Some Algebraic Theoretic Properties on Gamma 1 Non Deranged Permutation

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Abstract
This paper investigated some algebraic theoretic properties of fuzzy set on $G_p$ using constructed membership function of fuzzy set on $G_p$ and established the result for algebraic operators of fuzzy sets on $G_p$, which are algebraic sum, algebraic product, bounded sum and bounded difference, then constructed a relationship between the operators of fuzzy sets on $G_p$, and came about some propositions.

Keywords
Fuzzy set, Algebraic Sum, Algebraic Product, Bounded Sum, Bounded Difference, Membership function

I. Introduction
Zadeh (1965) introduced the concept of Fuzzy sets by defining them in terms of mapping from a set into a unit interval. Permutation pattern have been used in the past decade to study mathematical structures. For instance Audu (1986), Ibrahim (2006) studied the concept of permutation pattern using some elaborate scheme to determine the order of precedence and the position of each of the elements in a finite set of prime size. Similarly an idea of an embedment as an algebraic structure has yielded some interesting results by Ibrahim (2005). Garba and Ibrahim (2010), studied the structure and developed a scheme for the range of such cycles and use it to investigate further number theoretic and algebraic properties of $G_p$.

Furthermore, a group theoretical properties of $G_p$ was also investigated by Garba and Abubakar (2015), the concept of fuzzy nature and of $G_p$ alpha-level cut has also been studied by Aremu, Ejima and Abdullahi (2017). Garba, Zakari and Hassan (2019) investigated the fuzzy nature and modified fuzzy membership function on $G_p$ and established that the $\alpha$-cut level of the $G_p$ is the domain $G_p[\alpha_{p-1}$ and the support (supp) of the $G_p$ is the entire structure $G_p$.

II. Preliminaries

2.1 FUZZY SET
If $X$ is a Collection of objects and $A \subseteq X$, then the fuzzy set $\tilde{A} \subseteq X$ is a set of ordered pairs
$$\tilde{A} = \{(x, \mu_A(x)) : x \in X\}$$

Where $\mu_A(x)$ is a measure taking values in the unit interval $[0,1]$ called the membership of $x$ in $A$.

2.2 ALGEBRAIC SUM
Let $A$ and $B$ be two fuzzy sets, then the Algebraic Sum of two fuzzy Sets is given by
$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x)$$

Where $\mu_{A+B}(x)$ is equal to the difference between the addition and product of measures of two fuzzy sets.

2.3 ALGEBRAIC PRODUCT
The algebraic product of two fuzzy Sets is given by
$$\mu_{A\cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$$

Where $\mu_{A\cdot B}(x)$ is equal to the product of measures of two fuzzy sets.

2.4 BOUNDED SUM
The bounded sum of two fuzzy sets is given by
$$\mu_{A\oplus B} = \min[1, (\mu_A + \mu_B)]$$

Where $\mu_{A\oplus B}$ is equal to the minimum value between 1 and the addition of measures of two fuzzy sets.

2.5 BOUNDED DIFFERENCE
The bounded difference of two fuzzy Sets is given by

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\[\mu_{A \Delta B} = \max [0, (\mu_A - \mu_B)]\]

Where \(\mu_{A \Delta B}\) is equal to the maximum value between 0 and the difference of measures of two fuzzy sets.

### 2.6 CYCLE AND SUCCESSOR

Let \(\Omega\) be a non-empty, totally ordered and finite subset of \(N\). Let \(G_p = \{\omega_1, \omega_2, \omega_3, \ldots, \omega_{p-1}\}\) be a structure such that each \(\omega_i\) is generated from the arbitrary set \(\omega\) for any prime \(p \geq 5\), using the scheme \(\omega_i = ((1 + \ell)_{mp}(1 + 2\ell)_{mp} \ldots (1 + (p - 1)\ell)_{mp})\) Then each \(\omega_i\) is called a cycle and the elements in each \(\omega_i\) are distinct and called successors.

#### 2.6.1 \(n^{th}\) SUCCESSOR OF \(\omega_i\)

\(n^{th}\) Successor of a cycle \(\omega_i\) is given by \(a_{n\ell} = (1 + (n - 1)\ell)_{mp}\) Where \(1 \leq n \leq p\), and \(1 \leq i \leq p-1\). The number of distinct successors in a cycle is called the length of the cycle.

#### 2.6.2 DEFINITION OF \(G_p\)

Let \(G_p = \{\omega_1, \omega_2, \omega_3, \ldots, \omega_{p-1}\}\) be as defined as above then \(G_p^0 = G_p \cup \omega_p\) where \(\omega_p = (p, p, \ldots, p)\). That is \(G_p = \{\omega_1, \omega_2, \omega_3, \ldots, \omega_{p-1}, \omega_p\}\).

#### 2.6.3 RANGE OF A CYCLE

The range of a cycle \(\omega_i \in G_p\) is defined as 
\[\pi(\omega) = |A_{\ell}^i(\omega)|\]

Where \(A_{\ell}^i(\omega)\) is the difference between the last successor and the first successor, where \(l\) is the last successor and \(f\) is the first successor.

### III. Result And Discussion

In this section, the discussion in been carried out by figures and proofs.

#### 3.1 FUZZY NATURE OF \(G_p\)

Let \(G_p\) be a set, and \(G_p \subseteq G_p\), then we can define fuzzy set on \(G_p\) as
\[\tilde{G}_p = \{\mu_{G_p}(\omega_i) : \omega_i \in G_p\}\]

Where
\[\mu_{G_p}(\omega_i) = \left(\frac{i \pi(\omega_i)}{p + 2}\right) : i \leq p - 1\]  

and
\[\pi(\omega) = |A_{\ell}^i(\omega)|\]

Where \(l\) is the last successor and \(f\) is the first successor of a cycle.

Consider \(G_p\) and let \(G_p \subseteq G_p\) where \(p = 5\)

\[G_5 = \{\omega_1, \omega_2, \ldots, \omega_5\}\]

and

\[G_5 = \{\omega_1, \omega_2, \ldots, \omega_4\}\]

Where
\[\mu_{G_5}(\omega_1) = (1, 0.6)\]
\[\mu_{G_5}(\omega_2) = (2, 0.4)\]
\[\mu_{G_5}(\omega_3) = (3, 0.3)\]
\[\mu_{G_5}(\omega_4) = (4, 0.1)\]

Then the result follows.

#### 3.2 Proposition

Given \(G_p\) as an extended \(G_p\) where \(G_p \leq G_p\) and \(G_p\) is a fuzzy subset \(G_p\) and \(p + n\) is an immediate prime after \(p\). (\(\forall p \geq 5\)) where \(p\) is prime. Then bounded sum of \(\tilde{G}_p\) is greater than or equal to algebraic sum of \(\tilde{G}_p\). Which is

\[\mu_{\tilde{G}_p} \geq \mu_{\tilde{G}_p}^+\]

**Proof**

Since the \(\mu(\omega_i) \forall i \in \mathbb{N}, \mu(\omega_i) \in [0,1]\) then for any \(\mu_{\tilde{G}_p}\).

Let say \(\omega_x, \omega_y\) where \(\omega_x = \mu_x\) and \(\omega_y = \mu_y\)

\[\omega_x + \omega_y \geq \omega_x \omega_y\]

This clearly shows that

\[\omega_x + \omega_y - \omega_x \omega_y \leq \omega_x + \omega_y \forall \omega_x, \omega_y \in [0,1]\]

Then form the description below:

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(a) Algebraic sum of \( \omega_x, \omega_y = \omega_x + \omega_y - \omega_x \omega_y \)

(b) Bounded sum of \( \omega_x, \omega_y = \min[1, (\omega_x + \omega_y)] \)

Without loss of generality, we can see that (b) \( \geq \) (a) which follows from the claim that (b) \( \geq \) (a) and hence the proof.

3.3 Proposition

Given \( G_p \) as an extended \( G_p \) where \( G_p \subseteq G_p' \) and \( G_p \) is a fuzzy set and \( p + n \) is an immediate prime after \( p \) (\( \forall p \geq 5 \)) where \( p \) is prime. Then

(a) \( G_p \cap \hat{G}_{p+n} = \{0\} \)

(b) \( \mu(\hat{G}_{p+n} \cap \hat{G}_p) \leq \mu(\hat{G}_p) \)

**Proof**

(a) Let \( \omega_x \in \hat{G}_p \) and \( \omega_y \in \hat{G}_{p+n} \) where \( \omega_x = \mu_{\omega_x} \) and \( \omega_y = \mu_{\omega_y} \)

\[ \Rightarrow \omega_x, \omega_y \in [0, 1], \min = 0 \text{ and max is } 1. \]

For any \( \hat{G}_p \) and \( \hat{G}_{p+n} \) where \( \omega_x \in \hat{G}_p \) and \( \omega_y \in \hat{G}_{p+n} \)

Then \( \omega_x \leq \omega_y \) when \( x = y \). therefore since \( \omega_x \leq \omega_y \) this follows:

\[ \omega_x \cap \omega_y = \max[0, (\omega_x - \omega_y)] \]

Then the proof follows.

(b) The proof follows from (a) above, since \( \mu_{\hat{G}_p} \leq \mu_{\hat{G}_{p+n}} \) and also \( \mu_{\hat{G}_{p+n} - \hat{G}_p} \leq \mu_{\hat{G}_{p+n}} \)

\[ \Rightarrow \max[0, (\mu_{\hat{G}_{p+n} \cap \hat{G}_p})] \leq \mu(\hat{G}_{p+n}) \]

And hence the proof.

3.4 Proposition

Given \( G_p \) as a set and \( \hat{G}_p \) as a fuzzy set, where \( G_p \subseteq G_p' \), the algebraic product of \( \hat{G}_p \) and \( \hat{G}_{p+n} \), for any ” \( p + n \) ” where ” \( p + n \) ” is a prime, the membership function of \( \hat{G}_{p+n} \) is less than the membership function of \( \hat{G}_p \).

\[ \Rightarrow \mu(\hat{G}_{p+n} \cdot \hat{G}_p) \leq \mu(\hat{G}_p) \]

**Proof**

Since \( \forall x, y \in [0, 1] \)

(a) \( x \cdot y \leq x \)

(b) \( x \cdot y \leq y \)

Therefore \( x \cdot y \leq x + y \) and also \( x \cdot y \leq x + y \) if and only if \( x \neq 0 \) and \( y \neq 0 \).

\[ \Rightarrow \forall \mu_{\omega_x} \in [0, 1] \]

And also

\[ \mu(\omega_x \cdot \omega_y) \leq \mu_{\omega_y} \]

Let \( \mu_{\hat{G}_{p+n}} = a \) and \( a \) is a vector

And \( \mu_{\hat{G}_p} = b \) and \( b \) is a vector

then \( a, b \in [0, 1] \)

\[ \Rightarrow a \cdot b \in [0, 1] \text{ since } a, b \in [0, 1] \]

(c) this clearly shows that \( \forall a, b \in [0, 1], a \cdot b \leq a \) or \( a \cdot b < b \)

And hence the proof.

3.5 CONCLUSION

There are a lot of applications in different field of mathematics of constructed algebraic structures and investigating their algebraic properties which cannot be exhausted, in this paper we investigated some fuzzy nature of an algebraic structure \( G_p \) that was constructed earlier we discovered that \( \hat{G}_p \) is a fuzzy set, then we constructed algebraic operators of fuzzy sets on \( G_p' \) and a relationship between the operators of fuzzy sets on \( G_p' \), and came about some propositions.

**References**


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