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Centralizers on Semiprime Semiring

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Abstract: *Let S be a 2-torsion free semiprime semiring and $T: S \to S$ be an additive mapping. Then we prove that every Jordan left centralizer on S is a left centralizer on S. We also prove that every Jordan centralizer of a 2-torsion free Semiprime Semiring is a centralizer.*
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I. Introduction

Semiring was introduced by H.S. Vandiver in 1934. Herstein and Neumann investigated Centralizers in rings for a situation which arises in the study of rings with involutions. The study of centralizing mapping on Semiprime ring was established by H.E.Bell and W.S.Martindale. This research has been motivated by the work of Borut Zalar [4] who worked on centralizers of Semiprime rings and proved that Jordan centralizers and centralizers of this rings coincide. Joso Vukman[5],[6], developed some remarkable results using centralizers on prime and Semiprime rings. In [7] Md. Fazlul Hoque and A.C.Paul discussed the notion of Centralizers of Semiprime Gamma Rings. In this paper we introduce the notion of Centralizers of Semiprime Semirings. Throughout, S will represent Semiprime Semiring with center $Z(S)$.

II. Preliminaries

In this section, we recall some basic definitions and results that are needed for our work.

Definition 2.1

A Semiring is a nonempty set S followed with two binary operation ' $+$ ' and '$.$ 'such that
(1) $(S, +)$ is a commutative monoid with identity element '0'.
(2) $(S, .)$ is a monoid with identity element 1
(3) Multiplication distributes over addition from either side
That is $\forall\ a, b, c \in S$, $a.(b + c) = a.b + a.c$
$(b + c).a = b.a + c.a$

Definition 2.2

A Semiring S is said to be prime if $xSy = 0 \Longrightarrow x = 0$ or $y = 0\ \forall\ x, y \in S$

Definition 2.3

A Semiring S is said to be semiprime if $xSx = 0 \Longrightarrow x = 0\ \forall x \in S$

Definition 2.4

A Semiring S is said to be 2-torsion free if $2x = 0 \Longrightarrow x = 0\ \forall x \in S$

Definition 2.5

A Semiring S is said to be commutative Semiring, if $xy = yx\ \forall x, y \in S$, then the set $Z(S) = \{\ x \in S,\ xy = yx\ \ \forall y \in S\}$ is called the center of the Semiring S.

Definition 2.6

For any fixed $a \in S$, the mapping $T(x) = ax$ is a left centralizer and $T(x) = xa$ is a right centralizer.

Definition 2.7

An additive mapping $T: S \to S$ *is a left (right) centralizer if* $T(xy) = T(x)y, \big(T(xy) = xT(y)\big)\ \forall x, y \in S$
A centralizer is an additive mapping which is both left and right centralizer.

Definition 2.8

An additive mapping $T: S \to S$is Jordan left (right) Centralizer if
$T(xx) = T(x)x,\ (T(xx) = xT(x))\ \ \forall x \in S$
Every left centralizer is a Jordan left centralizer, but the converse is not in general true.

Definition 2.9

An additive mapping $T: S \to S$is a Jordan centralizer if $T(xy + yx) = T(x)y + yT(x)$. $\forall\ x, y \in S$
Every centralizer is a Jordan centralizer, but Jordan centralizer is not in general a centralizer.

Definition 2.10

An additive mapping $D: S \to S$ is called a derivation if $D(xy) = D(x)y + xD(y)\ \forall x, y \in S$ and is called a Jordan derivation if $D(xx) = D(x)x + xD(x)\ \forall x \in S$.

Definition 2.11

If S is a semiring then $[x, y] = xy + y'x$ is known as the commutator of x and y

The following are the basic commutator identities:
$[xy, z] = [x, z]y + x[y, z]$ *and* $[x, yz] = [x, y]z + y[x, z]$. $\forall x, y, z \in S$

According to [11] $for\ all\ a, b \in S$ we have
$(a + b)' = a' + b'$
$(ab)' = a'b = ab'$
$a'' = a$
$a'b = (a'b)' = (ab)'' = ab$
Also the following implication is valid.
$a + b = o\ implies\ a = b'$ and $a + a' = 0$

III. Left Centralizers Of Semiprime Semirings

Lemma 3.1[2]
Let S be a 2-torsion free Semiprime Semirings , a and b the elements of S, then the following are equivalent.
(i) $aSb = 0$
(ii) $bSa = 0$
(iii) $aSb + bSa = 0$
If one of these conditions are fulfilled, then $ab = ba = 0$

Lemma 3.2
Let S be a semiprime semiring and $A: S\ x\ S \to S$ biadditive mapping. If $A(x, y)wB(x, y) = 0\ \forall x, y \in S$. then $A(x, y)wB(s, t) = 0\ \forall x, y, s, t, w \in S$

Proof
By hypothesis $A(x, y)wB(x, y) = 0$, (1)
Replace $x\ by\ x + s$ in the above, we get
$A(x, y)wB(s, y) = A(s, y)w'B(x, y)$
This implies
$\big(A(x, y)wB(s, y)\big)z\big(A(x, y)wB(s, y)\big) = \big(A(s, y)w'B(x, y)\big)z\ (A(x, y)wB(s, y))$
$= A(s, y)w'\ (B(x, y)z\ A(x, y))\ \ w\ B(s, y)$
Using lemms 3.1 and by the semiprimeness of S implies,
$A(x, y)wB(s, y) = 0$
Now replacing y by $y + t$ in the last equation and using similar approach we get the required result.

Lemma 3.3
Let S be a Semiprime Semiring and $a \in S$ be some fixed element. If $a[x, y] = 0$ for $x, y \in S$, then there exists an ideal U of S such that $a \in U \subset Z(S)$ holds.

Proof Let $x, y, z \in S$
Consider $[z, a]x[z, a]$ $= (za + a'z)x[z, a]$
$= zax[z, a] + a'zx[z, a]$
$= za\{[z, xa] + [z, x]a'\} + a'\{[z, zxa] + [z, zx]a'\}$
$= za\{[z, xa] + [z, x]a'\} + a'[z, zxa] + a'\ [z, zx]a'\}$
$= za\{x[z, a] + [z, x]a + [z, x]a'\} + a'\{[z, z]xa + z[z, x]a + zx[z, a]\}$
$+a'(z[z, x] + [z, z]x)a'$
$=0$
Since S is semiprime, $[z, a] = 0$
$za + a'z = 0$.This implies $a \in Z(S)$
Also, $zaw[x, y] = zwa[x, y]\ \forall\ x, y, z, w \in S$
$=0$ for all $x, y, z, w \in S$
By similar arguments we can show that $zaw \in Z(S)$
Thus we obtain $SaS \subset Z(S)$, and it is easy to see that ideal generated by a is central.

Lemma 3.4

Let S be a semiprime semiring and let $T:S \to S$ be a Jordan left centralizer. Then

(a) $T(xy+yx) = T(x)y + T(y)x \ \forall x,y \in S$

(b) If S is a 2- torsion free Semiring, then

(i) $T(xyx) = T(x)yx$

(ii) $T(xyz+zyx) = T(x)yz + T(z)yx \ \forall \ x,y,z \in S$

Proof T is a Jordon left centralizer then $T(xx) = T(x)x$ (2)

Replace x by $x+y$

$$T[(x+y)(x+y)] = [T(x+y)](x+y)$$

$$T(xx+xy+yx+yy] = [T(x)+T(y)]\ (x+y)$$

$$T(xx) + T(xy+yx) + T(yy) = T(x)x + T(x)y + T(y)x + T(y)y$$

$$T(x)x + T(xy+yx) + T(y)y = T(x)x + T(x)y + T(y)x + T(y)y$$

$$T(xy+yx) = T(x)y + T(y)x \quad (3)$$

Hence proved (a)

Now replacing y by $xy+yx$ in (3) we get

$$T[x(xy+yx)+(xy+yx)x] = T(x)[xy+yx] + T(xy+yx)x$$

$$T(x^2y+xyx+xyx+yx^2) = [T(x)xy + T(x)yx + T(xy+yx)x$$

$$T(x^2)y + T(y)x^2 + 2T(xyx) = T(x)\,xy + T(x)yx + [T(x)y+T(y)x]x$$

$$T(x^2)y + T(y)x^2 + 2T(xyx) = T(x^2)y + T(x)yx + T(x)yx + T(y)x^2$$

$$2T(xyx) = 2T(x)yx$$

Adding $2T(x)yx'$ on bothsides, we get $2T(xyx) + 2T(x)yx' = 0$

Since S is 2-torsion free, $T(xyx) + T(x)yx' = 0$

This implies, $T(xyx) = T(x)yx$ (4)

Hence proved b(i)

If we linearize (4), we get

$$T(xyz+zyx) = T(x)yz + T(z)yx \ \forall \ x,y,z \in S$$

Hence b(ii) is proved.

Theorem 3.1

Let S be a 2- torsion free Semiprime Semiring and $T:S \to S$ be a Jordan Left centralizer on S. Then T is a left centralizer.

Proof

T is a Jordan left centralizer then

$$T(xy+yx) = T(x)y + T(y)x \quad (5)$$

First we shall compute

$J = T(xyzyx + yxzxy)$in two different ways

Using Lemma 3.4. b(i), we have

$$J = T(x)yzyx + T(y)xzxy \quad (6)$$

Using Lemma 3.4. b(ii), we have

$$J = T(xy)zyx + T(yx)zxy \quad (7)$$

Comparing (1) and (2)

$$T(x)yzyx + T(y)xzxy = T(xy)zyx + T(yx)zxy$$

$$T(y)xzxy = T(xy)zyx + T'(x)yzyx + T(yx)zxy$$

$$T(y)xzxy = (T(xy) + T(x)y')zyx + T(yx)zxy$$

Adding $T(y)xzxy'$ on both sides, we get

$$(T(xy) + T(x)y')zyx + T(yx)zxy + T(y)xzxy' = 0$$

$$(T(xy) + T(x)y')zyx + (T(yx) + T(y)x')zxy = 0$$

Introducing a biadditive mapping $B: SXS \to S$ by $B(x,y) = T(xy) + T(x)y'$

From the above relation we arrive at $B(x,y)zyx + B(y,x)zxy = 0$ (8)

Using $B(y,x) = B^{'}(x,y)$ in the above, we get

$B(x,y)z[x,y] = 0$

Using Lemma 3.2 we get, $B(x,y)z[s,t] = 0$

Using Lemma 3.1 we obtain $B(x,y)[s,t] = 0$

Now fix some $x, y \in S$. Write $b = B(x, y),\ b \in S\ \ then\ \ b[s, t] = 0,\ for\ \ s, t \in S$. $By\ lemma$ 3.3 there exist an ideal U such that $b \in U \subset Z(S)$ holds. In particular $by, yb \in Z(S),\ y \in S$, This gives us

$x(b^2y) = (b^2y)x$
$= (yb^2)x$
$xb^2y = yb^2x$
$T(xb^2y) = T(yb^2x)$

Linearizing the above

$T(xb^2y + yb^2x) = T(yb^2x + xb^2y)$
$T(xb^2y + xb^2y) = T(yb^2x + yb^2x)$
$2T(xb^2y) = 2T(yb^2x)$
$This\ gives\ us\ 4T(xb^2y) = 4T(yb^2x)$

Now we will compute each side of this equality by using (5) and the above notation

$4T(xb^2y) = 2T[xb^2y + xb^2y]$
$= 2T[xb^2y + yb^2x]$
$= 2T[xb^2y + b^2yx]$

Similarly $4T(yb^2x) = 2T(yb^2x + yb^2x)$
$= 2T(yb^2x + xb^2y)$
$= 2T(yb^2x + b^2xy)$

Therefore we get $2T(xb^2y + b^2yx) = 2T(yb^2x + b^2xy)$
$2[T(x)b^2y + T(b^2y)x] = 2[T(y)b^2x + T(b^2x)y]$
$2T(x)b^2y + 2T(b^2y)x = 2T(y)b^2x + 2T(b^2x)y$
$2T(x)b^2y + T[b^2y + b^2y]x = 2T(y)b^2x + T[b^2x + b^2x]y$
$2T(x)b^2y + T[b^2y + yb^2]x = 2T(y)b^2x + T[b^2x + xb^2]y$
$2T(x)b^2y + [T(b^2)y + T(y)b^2]x = 2T(y)b^2x + [T(b^2)x + T(x)b^2]y$
$2T(x)b^2y + T(b^2)yx + T(y)b^2x = 2T(y)b^2x + T(b^2)xy + T(x)b^2y$
$T(x)b^2y + T(b)byx = T(y)b^2x + T(b)bxy$
$T(x)b^2y + T(b)bxy = T(y)b^2x + T(b)bxy$
$T(x)b^2y = T(y)b^2x$ (9)

On the other hand we also have

$x(b^2y) = xbby$
$xb^2y = xbyb$
$xyb^2 = xbyb$
$T(xyb^2) = T(xbyb)$

Linearizing the above and using (5) and (9), we get

$T(xyb^2 + yxb^2) = T(xbyb + ybxb)$
$T(xyb^2 + xyb^2) = T(xbyb + xbyb)$
$2T(xyb^2) = 2T(xbyb)$

This gives us

$4T(xyb^2) = 4T(xbyb)$
$2T(xyb^2 + xyb^2) = 2T(xbyb + xbyb)$
$2T(xyb^2 + b^2xy) = 2T(xbyb + ybxb)$
$2T(xy)b^2 + 2T(b^2)xy = 2T(xb)yb + 2T(yb)xb$
$2T(xy)b^2 + 2T(b)bxy = T(xb + xb)yb + T(yb + yb)xb$
$= T(xb + bx)by + T(yb + by)bx$
$= [T(x)b + T(b)x]by + [T(y)b + T(b)y]bx$
$= T(x)b^2y + T(b)bxy + T(y)b^2x + T(b)bxy$
$2T(xy)b^2 = T(x)b^2y + T(y)b^2x$
$2T(xy)b^2 = T(x)b^2y + T(x)b^2y$

Adding $2T(x)b^2y'$ on both sides

$2T(xy)b^2 + 2T(x)b^2y' = 0$

Since S is 2 – torsion, $[T(xy) + T(x)y']b^2] = 0$

$bb^2 = 0,$
$b^3 = 0,$

So that, $b^2Sb^2 = Sb^2b^2$

$$= Sbb^3$$
$$= 0$$

Since S is Semiprime $b^2 = 0$

Also $bSb = Sbb$
$$= Sb^2$$
$$= 0$$

Since S is Semiprime, $b = 0$

$$\mathrm{T}(xy) + T(x)y' = 0$$
$$T(xy) = T(x)y$$

Hence T is a left centralizer.

IV. The Centralizers Of Semiprime Semirings

Lemma 4.1

Let S be a semiprime semiring. D is a deveriation of S and $a \in S$ be some fixed element. Then

(i) $D(x)D(y) = 0 \ \forall\ x, y \in S \implies D = 0$

(ii) $ax + x'a \in Z(S) \ \forall\ x \in S \implies a \in Z(S)$

Proof

i) Consider $D(x)yD(x) = D(x)[D(yx) + D(y)x']$
$$= D(x)D(yx) + D(x)D(y)x'$$

Replace yx by $z \ \forall\ x, y, z \in S$.

$$D(x)yD(x) = D(x)D(z) + D(x)D(y)x'$$
$$= 0$$

Since S is Semiprime, $D = 0$, hence Proved (i)

ii) Define $D(x) = ax + x'a \in Z(S)$

$D(xy) = axy + x'ya$, since D is a derivation, we get

$D(x)y + xD(y) = axy + x'ya$

Since $D(x) \in Z(S)$, $\forall\ x \in S$ we have $D(y)x = xD(y)$

and also $D(yz)x = xD(yz)$

$$[D(y)z + yD(z)]x = x[D(y)z + yD(z)]$$
$$D(y)zx + yD(z)x = xD(y)z + xyD(z)$$
$$D(y)zx = xyD(z) + y'D(z)x + xD(y)z$$
$$= xD(y)z + xD(z)y + D(z)y'x$$
$$= xD(y)z + D(z)xy + D(z)y'x$$
$$= xD(y)z + D(z)[x, y]$$

Obviously $D(a) = 0$, now take $z = a$, we get $D(y)ax = xD(y)a + D(a)[x, y]$
$$D(y)ax = xD(y)a$$

Adding $x'D(y)a$ on both sides, We get $D(y)ax + x'D(y)a = 0$

$D(y)ax + D(y)x'a = 0$
$$D(y)D(x) = 0$$

By (i) $D = 0$

$$D(x) = 0$$
$$ax + x'a = 0$$
$$a \in Z(S)$$

Lemma 4.2

Let S be a semiprime semiring and $a \in S$ be some fixed element. If $T(x) = ax + xa$ is a Jordan centralizer, then $a \in Z(S)$.

Proof

By hypothesis, T is a Jordan centralizer we have $T(xy + yx) = T(x)y + yT(x)$ (10)

Given $T(x) = ax + xa$ then replace $x = xy + yx$

$$T(xy + yx) = a(xy + yx) + (xy + yx)a \quad (11)$$

Comparing (10) and (11)

$$a(xy + yx) + (xy + yx)a = T(x)y + yT(x)$$
$$a(xy + yx) + (xy + yx)a = [ax + xa]y + y[ax + xa]$$

$ayx + xya = xay + yax$ adding $x'ay + yax'$ on both sides
$ayx + xya + x'ay + yax' = 0$
$(ay + y'a)x + x'(ay + y'a) = 0$
$ay + y'a \in Z(S)$
by Lemma 4.1, (ii) gives us $a \in Z(S)$. Hence the proof is complete.

Lemma 4.3

Let S be a semiprime semiring. Then every Jordon centralizer of S maps $Z(S)$ into $Z(S)$

Proof Take any $c \in Z(S)$and $a = T(c)$, then we have

$$
\begin{aligned}
2T(cx) &= T(cx + cx) \\
&= T(cx + xc) \\
&= T(c)x + x\,T(c) \\
&= ax + xa
\end{aligned}
$$

Let$f(x) = 2T(cx)$
Replace $x = xy + yx$

$$
\begin{aligned}
f(xy + yx) &= 2T[c(xy + yx)] \\
&= 2T[cxy + ycx] \\
&= 2[T(cx)y + yT(cx)] \\
&= 2T(cx)y + y2T(cx) \\
&= f(x)y + yf(x)
\end{aligned}
$$

Hence$f(x)$ is a Jordan centralizer.
Therefore $f(x) = ax + xa$ is a Jordan centralizer.
By Lemma 4.2, $a \in Z(S)$
$\Rightarrow T(c) \in Z(S)$, hence complete the proof.

Lemma 4.4

Let S be a Semiprime Semiring and $a, b \in S$ be two fixed elements. If $ax + x'b = 0 \ \forall x \in S$, then $a = b \in Z(S)$

Proof

By hypothesis $ax + x'b = 0$, we have $ax = xb$
Replace x by xy
$axy + x'yb = 0$
$x[b, y] = 0, \forall\, b, x, y \in S$
Hence $[b, y]x[b, y] = 0$
Since S is semipime, $[b, y] = 0$
$by + y'b = 0$
$b \in Z(S)$
We have $ax + x'b = 0$
$ax + b'x = 0$
$(a + b')x = 0$ hence $(a + b')x(a + b') = 0$
By the semiprimeness of S implies $a + b' = 0$

$a = b \in Z(S)$

Theorem 4.1

Every Jordan Centralizer of a 2-torsion *free Semiprime Semiring is a centralizer.*

Proof

Suppose T is a Jordan Centralizer, Then
$T(xy + yx) = T(x)y + yT(x) = xT(y) + T(y)x$
$T(x)y + yT(x) = xT(y) + T(y)x$,
Replace y by $xy + yx$, we get
$T(x)(xy + yx) + (xy + yx)T(x) = xT(xy + yx) + T(xy + yx)x$
$T(x)xy + T(x)yx + xyT(x) + yxT(x) = xT(x)y + xyT(x) + T(x)yx + yT(x)x$
$T(x)xy = xT(x)y + yT(x)x + yx'T(x)$
$T(x)xy = xT(x)y + y[T(x), x]$

Adding $x'T(x)y$ on both sides;
$T(x)xy + x'T(x)y = y[T(x), x]$
$[T(x), x]$y=$y[T(x), x]$
$[T(x), x] \in Z(S)$

Next our goal is to show that $[T(x), x] = 0$
Take any $c \in Z(S)$. Then
$2T(cx) = T(cx + cx)$
$= T(cx + xc)$
$= cT(x) + T(x)c$
$= T(x)c + T(x)c$
$= 2T(x)c$
$Adding\ 2T'(x)c$ on both sides
$2T(cx) + 2T'(x)c = 0$, this implies
$T(cx) = T(x)c$

By lemma 4.3, T(C)∈ $Z(S)$, Hence $T(c)x = xT(c)$
Hence $T(cx) = T(c)x = T(x)c$
$[T(x), x]c = [T(x)x + x'T(x)]c$
$= T(x)xc + x'T(x)c$
$= T(x)cx + x'T(c)x$
$= T(c)xx + T(c)xx'$
$= 0$
$Hence\ [T(x), x]c\ [T(x), x] = 0$
Thus $[T(x), x]$ itself is central element, our goal is achieved
$2T(xx) = T(xx + xx)$
$= T(x)x + xT(x)$
$= T(x)x + T(x)x$
$= 2T(x)x$
$Adding\ 2T(x)x'$ on both sides

$T(xx) + T(x)x^{'} = 0$
Since $S\ is\ 2\ torsion \quad T(xx) = T(x)x$
Hence T is Jordan left centralzer.
By theorem 3.1, T is a left centralizer. Similarly, we have prove that T is a right centralizer. Therefore T is a centralizer

Example 4.1
Let S be a semiring. Define, $S_1 = \{(x, x): x \in S\}$. Let the operations of addition and multiplication on S_1 be defined by $(x_1, x_1) + (x_2, x_2) = (x_1 + x_2, x_1 + x_2)$
$(x_1, x_1)(x_2, x_2) = (x_1x_2, x_1x_2)$ for every $x_1, x_2 \in S$. Then it can easily seen that S_1 is a semiring.
Let $d_1: S \to S$is a left centralizing mapping and $d_2: S \to S$ be a right centralizing and commutating mapping . Let $T: S_1 \to S_1$ be the additive mapping defined by $T(x, x) = (d_1(x), d_2(x))$
Let $(x, x) = a \in S_1$.
We have$T(a, a) = T((x, x), (x, x))$
$= T(xx, xx)$
$= (d_1(xx), d_2(xx))$
$= (d1(x)x, xd_2(x)$
$= (d_1(x)x, d_2(x)x\ ]$
$= ((d1(x), d_2(x))(x, x)$
$= T(x, x)(x, x)$
$= T(a)a$

Hence T is a Jordan left centralizer, which is not a centralizer.

Example 4.2
Let S be a semiring. Define $S_1 = \{(x, x): x \in S\}$
Let the operations of addition and multiplication on S_1 be defined by

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$(x_1, x_1) + (x_2, x_2) = (x_1 + x_2, x_1 + x_2)$
$(x_1, x_1)(x_2, x_2) = (x_1x_2, x_1x_2)$ for every $x_{1,}x_2 \in S$

Then it can easily seen that S_1 is a semiring.

Let$d_1: S \to S$is a left centralizing mapping and $d_2: S \to S$be a right centralizing and commutating mapping.

Let $T: S_1 \to S_1$be the additive mapping defined by $T(x, x) = (d_1(x), d_2(x))$

Let $(x, x) = a$ and $(y, y) = b$

$$
\begin{aligned}
T(ab + ba) &= T[(x, x)(y, y) + (y, y)(x, x)] \\
&= T(xy, xy) + (yx, yx) \\
&= T[(xy + yx), (xy + yx)] \\
&= (d_1(xy + yx), d_2(xy + yx)) \\
&= [(d_1(x)y + d_1(y)x), (xd_2(y) + yd_2(x))] \\
&= [(d_1(x)y + d_1(y)x), (d_2(y)x + d_2(x)y] \\
&= [d_1(x)y + d_1(y)x, (d_2(x)y + d_2(y)x] \\
&= (d_1(x)y, d_2(x)y) + (d_1(y)xd_2(y)x) \\
&= \left(d_1(x), d_2(x)\right)(y, y) + (d_1(y)d_2(y))(x, x) \\
&= T(x, x)(y, y) + T(y, y)(x, x) \\
&= T(a)b + T(b)a
\end{aligned}
$$

Hence T is Jordan Centralizer Which is not a centralizer.

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