_2^3 r^6}{2^2 4^2 6^2} + \frac{\beta_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (32)$$

The particular solution for the pulsatile state blood flow velocity in equation (16) is expressed as

$$u_{pp} = L_0 + L_1r^2 + L_2r^4 + L_3r^6 + L_4r^8 \quad (33)$$

The solution for the blood flow velocity in equation (17) is the combination of equation (32) and (33).

$$u_p = C_4 \left[1 + \frac{\beta_2 r^2}{2^2} + \frac{\beta_2^2 r^4}{2^2 4^2} + \frac{\beta_2^3 r^6}{2^2 4^2 6^2} + \frac{\beta_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + L_0 + L_1r^2 + L_2r^4 + L_3r^6 + L_4r^8 \quad (34)$$

Where $C_4 = -$

$$\frac{[h(L_0 + L_1R^2 + L_2R^4 + L_3R^6 + L_4R^8) + 2L_1R + 4L_2R^3 + 6L_3R^5 + 8L_4R^7]}{\left[\frac{\beta_2 R^2}{2} + \frac{\beta_2^2 R^3}{2^2 4} + \frac{\beta_2^3 R^5}{2^2 4^2 6} + \frac{\beta_2^4 R^7}{2^2 4^2 6^2 8} + \dots \right] + h \left[1 + \frac{\beta_2 R^2}{2^2} + \frac{\beta_2^2 R^4}{2^2 4^2} + \frac{\beta_2^3 R^6}{2^2 4^2 6^2} + \frac{\beta_2^4 R^8}{2^2 4^2 6^2 8^2} + \dots \right]}$$

The solution to the blood flow velocity in equation (9) by substituting equation (30) and (34) into equation (12) is expressed as

$$u(r, t) = C_3 \left[1 + \frac{\beta_1 r^2}{2^2} + \frac{\beta_1^2 r^4}{2^2 4^2} + \frac{\beta_1^3 r^6}{2^2 4^2 6^2} + \frac{\beta_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + M_0 + M_1r^2 + M_2r^4 + M_3r^6 + M_4r^8 + \left(C_4 \left[1 + \frac{\beta_2 r^2}{2^2} + \frac{\beta_2^2 r^4}{2^2 4^2} + \frac{\beta_2^3 r^6}{2^2 4^2 6^2} + \frac{\beta_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + L_0 + L_1r^2 + L_2r^4 + L_3r^6 + L_4r^8 \right) \varepsilon e^{i\omega t} \quad (35)$$

The Solution for the Fluid Acceleration equation

$$F(r, t) = \frac{du}{dt} = i\omega \varepsilon e^{i\omega t} \left(C_4 \left[1 + \frac{\beta_2 r^2}{2^2} + \frac{\beta_2^2 r^4}{2^2 4^2} + \frac{\beta_2^3 r^6}{2^2 4^2 6^2} + \frac{\beta_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] + \frac{4L_1}{\beta_2} + \frac{GrC_2}{\beta_2} + L_1r^2 + L_2r^4 + L_3r^6 + L_4r^8 \right) + \varepsilon e^{i\omega t} \frac{P_1}{\beta_2} (i\omega \cos t - \sin t) + \varepsilon e^{i\omega t} \frac{G_0}{\beta_2} (i\omega \cos (bt + \varphi) - b \sin(bt + \varphi)) \quad (36)$$

The Solution for the Wall Shear Stress equation

$$\frac{du}{dr} = C_3 \left[\frac{\beta_1 r}{2} + \frac{\beta_1^2 r^3}{2^2 4} + \frac{\beta_1^3 r^5}{2^2 4^2 6} + \frac{\beta_1^4 r^7}{2^2 4^2 6^2 8} + \dots \right] + 2M_1 r + 4M_2 r^3 + 6M_3 r^5 + 8M_4 r^7 + \left(C_4 \left[\frac{\beta_2 r}{2^2} + \frac{\beta_2^2 r^3}{2^2 4} + \frac{\beta_2^3 r^5}{2^2 4^2 6} + \frac{\beta_2^4 r^7}{2^2 4^2 6^2 8} + \dots \right] + 2L_1 r + 4L_2 r^3 + 6L_3 r^5 + 8L_4 r^7 \right) \epsilon e^{i\omega t} \quad (37)$$

The Solution for the Volumetric Flow Rate equation

$$Q(r, t) = 2\pi \int_0^a r u(r, t) dr = 2\pi \left\{ \begin{aligned} & C_3 \left[\frac{a^2}{2} + \frac{\beta_1 a^4}{2^2 4} + \frac{\beta_1^2 a^6}{2^2 4^2 6} + \frac{\beta_1^3 a^8}{2^2 4^2 6^2 8} + \frac{\beta_1^4 a^{10}}{2^2 4^2 6^2 8^2 10} + \dots \right] + \\ & \left(\frac{M_0 a^2}{2} + \frac{M_1 a^4}{4} + \frac{M_2 a^6}{6} + \frac{M_3 a^8}{8} + \frac{M_4 a^{10}}{10} + \left(C_4 \left[\frac{a^2}{2} + \frac{\beta_2 a^4}{2^2 4} + \frac{\beta_2^2 a^6}{2^2 4^2 6} + \frac{\beta_2^3 a^8}{2^2 4^2 6^2 8} + \frac{\beta_2^4 a^{10}}{2^2 4^2 6^2 8^2 10} + \dots \right] + \right. \right. \\ & \left. \left. + \frac{L_0 a^2}{2} + \frac{L_1 a^4}{4} + \frac{L_2 a^6}{6} + \frac{L_3 a^8}{8} + \frac{L_4 a^{10}}{10} \right) \epsilon e^{i\omega t} \right\} \quad (38)$$

VI. Graphical Results and Discussion.

In Figure 2.0, it was observed that the higher the heat source H from $0.5 \leq H \leq 2$, resulted to an increase in the blood flow velocity at the artery center but approaches zero at the wall of the stenotic artery. This is because the increased heat reduces the blood viscosity hence causing an increase in blood flow. The increase in the blood flow at the wall of the artery results to an increase in both the blood acceleration and the volumetric flow rate in figure 3.0 and figure 5.0 but a decrease in the shear stress at the artery wall in figure 4.0.

In figure 6.0, it was observed that the higher the artery inclination ϕ from $15^\circ \leq \phi \leq 60^\circ$, resulted to the increase in the blood flow velocity at the center of the artery but tends to zero at the artery wall with stenosis. A fluctuating behavior was observed for the blood acceleration in figure 7.0, while the blood flow velocity increase caused an increase in the wall shear stress and volumetric flow rate in figure 8.0 and figure 9.0.

Figure 10.0, showed that an increased body acceleration G_0 caused an increase in blood flow due because the heart work rate increase which causes more blood to be pumped from the heart to the muscles. This increase results to an increase in the blood acceleration, shear stress at the walls of the artery with stenosis and volumetric flow rate in figure 11.0 to Figure 13.0.

In figure 14.0, it was observed that the increased Wormersely number caused the velocity of the blood flow to decrease. This decrease causes a decrease in the acceleration of blood, wall shear stress at the stenotic wall and volumetric flow rate from figure 15.0 to Figure 17.0.

In figure 14.0, it was seen that an as the body acceleration frequency b increased, the velocity of the blood flow decreased. This decrease results to an increase in the blood acceleration in figure 15.0 but a decrease in the shear stress at the artery walls and volumetric flow rate from figure 16.0 to Figure 17.0.

Figure 18.0, showed that increased permeability of porous wall k caused an increase the blood flow velocity because of the reduced viscous force at the artery walls. The increase causes an increase in acceleration of blood, shear stress at the wall of the artery and the volumetric flow rate from figure 19.0 to Figure 21.0. This conforms to study done by Tripathi and Sharma [21] and Shina et al. [12].

Figure 22.0, showed that increased magnetic field M caused a decrease in the velocity of the blood flow caused by increased Lorentz force resist the flow of blood. This will decrease the acceleration of the blood, wall shear stress of the stenotic artery and the volumetric flow rate from figure 23.0 to Figure 25.0. This conforms to study done by Tripathi and Sharma [20], Wahab and Salemi [22] and Shina et al. [12].

An increase in heart work rate caused by the body acceleration increase, results to an increase in the pulsatile pressure Pl which increases the velocity of the blood flow in figure 26.0. The increase caused an increase in the acceleration of the blood, wall shear stress of the stenotic artery and the volumetric flow rate from figure 27.0 to figure 29.0.

An increased slip value h at the stenotic artery wall caused a decrease in the velocity and acceleration of the blood flow from figure 30.0 to figure 31.0. This was because the slip induced at the wall also opposes the blood flow hence reduces the blood flow and blood acceleration. This conforms to study done by Shina et al. [12]. The shear stress at the wall of the artery with stenosis becomes less in figure 32.0. The volumetric flow rate also reduced at the artery walls in figure 33.0. This conforms to study done by Tripathi and Sharma [21].

Over a prolonged time t in figure 34.0, the velocity of the blood flow decreased but the acceleration of the blood increased in figure 35.0 in figure 36.0 and figure 37.0, the shear stress and the volumetric flow rate decreases at the artery wall with stenosis.

An increased radius of stenosis R at the stenotic artery wall leads to reduction in stenosis height which caused an increase in the velocity and acceleration of the blood flow from figure 38.0 to figure 39.0. This was because the artery becomes open due to reduction in stenosis at the wall, hence aiding blood flow and blood acceleration. This conforms to study done by Tripathi and Sharma [21]. The shear stress at the wall of the artery with stenosis reduces in figure 40.0 while the volumetric flow rate increases at the artery walls in figure 41.0.

Finally the increase in the magnetic field and heat source increased the blood temperature in figure 42.0 and figure 43.0

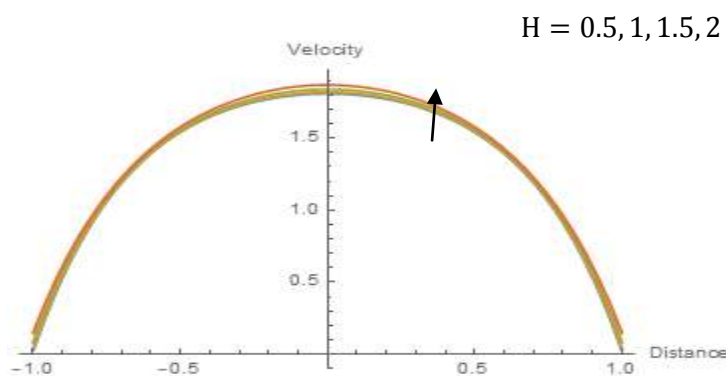


Figure 2.0 Graph for the velocity of Blood flow with increasing values of Heat Source when $Gr = 2, Pe = 1, Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 30^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

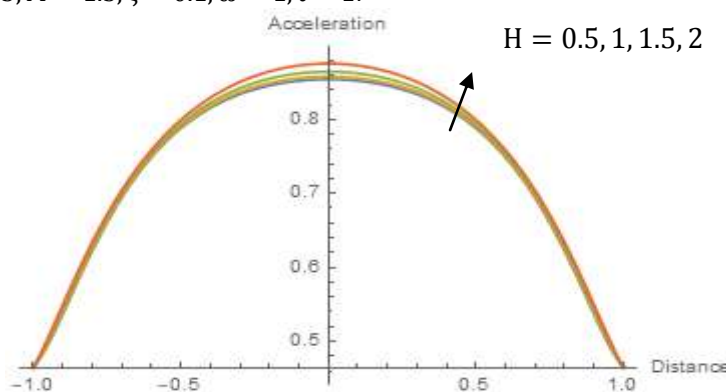


Figure 3.0 Graph for the Blood acceleration with increasing values of Heat Source when $Sc = 1, Kr = 1, Gc = 3, Gr = 2, Pe = 1, Po = 2, Pl = 4, Go = 3, b = 2, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

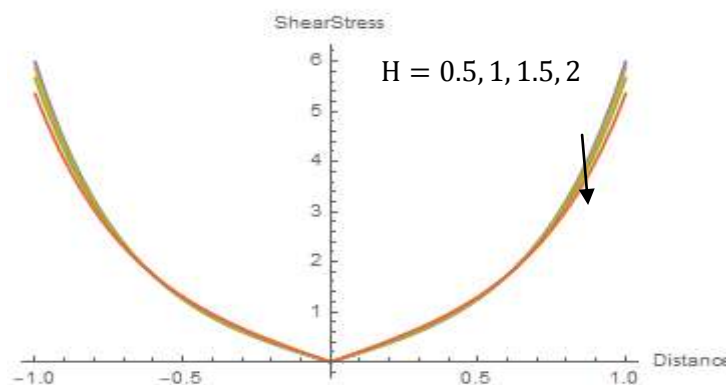


Figure 4.0 Graph for the shear stress at the wall with increasing values of Heat Source $H = 0.5, 1, 1.5, 2$, when $Gr = 2, Pe = 1, Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 30^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

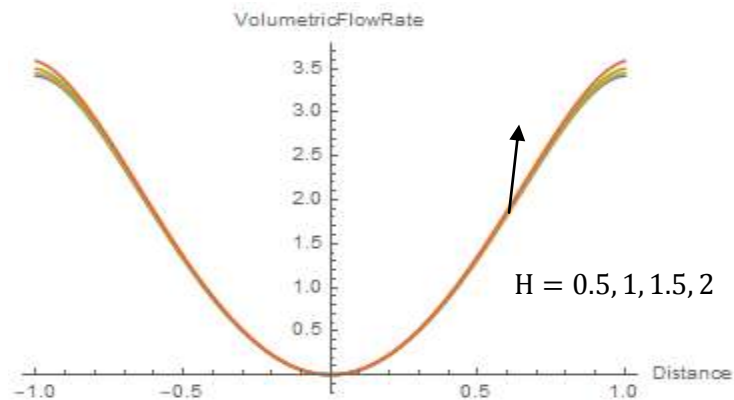


Figure 5.0 Graph for the Volumetric Flow rate with increasing values of Heat Source $H = 0.5, 1, 1.5, 2$, when $Gr = 2, Pe = 1, Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 30^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

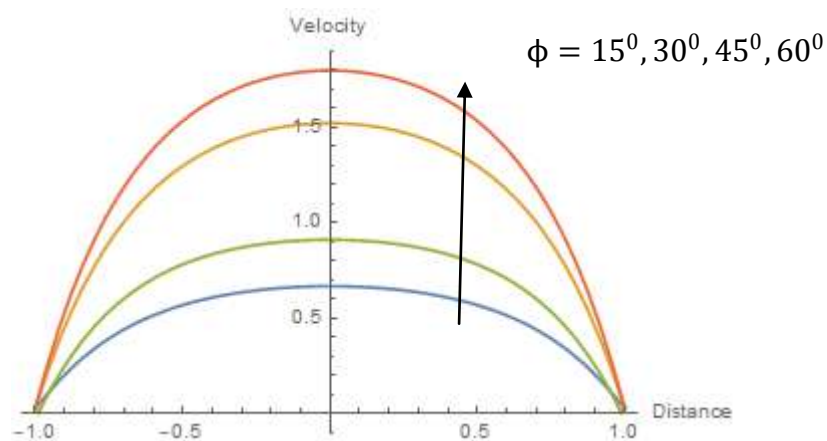


Figure 6.0 Blood flow velocity distribution for increase in the inclined artery ϕ when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

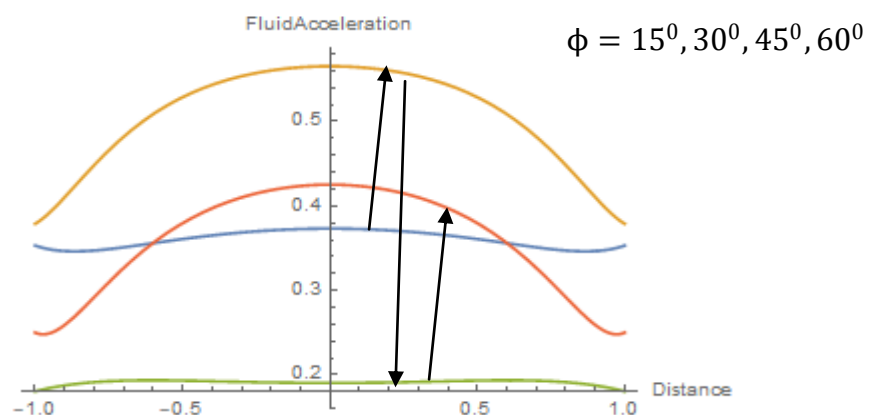


Figure 7.0 Blood acceleration profile for increase in the inclined artery angle ϕ when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

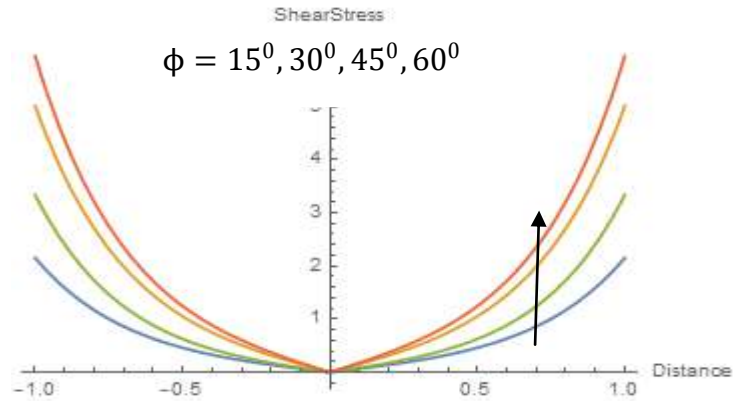


Figure 8.0 Wall shear stress profile for increase in the inclined artery ϕ when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

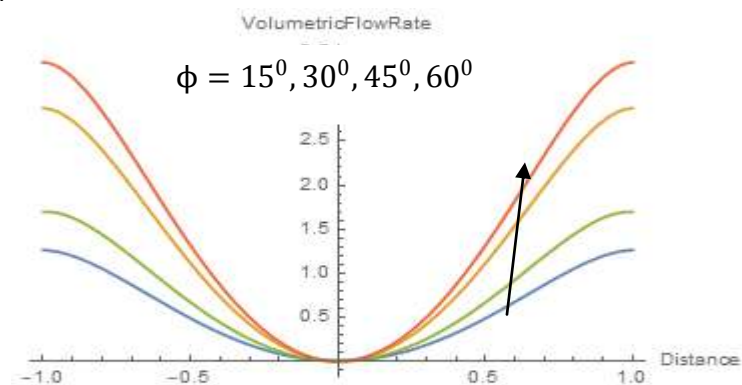


Figure 9.0 Volumetric flow rate profile for increase in the inclined artery ϕ when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

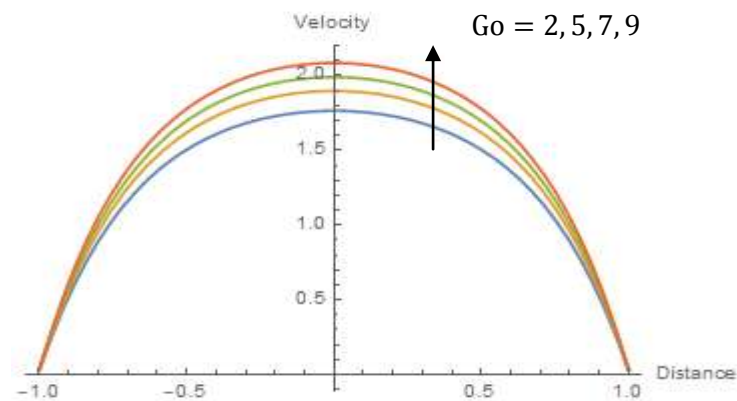


Figure 10.0 Blood flow velocity distribution for increase in the Body acceleration Go when $Po = 2, Pl = 4, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

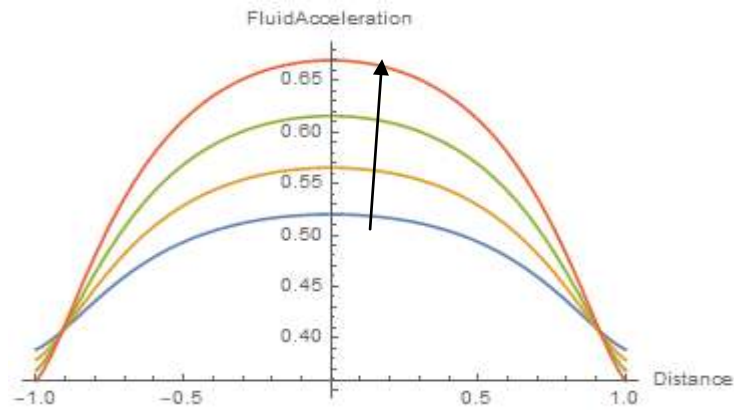


Figure 11.0 Blood acceleration profile for increase in the Body acceleration G_0 when $Po = 2, Pl = 4, Fr = 0.05, b = 2, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

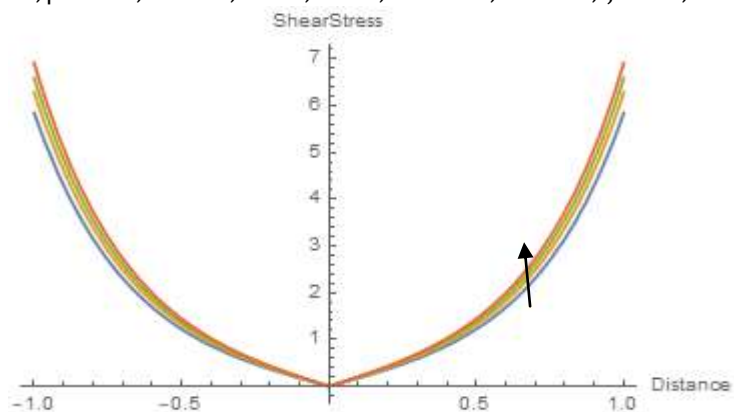


Figure 12.0 Wall shear stress profile for increase in the Body acceleration G_0 when $Po = 2, Pl = 4, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

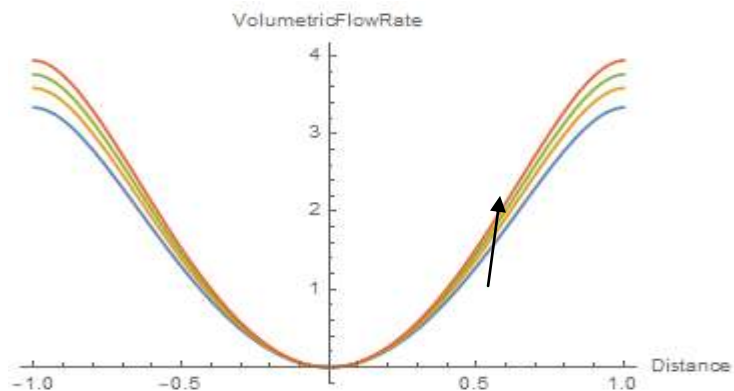


Figure 13.0 Volumetric flow rate profile for increase in the Body acceleration G_0 when $Po = 2, Pl = 4, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

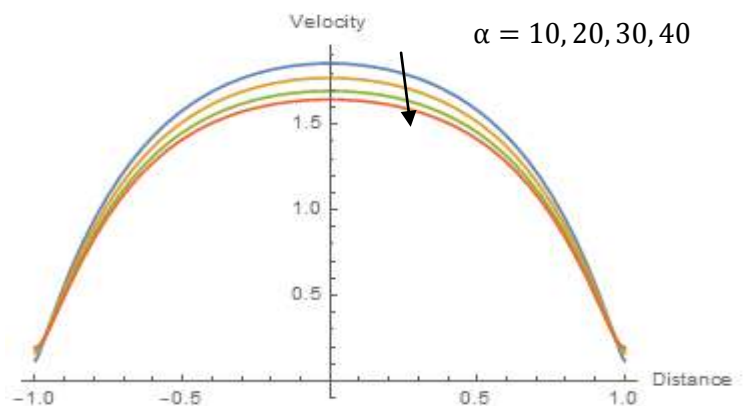


Figure 14.0 Blood flow velocity distribution for increase in the Womersley Number α when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

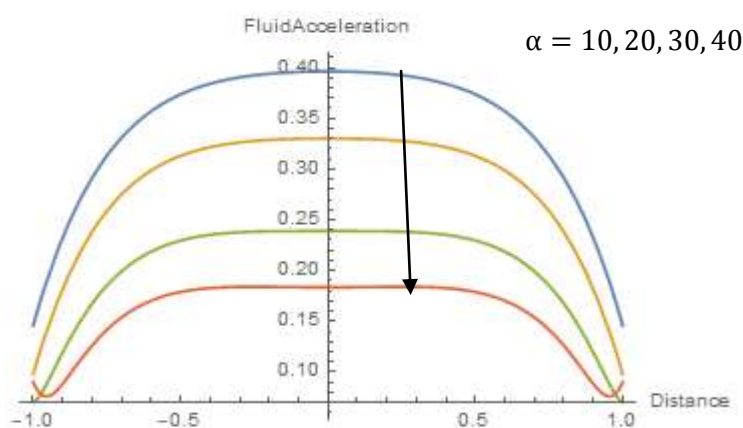


Figure 15.0 Blood acceleration profile for increase in the Womersley Number α when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

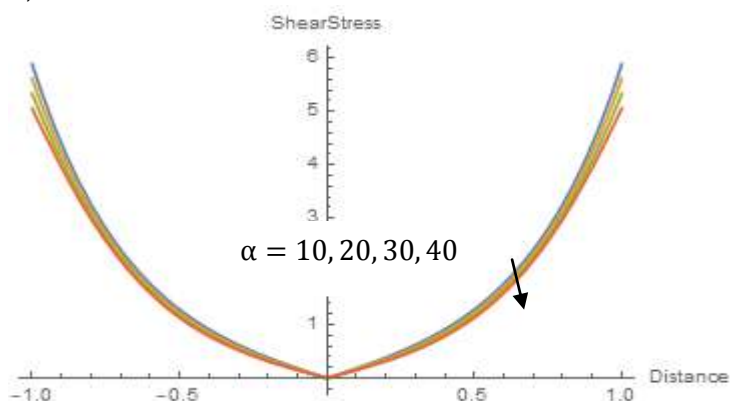


Figure 16.0 Wall shear stress profile for increase in the Womersley Number α when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

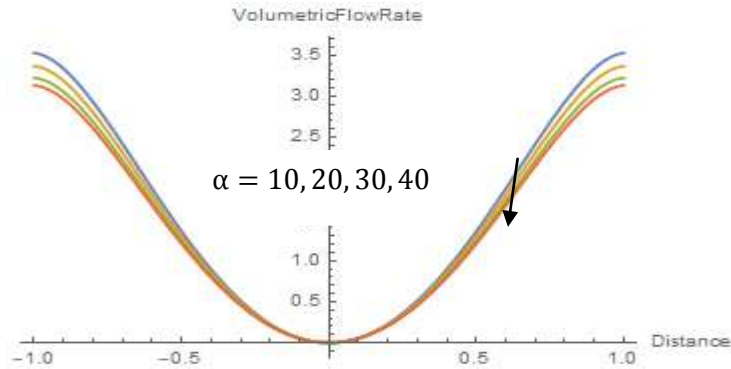


Figure 17.0 Volumetric flow rate profile for increase in the Womersley Number α when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

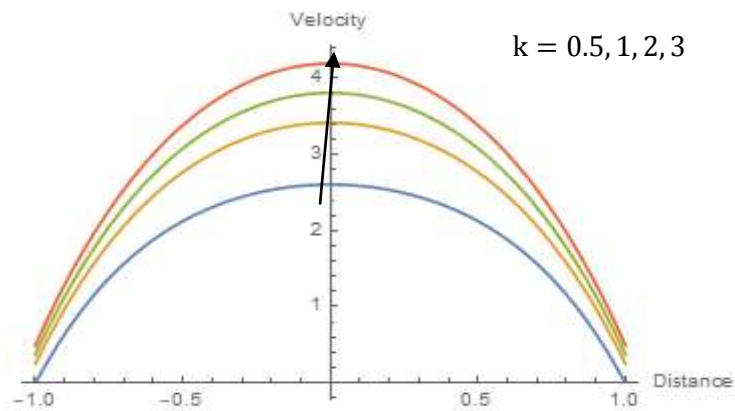


Figure 18.0 Blood flow velocity distribution for increase in the Permeability of the porous wall k when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

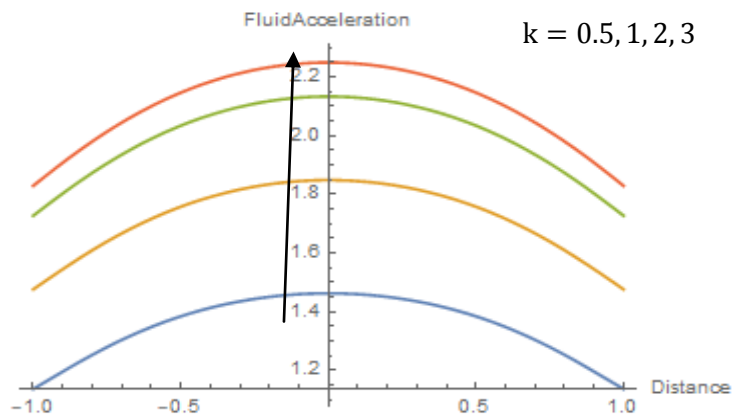


Figure 19.0 Blood acceleration profile for increase in the Permeability of the porous wall k when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \beta = 30^\circ, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

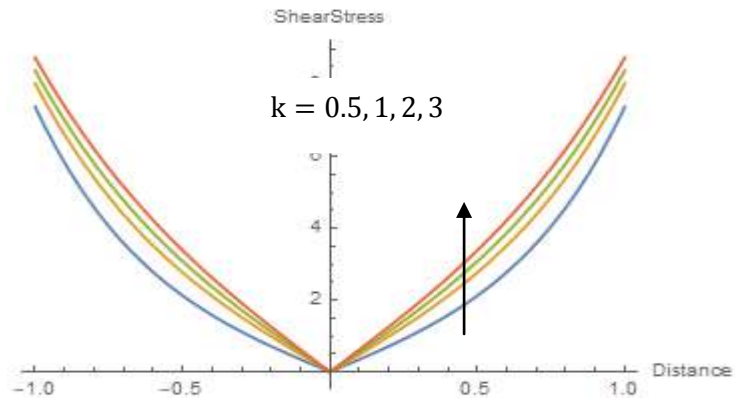


Figure 20.0 Wall shear stress profile for increase in the Permeability of the porous wall k when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

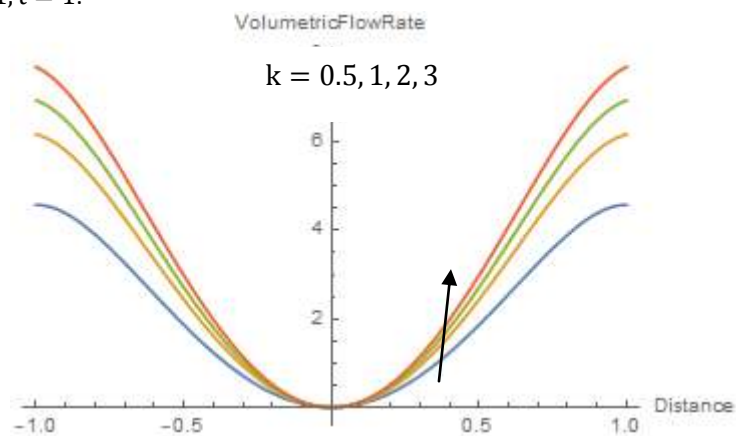


Figure 21.0 Volumetric flow rate profile for increase in the Permeability of the porous wall k when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

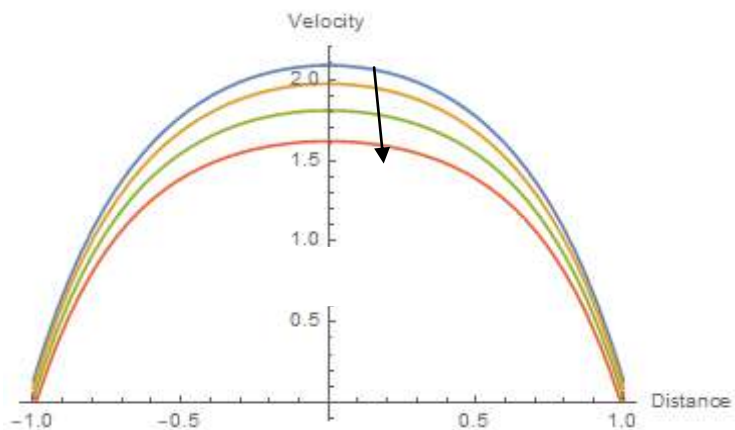


Figure 22.0 Blood flow velocity distribution for increase in the Magnetic field M when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, \xi = 0.1, \omega = 1, t = 1$.

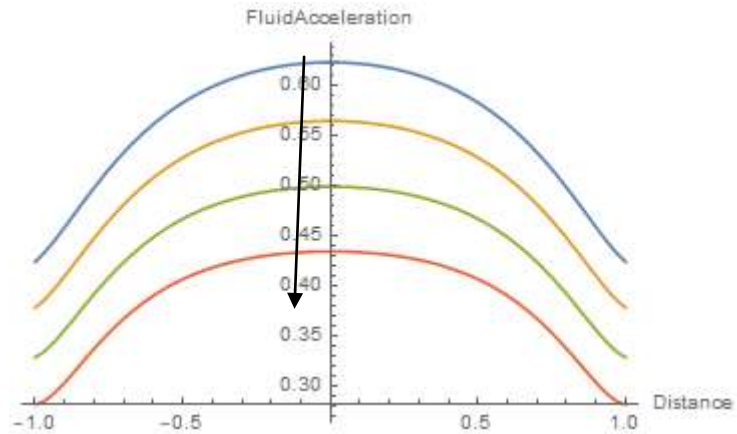


Figure 23.0 Blood acceleration profile for increase in the Magnetic field M when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, \xi = 0.1, \omega = 1, t = 1$.

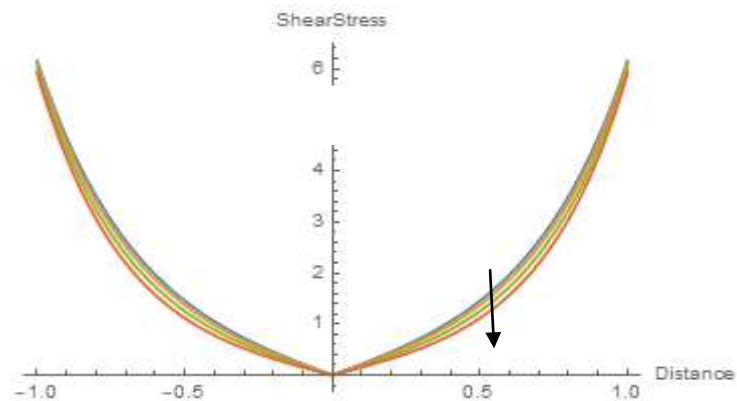


Figure 24.0 Wall shear stress profile for increase in the Magnetic field M when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, a = 1, \xi = 0.1, \omega = 1, t = 1$.

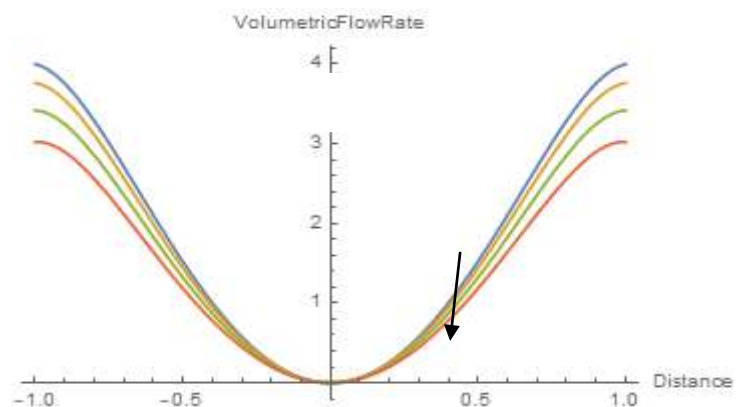


Figure 25.0 Volumetric flow rate profile for increase in the Magnetic field M when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, a = 1, \xi = 0.1, \omega = 1, t = 1$.

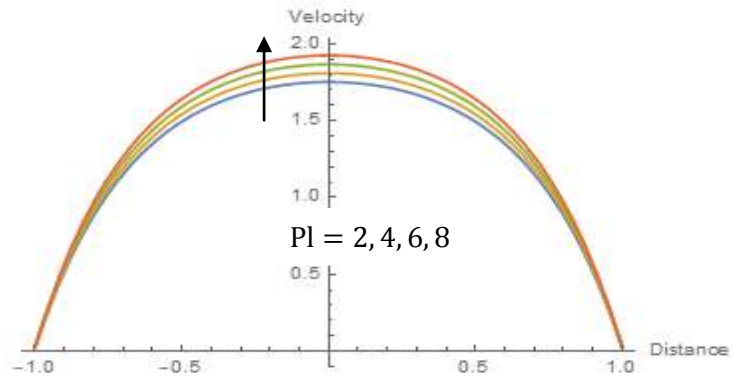


Figure 26.0 Blood flow velocity distribution for increase in the Pulsatile pressure PI when $Po = 2, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

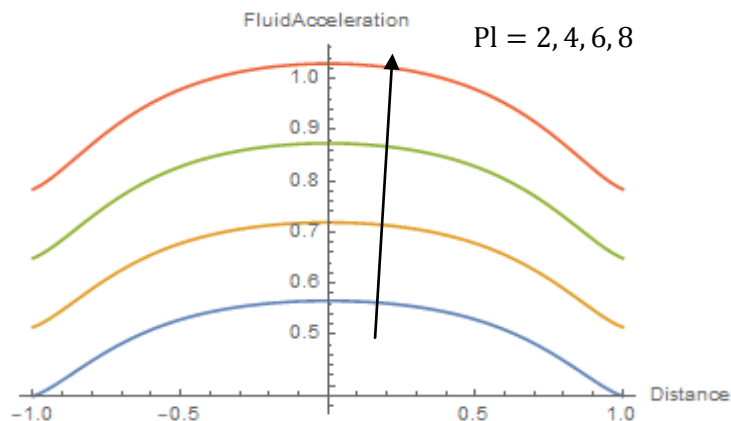


Figure 27.0 Blood acceleration profile for increase in the Pulsatile pressure PI when $Po = 2, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

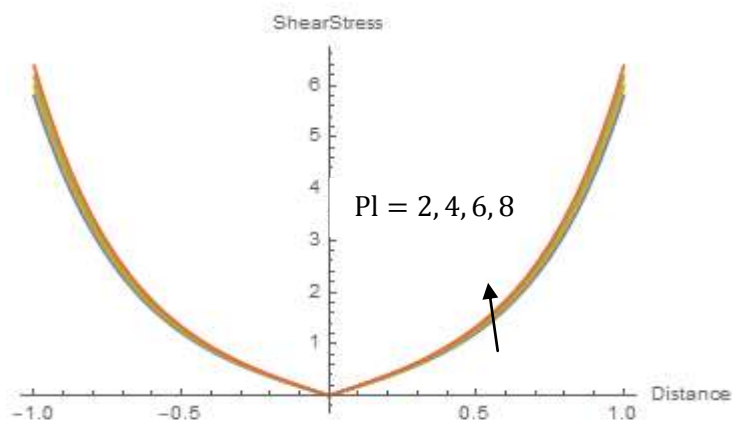


Figure 28.0 Wall shear stress profile for increase in the Pulsatile pressure PI when $Po = 2, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

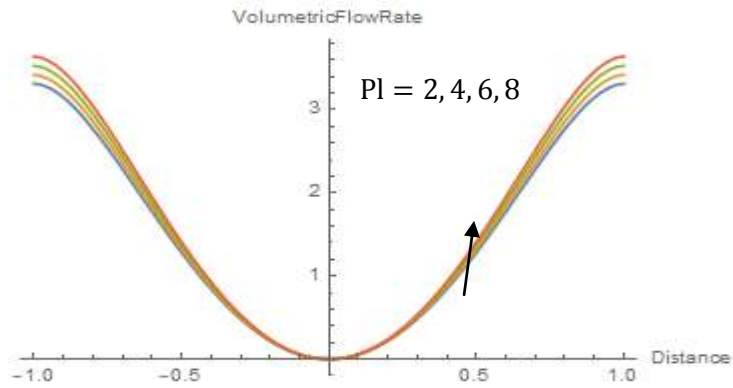


Figure 29.0 Volumetric flow rate profile for increase in the Pulsatile pressure Pl when $Po = 2, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

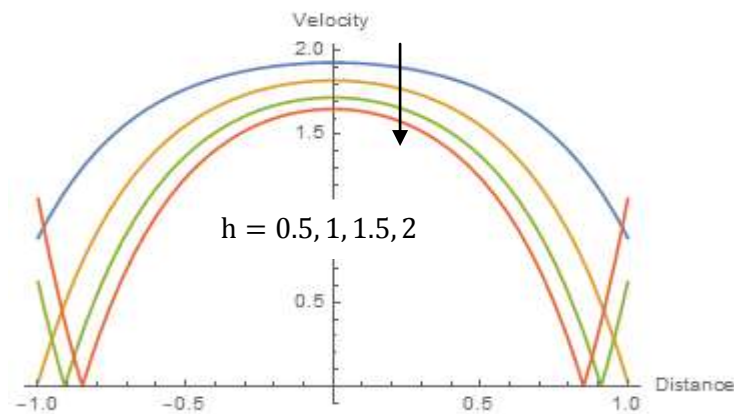


Figure 30.0 Blood flow velocity distribution for increase in the Slip Parameter h when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

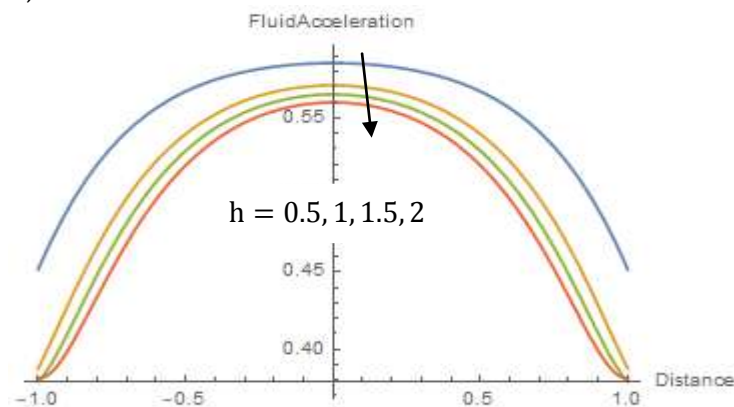


Figure 31.0 Blood acceleration profile for increase in the Slip Parameter h when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \beta = 30^\circ, k = 0.1, \alpha = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

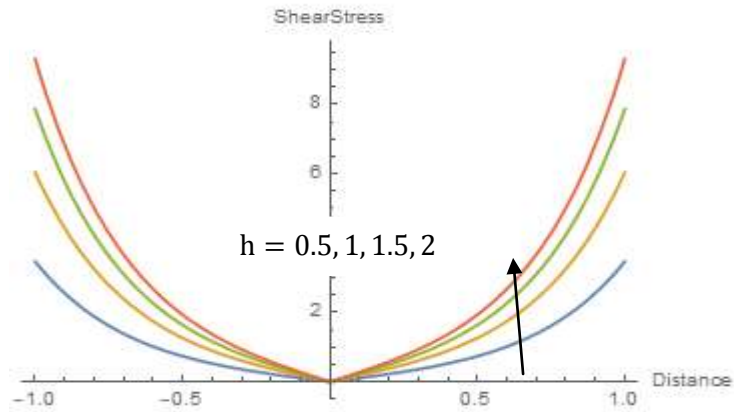


Figure 32.0 Wall shear stress profile for increase in the Slip Parameter h when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

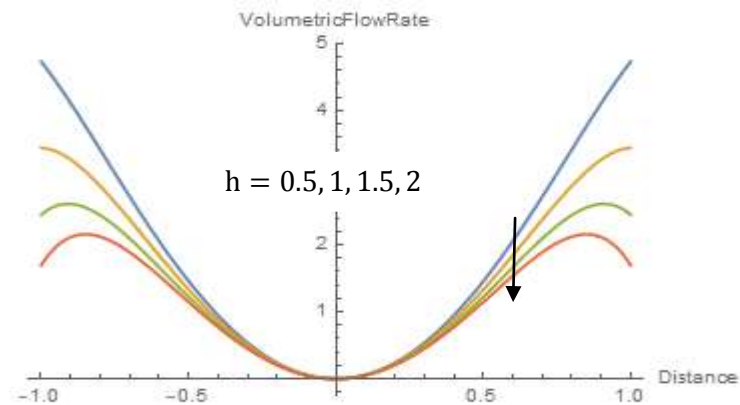


Figure 33.0 Volumetric flow rate profile for increase in the Slip Parameter h when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

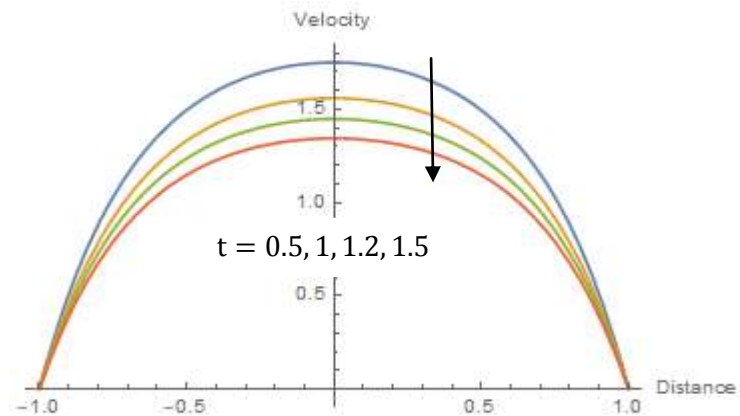


Figure 34.0 Blood flow velocity distribution for increase in the time t when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1$.

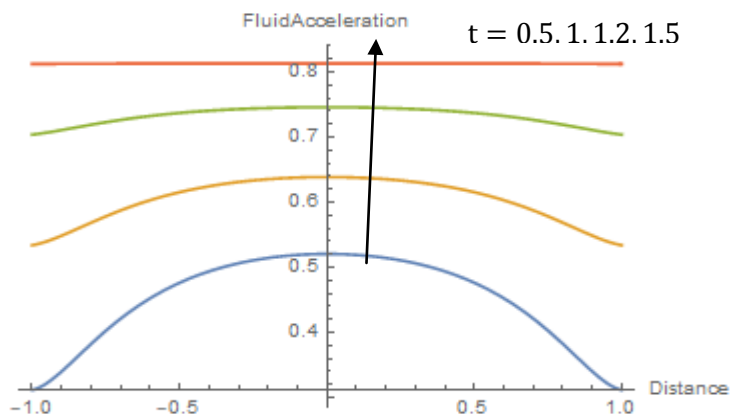


Figure 35.0 Blood acceleration profile for increase in the timet when $Po = 2, Pl = 4, Go = 3, b = 2, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1$.

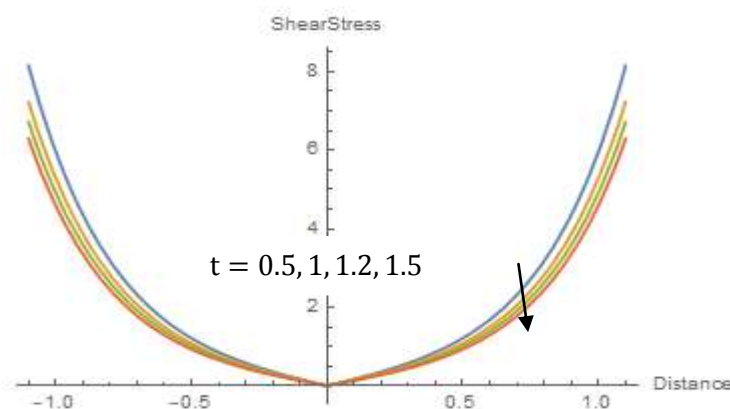


Figure 36.0 Wall shear stress profile for increase in the timet = when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1$.

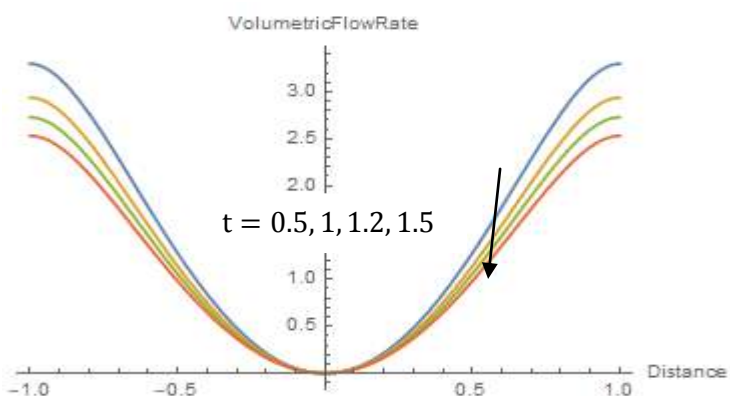


Figure 37.0 Volumetric flow rate profile for increase in the timet when $Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 45^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1$.

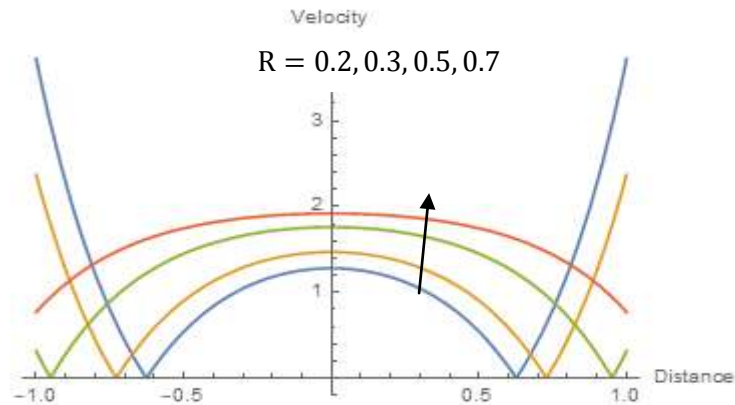


Figure 38.0 Graph for the velocity of Blood flow with increasing values of Radius of stenosis $R = 0.2, 0.3, 0.5, 0.7$, when $Gr = 2, H = 0.5, Pe = 1, Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 30^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

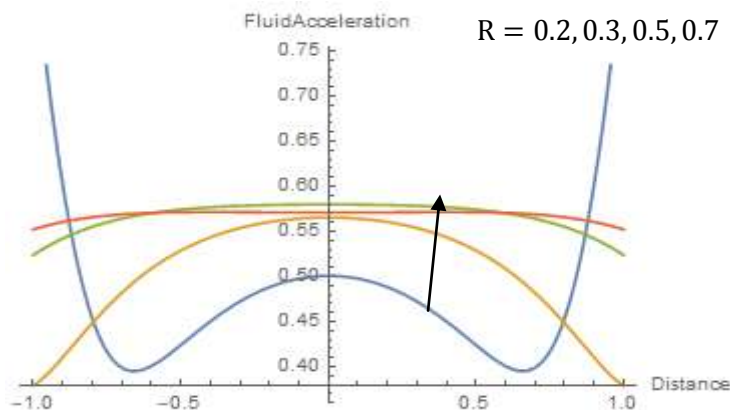


Figure 39.0 Graph for the Blood acceleration with increasing values of Radius of stenosis $R = 0.2, 0.3, 0.5, 0.7$, when $Gr = 2, H = 0.5, Pe = 1, Po = 2, Pl = 4, Go = 3, b = 2, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, M = 1.5, \xi = 0.1, \omega = 1, t = 1$.

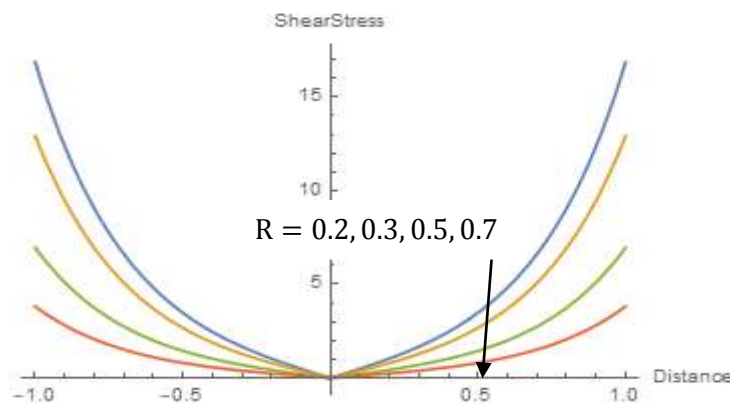


Figure 40.0 Graph for the shear stress at the wall with increasing values of Radius of stenosis $R = 0.2, 0.3, 0.5, 0.7$, when $Gr = 2, H = 0.5, Pe = 1, Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 30^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

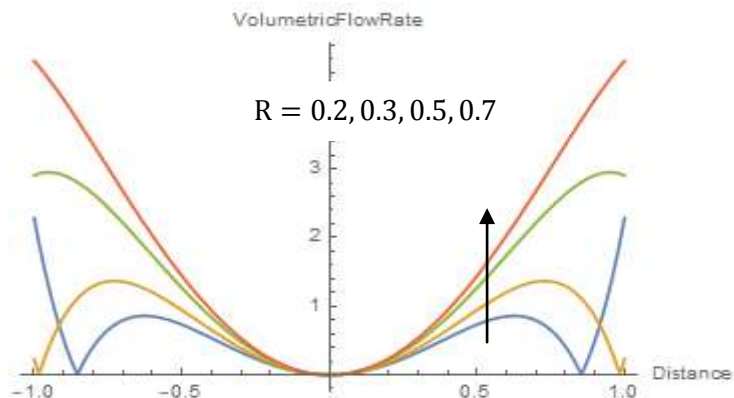


Figure 41.0 Graph for the Volumetric Flow rate with increasing values of Radius of stenosis when $Gr = 2, H = 0.5, Pe = 1, Po = 2, Pl = 4, Go = 3, Fr = 0.05, b = 2, \phi = 30^\circ, \beta = 30^\circ, k = 0.1, \alpha = 1, h = 1, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1$.

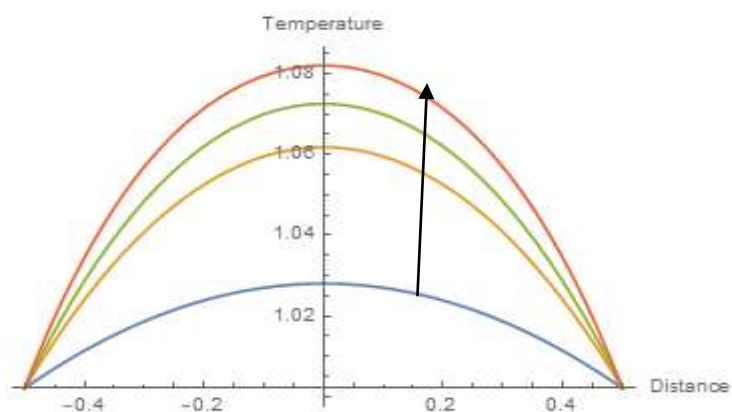


Figure 42.0 Graph for the Temperature with increasing values of Magnetic field when $\alpha = 1, R = 0.55, \xi = 0.1, \omega = 1, t = 1, \theta_a = 1$.

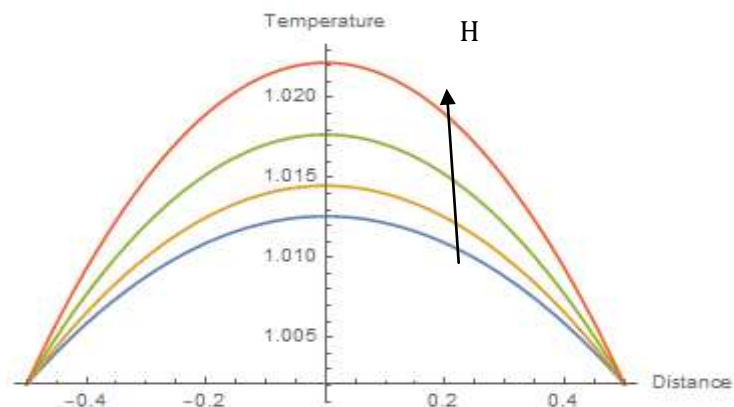


Figure 43.0 Graph for the Temperature with increasing values of Heat Source H when $\alpha = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1, \theta_a = 1$

VII. Conclusion

This study has analyzed theoretically the effect of heat source, slip, pulsatile pressure of the blood flow and body acceleration on non-Newtonian steady and unsteady blood flowing through a stenotic artery with stenosis and permeable walls. The summary of the results from the study showed that,

- (i) The body acceleration Go increase caused an increase in the velocity of the blood, flow blood acceleration, shear stress at the stenotic artery wall and the volumetric flow rate. Hypertensive patients are encouraged minimize their movement by land, air and water, minimize activities with stress which could increase the work rate of the heart leading to cardiac arrest. Increased body acceleration could improve patient's health with hypotension.

- (ii) The increase in the heat source caused the blood viscosity to reduce hence increasing the velocity, acceleration and volumetric flow rate of the blood while there was a reduced effect on the shear stress at the artery walls with stenosis.
- (iii) Increased inclination artery ϕ results to an increase in the velocity of the blood flow velocity, shear stress and the volumetric flow rate at the artery wall with stenosis but a fluctuating pattern in the acceleration of the blood.
- (iv) The permeability of the porous wall k increase, caused an increase in the velocity and acceleration of the blood flow, shear stress at the stenotic artery wall and volumetric flow rate.
- (v) Magnetic field M increase caused a decrease in the velocity and acceleration of the blood flow, shear stress at the stenotic artery wall and volumetric flow rate. Health practitioners could adopt this in treatment of patients with hypertension with the use of a magnetic jacket that will aid the decrease in the flow of blood to the body muscles from the heart.
- (vi) Pulsatile pressure PI increase causes an increase in the velocity and acceleration of the blood flow, shear stress at the stenotic artery wall and volumetric flow rate. The heart when overworked will pump more blood to muscles in the body pressure is increased. Hence a lot of rest could be recommended to patients with high blood pressure (hypertension).
- (vii) Increase in slip h induced at the artery wall with stenosis, decreases the velocity, acceleration and volumetric flow rate of the blood flow but decreases the shear stress at the stenotic artery wall. This could be adopted to treat hypertensive patients when injected drug induces a slip at the wall of the stenotic artery. This could help in treatment of heart disease by cleaning the heart valves and cavities such that the reduction in the radius stenosis will cause the increase of velocity, acceleration and volumetric flow rate of the blood and a reduced effect in the shear stress at the wall.

Conclusively, this study could be of immense benefit to the medical and health practitioners when administering treatment to patients because of the prediction of the outcome of the study shown in the results.

References

- [1]. Rabby, M. G., Razzak, A. and Molla, M. M. (2013). Pulsatile non-Newtonian blood flow through a model of arterial stenosis. *Procedia Engineering*, volume 56, number 5, page 225-231.
- [2]. Ellahi, R., Rahman, S. U. and Nadeen, S. (2014). Blood flow of Jeffery fluid in a catheterized tapered artery with the suspension of nanoparticles. *Physics Letters A*, volume 378, number 40, page 2973-2980.
- [3]. Pralhad, R. N. and Schultz, D. H. (2004). Modelling of arterial stenosis and its application to blood disease, *mathematical Biosciences*, volume 190, number 2, page 203 – 220.
- [4]. Ellahi, R., Rahman, S. U., Gulzar, M. M., Nadeem, S. and Vafai, K. (2014). A mathematical study of non-Newtonian micro-polar fluid in arterial blood flow through composite stenosis. *Applied Mathematics and Information Science*, volume 8, number 4, page 1567-1573.
- [5]. Haik, Y., Pai, V. and Chen, C. J. [1999]. Development of magnetic device for cell separation. *Physics of fluids [1994-present]*, vol.17, No.7, 077103-077118.
- [6]. Abdullah, I., Amin, N. & Hayat, T. (2010). Magneto-hydrodynamic effects on blood flow through an irregular stenosis. *International Journal for numerical method in fluids*, 67, 1624-1636, DOI:10.1002/fld.2436.
- [7]. Prakash, O., Singh, S. P., Devendara, K. & Dwivedi, Y. K. (2011). A Study of the Effects of Heat Source on MHD Blood Flow through Bifurcated Arteries. *AIP Advances I*, 042128.
- [8]. Srivastava, N. (2014). Analysis of Flow Characteristics of the Blood Flowing through an Inclined Tapered Porous Artery with Mild Stenosis under the Influence of an Inclined Magnetic Field. *Journal of Biophysics*, doi.org/10.1155/2014/797142.
- [9]. Eldesoky, M. I. (2012). Mathematical Analysis of Unsteady MHD Blood Flow through Parallel Plate Channel with Heat Source. *World Journal of Mechanics*, 2, 131, 131 – 137.
- [10]. Saddiqui, S. U., Shah, S. R. and Geeta (2014). Effect of body acceleration and slip velocity on the pulsatile flow through a cationic fluid through stenosed artery, *Adv. Appl. Sci. Res.* Vol. 5, No. 3, pp. 213-225.
- [11]. Sinha, A., Shit, G. C. & Kundu, P. K. (2013). Slip Effect on Pulsatile Flow of Blood through a Stenosed Arterial Segment under Periodic Body Acceleration. *ISRN Biomedical Engineering*, doi.org/10.1155/2013/925876.
- [12]. Sinha, A., Misra, J. C. & Shit, G. C. (2016). Effect of heat transfer on unsteady MHD flow of blood in a permeable vessel in the presence of non-uniform heat source. *Alexandria Engineering Journal* 55, 2023-2033, www.sciencedirect.com
- [13]. Tripathi, B. & Sharma, B. K. (2018). Effect of heat transfer on MHD blood flow through an inclined stenosed porous artery with variable viscosity and heat source. *Romanian Journal of Biophysics*, 28(3), 89 – 102.
- [14]. Karthikeyan, D. & Jeevitha, G. (2019). Heat and Mass Transfer on MHD Two Phase Blood Flow through a Stenosed Artery with Permeable Wall. *International Journal of Innovative Technology and Exploring Engineering (IJITEE)*, 8(7), ISSN: 2278-3075.
- [15]. Abubakar, J. U. & Adeoye, A. D. (2020). Effects of Radiative Heat and Magnetic Field on Blood Flow in an Inclined Tapered Stenosed Porous Artery. *Journal of Taibah University for Science*, 14:1, 77 – 86, DOI: 10.1080/16583655, 2019, 1701397.
- [16]. Amos, E. & Ogulu, A. (2003). Magnetic Effect on Pulsatile Flow in a Constricted Axis-symmetric Tube. *Indian Journal Pure and Applied Mathematics*, 34 (9): 1315-1326, September.
- [17]. Bunonyo, W. K. & Amos, E. (2020). Blood flow through an inclined arterial channel with magnetic field. *Mathematical Modelling and Application*, 5(3), 129 - 137
- [18]. Eldesoky, M. I. (2012). Slip Effects on Unsteady MHD Pulsatile Blood Flow through Porous Medium in an Artery under the Effect of Body Acceleration. *International Journal of Mathematics and Mathematical Sciences*, Volume 2012, ID 860239, doi:10.1155/2012/860239.
- [19]. Kumar, A., Chandel, R. S., Shrivastava, R., Shrivastava, K. & Kumar, S. (2016). Mathematical Modelling of blood flow in an inclined tapered artery under MHD effect through porous medium. *International Journal of Pure and Applied Mathematical Science*, 9(1), 75 – 88, ISSN 0972 – 9828, www.ripublication.com.

- [20]. Nadeem, S., Noreen, S. A., Hayat, T. &Awatif, A. H. (2012). Influence of Heat and Mass Transfer on Newtonian Bio magnetic Fluid of Blood Flow through a Tapered Porous Artery with Stenosis. *Transport in Porous Medium*, 91(1):81-100.
- [21]. Tripathi, B. & Sharma, B. K. (2018). Effect of Variable Viscosity on MHD Inclined Arterial Blood Flow with Chemical Reaction. *International Journal of Applied Mechanics and Engineering*, Volume 23, Number 3, pp. 767 – 785.
- [22]. Abel Wahab, M. and Salemi, S. I. (2012). Magneto-hydrodynamic Blood flow in a Narrow Tube. *World Research Journal of Biomaterials*, ISSN: 2278-7046, E-ISSN: 2278-7054, Volume 1, Issue 1, page 01-07.

APPENDIX

$$L_4 = \frac{G_r C_2 \alpha_1^4}{\beta_2 2^2 4^2 6^2 8^2}; L_3 = \frac{1}{\beta_2} \left(64L_4 - \frac{G_r C_2 \alpha_1^3}{2^2 4^2 6^2} \right); L_2 = \frac{1}{\beta_2} \left(36L_3 + \frac{G_r C_2 \alpha_1^2}{2^2 4^2} \right); L_1 = \frac{1}{\beta_2} \left(16L_2 - \frac{G_r C_2 \alpha_1}{2^2} \right);$$

$$L_0 = \frac{1}{\beta_2} (4L_1 + F + G_r C_2); M_4 = \frac{G_r C_1 N^8}{\beta_1 2^2 4^2 6^2 8^2}; M_3 = \frac{1}{\beta_1} \left(64M_4 - \frac{G_r C_1 N^6}{2^2 4^2 6^2} \right); M_2 = \frac{1}{\beta_1} \left(36M_3 + \frac{G_r C_1 N^4}{2^2 4^2} \right); M_1 = \frac{1}{\beta_1} \left(16M_2 - \frac{G_r C_1 N^2}{2^2} \right); M_0 = \frac{1}{\beta_1} (4M_1 + G + G_r C_1)$$

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