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Hippocratean Squaring Of Lunes, Semicircle and Circle

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Abstract: *Hippocrates has squared lunes, circle and a semicircle. He is the first man and a last man. S. Ramanujan is the second mathematician who has squared a circle upto a few decimals of π equal to 3.1415926… The squaring of curvature entities implies that lune, circle are finite entities having a finite magnitude to be represented by a finite number.*
Keywords: *Squaring, lune, circle, Hippocrates, S. Ramanujan, π, algebraic number.*

I. Introduction

Hippocrates of Chios was a Greek mathematician, geometer and astronomer, who lived from 470 until 410 BC. He wrote a systematically organized geometry text book **Stoicheia Elements**. It is the first book. And hence he is called the Founding Father of Mathematics. This book was the basis for **Euclid's Elements**.

In his days the π value was 3 of the **Holy Bible**. He is famous for squaring of lunes. The lunes are called Hippocratic lunes, or the lune of Hippocrates, which was part of a research project on the calculation of the area of a circle, referred to as the 'quadrature of the circle'. What is a lune ? It is the area present between two intersecting circles. It is based on the theorem that the areas of two circles have the same ratio as the squares of their radii.

His work is written by **Eudemus** of Rhodes (335 BC) with elaborate proofs and has been preserved by **Simplicius**.

Some believe he has not squared a circle. This view has become very strong with the number 3.1415926… a polygon's value attributed to circle, arrived at, from the Exhaustion method (EM) prevailing before **Archimedes** (240 BC) of Syracuse, Greece, and refined it by him, hence the EM is also known as Archimedean method. This number 3.1415926… has become much stronger as π value, and has been **dissociated** from circle-polygon composite construction, with the introduction of infinite series of **Madhavan** (1450) of South India, and **independently** by later mathematicians **John Wallis** (1660) of England, **James Gregory** (1660) of Scotland.

With the progressive gaining of the importance of 3.1415926… as π value from infinite series, the work of 'squaring of circle' of Hippocrates has gone into oblivion. When the prevailing situation is so, in the mean time, a great mathematician **Leonhard Euler** (1707-1783) of Switzerland has come. His record-setting output is about 530 books and articles during his lifetime, and many more manuscripts are left to posterity. He had created an interesting formula $e^{i\pi} +1 = 0$ and based on his formula, **Carl Louis Ferdinand Lindemann** (1852-1939) of Germany proved in 1882 that π was a type of nonrational number called a transcendental number. (It means, it is one that is not the root of a polynomial equation with rational coefficients. Another way of saying this is that it is a number that cannot be expressed as a combination of the four basic arithmetic operations and root extraction. In other words, it is a number that cannot be expressed algebraically). Interestingly, the term *transcendental number* is introduced by **Euler**.

When all these happened, naturally, the work on the **Squaring of circle** by **Hippocrates** was almost buried permanently.

This author with his discovery in March 1998 of a number $\frac{14-\sqrt{2}}{4}$ = 3.1464466… from Gayatri method, and its confirmation as π value, from Siva method, Jesus proof etc. later, has made the revival of the work of Hippocrates. Hence, this submission of this paper and restoring the **golden throne** of greatness to **Hippocrates** has become all the more a bounden duty of this author and the mathematics community.

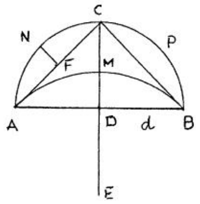
II. Procedure

I. Squaring of Lunes-(1)

Hippocrates has squared many types of lunes. In this paper four types of lunes are studied.

"Consider a semi-circle ACB with diameter AB. Let us inscribe in this semi-circle an isosceles triangle ACB, and then draw the circular are AMB which touches the lines CA and CB at A and B respectively. The segments ANC, CPB and AMB are similar. Their areas are therefore proportional to the squares of AC, CB and

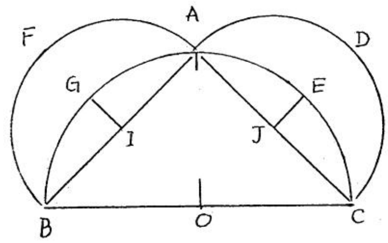
AB respectively, and from Pythagora's theorem the greater segment is equivalent to the sum of the other two. Therefore the lune ACBMA is equivalent to the triangle ACB. It can therefore be squared."



The circular arc AMB which touches the lines CA and CB at A and B respectively can be drawn by taking E as the centre and radius equal to EA or EB.

1. AB = diameter = d
2. DE = DC = radius = d/2
3. F = midpoint of AC
4. N = midpoint of arc AC.
5. NF = $\frac{\sqrt{2}d-d}{2\sqrt{2}}$, DM = $\frac{\sqrt{2}d-d}{2}$, MC = $\frac{\sqrt{2}d-d}{\sqrt{2}}$
6. Area of ANC = Area of CPB = $\frac{d^2}{16}(\pi-2)$
7. Area of AMB = Areas of ANC + CPB = $\frac{d^2}{8}(\pi-2)$
8. Area of ACM = Area of BCM = $\frac{d^2}{16}(4-\pi)$
9. Area of ACB triangle = $\frac{1}{2}\times\frac{d}{2}\times d=\frac{d^2}{4}$
10. According to Hippocrates the area of the lune ACBMA is equal to the area of the triangle ACB.
    Area of Lune ACBMA = Area of triangle ACB
    (ANC + CPB + ACM + BCM)
    $$2\left\{\frac{d^2}{16}(\pi-2)\right\}+2\left\{\frac{d^2}{16}(4-\pi)\right\}=\frac{d^2}{4}$$

Squaring of Lunes-(2)



11. "Let ABC be an isosceles right angled triangle inscribed in the semicircle ABOC, whose centre is O. On AB and AC as diameters described semicircles as in the figure. Then, since by Ecu. I, 47,
    Sq. on BC = Sq. on AC + Sq on AB.
    Therefore, by Euc. XII, 2,
    Area semicircle on BC = Area semicircle on AC + Area semicircle on AB.
    Take away the common parts
    $\therefore$ Area triangle ABC = Sum of areas of lunes AECD and AFBG.
    Hence the area of the lune AECD is equal to half that of the triangle ABC".
12. BC = diameter = d,
13. OB = OC = radius = d/2
14. AB = AC = $\frac{\sqrt{2}d}{2}$ = diameter of the semicircle ABF = ACD
15. GI = EJ = $\frac{2d-\sqrt{2}d}{4}$
16. Sq. on BC = Sq. on AC + Sq. on AB
17. Area of the larger semicircle = BAC = $\frac{\pi d^2}{8}$
18. Area of the smaller semicircle = ABF = ACD
    Diameter = $\frac{\sqrt{2}d}{2}$

$$\text{Area} = \frac{\pi d^2}{8} = \frac{\pi\left(\frac{\sqrt{2}d}{2}\right)^2}{8} = \frac{\pi d^2}{16}$$

18. a) Areas of two smaller semicircles $= 2\times\frac{\pi d^2}{16} = \frac{\pi d^2}{8}$

19. Area of the triangle ABC = $\frac{1}{2}\times\text{base}\times\text{altitude}$

Base = BC = d, Altitude = $\text{OA} = \frac{d}{2}$

Area = $\frac{1}{2}\times d\times\frac{d}{2} = \frac{d^2}{4}$

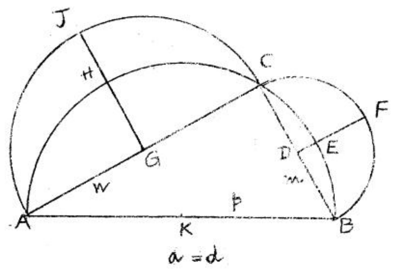
20. Segment AIBG = Segment AJCE

Areas of AIBG + AJCE = $\frac{d^2}{16}(\pi-2)+\frac{d^2}{16}(\pi-2) = \frac{d^2}{8}(\pi-2)$

21. Lune AGBF = lune AECD

22. Area of the lune (AGBF or AECD)
= Semicircles (ABF & ACD) – Segments (AIBG & AJCE)
$\left(\frac{\pi}{8}\right)d^2 - \left(\frac{\pi-2}{8}\right)d^2 = \frac{d^2}{4}$

Squaring of lunes-(3)



"There are also some famous moonshaped figures. The best known of these are the *crescents (or lunulae) of Hippocrates*. By the theorem of Thales the triangle ABC in the first figure is right angled: Thus $p^2 = m^2 + n^2$. The semicircle on AB = p has the area

$A_{AB} = \pi p^2/8$; the sum of the areas of the semicircles on AC and BC is $A_{AC}+A_{BC} = \pi(n^2 + m^2)/8$ and is thus equal to A_{AB}. From this it follows that:

The sum of the areas of the two crescents is the area of the triangle."

23. AB = diameter = d

24. BC = radius = d/2

25. AC = $\frac{\sqrt{3}d}{2}$

26. DF = d/4

27. DE = $\frac{2d-\sqrt{3}d}{4}$

28. EF = $\frac{\sqrt{3}d-d}{4}$

29. GH = d/4

30. GJ = $\frac{\sqrt{3}d}{4}$

31. HJ = GJ – GH = $\frac{\sqrt{3}d}{4}-\frac{d}{4} = \frac{\sqrt{3}d-d}{4}$

32. Area of the semicircle BDCF
$= \pi\times\frac{d}{2}\times\frac{d}{2}\times\frac{1}{8} = \left(\frac{\pi}{32}\right)d^2$ where BC = diameter = $\frac{d}{2}$

33. Area of the semicircle AGCJ
$= \pi\times\frac{\sqrt{3}d}{2}\times\frac{\sqrt{3}d}{2}\times\frac{1}{8} = \left(\frac{3\pi}{32}\right)d^2$ where AC = diameter = $\frac{\sqrt{3}d}{2}$

34. Area of the triangle ABC = $\frac{1}{2}\times AC\times BC=\frac{1}{2}\frac{\sqrt{3}d}{2}\times\frac{d}{2}$ = $\left(\frac{\sqrt{3}}{8}\right)d^2$

35. Area of the curvature entity BDCE = $\left(\frac{2\pi-3\sqrt{3}}{48}\right)d^2$

36. Area of the curvature entity AGCH = $\left(\text{Circle}-\text{AGCH}\right)\frac{1}{3}$

$\left\{\frac{\pi d^2}{4}-\left(\frac{3\sqrt{3}}{16}\right)d^2\right\}\frac{1}{3}$ = $\left(\frac{4\pi-3\sqrt{3}}{48}\right)d^2$

Area of the triangle = $\left(\frac{3\sqrt{3}}{16}\right)d^2$, where side = AC = $\left(\frac{\sqrt{3}}{2}\right)d$

37. Area of the lune BECF =

Semicircle BDCF – BDCE segment = $\left(\frac{\pi}{32}\right)d^2-\left(\frac{2\pi-3\sqrt{3}}{48}\right)d^2$ = $\left(\frac{6\sqrt{3}-\pi}{96}\right)d^2$

(S.No. 32) (S.No. 35)

38. Area of the lune AHCJ

Semicircle AGCJ – AGCH segment = $\left(\frac{3\pi}{32}\right)d^2-\left(\frac{4\pi-3\sqrt{3}}{48}\right)d^2=\left(\frac{\pi+6\sqrt{3}}{96}\right)d^2$

(S.No. 33) (S.No. 36)

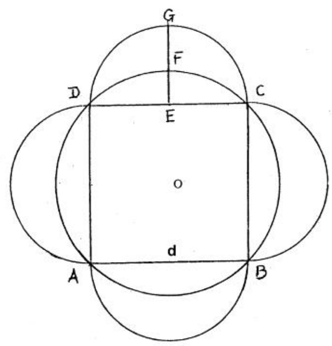
39. Sum of the areas of two lunes = area of the triangle

(S.No. 37) + (S.No. 38) (S.No. 34)

= $\left(\frac{6\sqrt{3}-\pi}{96}\right)d^2+\left(\frac{\pi+6\sqrt{3}}{96}\right)d^2$ = $\left(\frac{\sqrt{3}}{8}\right)d^2$

Squaring of lunes – (4)

The sum of the areas of the lunes is eqal to the area of the square.



40. AB = side = d

41. DE = EC = d/2

42. AO = OC = $\frac{\sqrt{2}d}{2}$

43. EF = $\frac{\sqrt{2}d-d}{2}$

44. FG = $\frac{\sqrt{2}d-d}{\sqrt{2}}$

45. Area of the circle = $\frac{\pi d^2}{4}$

Where diameter = $\sqrt{2}d$

$\pi\times\sqrt{2}d\times\sqrt{2}d\times\frac{1}{4}$ = $\frac{\pi d^2}{2}=\left(\frac{\pi}{2}\right)d^2$

46. Area of the semicircle DECG

Where DC = diameter = d = $\frac{\pi d^2}{8}=\left(\frac{\pi}{8}\right)d^2$

47. Area of the curvature entity DECF = $\left(\frac{\pi-2}{8}\right)d^2$

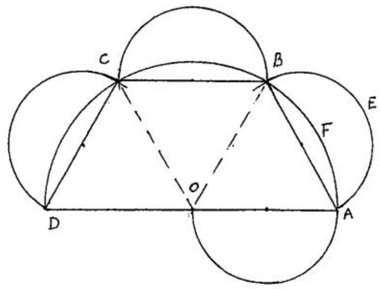
48. Area of the lune DFCG

Semicircle DECG – Curvature entity DECF = $\left(\frac{\pi}{8}\right)d^2-\left(\frac{\pi-2}{8}\right)d^2=\frac{d^2}{4}$

49. The sum of the areas of 4 lunes = the area of the square

$$4\left\{\left(\frac{\pi}{8}\right)d^2-\left(\frac{\pi-2}{8}\right)d^2\right\}=d^2$$

III. Squaring of a semicircle



"Hippocrates next inscribed half a regular hexagon ABCD in a semicircle whose centre was O, and on OA, AB, BC, and CD as diameters described semicircles. The AD is double any of the lines OA, AB, BC and CD,

$\therefore$ Sq. on AD = Sum of sqs. On OA, AB, BC and CD,

$\therefore$ Area semicircle ABCD = sum of areas of semicircles on OA, AB, BC and CD.

Take away the common parts.

$\therefore$ Area trapezium ABCD = 3 lune AEFB + Semicircle on OA".

50. DA = diameter = d

51. Area of the semicircle DABC = $\frac{\pi d^2}{8}=\left(\frac{\pi}{8}\right)d^2$

52. $\frac{DA}{2}=$ radius of larger semi circle = $\frac{d}{2}=AB$

53. AB = $\frac{d}{2}$ = diameter of smaller semi circle ABE

$$\frac{\pi d^2}{8}=\pi\times\frac{d}{2}\times\frac{d}{2}\times\frac{1}{8}=\left(\frac{\pi}{32}\right)d^2$$

54. Areas of semicircle on OA, AB, BC and CD = $\left(\frac{\pi}{32}\right)d^2$

55. Area of sector OAFB = $\left(\frac{\pi d^2}{4}\right)\frac{1}{6}=\left(\frac{\pi}{24}\right)d^2$

56. Area of the triangle OAB = $\left(\frac{\sqrt{3}}{16}\right)d^2$

57. Area of the segment AFB = Sector – Triangle

$$\left(\frac{\pi}{24}\right)d^2-\left(\frac{\sqrt{3}}{16}\right)d^2=\left(\frac{2\pi-3\sqrt{3}}{48}\right)d^2$$

58. Area of lune AEBF = Semicircle on AB – AFB segment

$$=\left(\frac{\pi}{32}\right)d^2-\left(\frac{2\pi-3\sqrt{3}}{48}\right)d^2=\left\{\frac{\left(6\sqrt{3}+3\pi\right)-4\pi}{96}\right\}d^2=x$$

Area of one lune = x

59. Area of 3 lunes = $3\left\{\frac{\left(6\sqrt{3}+3\pi\right)-4\pi}{96}\right\}d^2=\left\{\frac{\left(6\sqrt{3}+3\pi\right)-4\pi}{32}\right\}d^2$

60. Area of 3 lunes + semicircle on OA

$$=\left\{\frac{\left\{\left(6\sqrt{3}+3\pi\right)-4\pi\right\}}{32}\right\}d^2+\left(\frac{\pi}{32}\right)d^2=\left\{\frac{\left\{\left(6\sqrt{3}+3\pi\right)-4\pi\right\}+\pi}{32}\right\}d^2=\left(\frac{6\sqrt{3}}{32}\right)d^2=\left(\frac{3\sqrt{3}}{16}\right)d^2$$

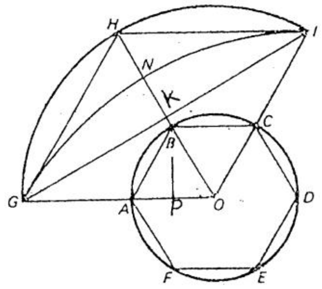
61. Area of trapezium = 3 x OA triangle
(S.No. 56)

$$=3\times\left(\frac{\sqrt{3}}{16}\right)d^2=\left(\frac{3\sqrt{3}}{16}\right)d^2$$

62. Area of 3 lunes + semicircle on OA =Area of trapezium = $\left(\frac{3\sqrt{3}}{16}\right)d^2=\left(\frac{3\sqrt{3}}{16}\right)d^2$

(S.No. 60)

IV. Squaring of circle



"Consider two concentric circles with common centre O and radii such that the square of the radius of the larger circle is six times the square of the radius of the smaller one. Let us inscribe in the smaller circle the regular hexagon ABCDEF. Let OA cut the larger circle in G, the line OB in H and the line OC in I. On the line GI we construct a circular segment GNI similar to the segment GH. Hippocrates shows that the lune GHIN plus the smaller circle is equivalent to the triangle GHI plus t he hexagon."

63. OA = radius of the smaller circle = $\frac{d}{2}$

64. OH = radius of the larger circle = $\sqrt{6\left(\frac{d}{2}\right)^2} = \left(\frac{\sqrt{6}}{2}\right)d$

65. Third circle: GI = radius = GK + KI

66. OH = OI = $\left(\frac{\sqrt{6}}{2}\right)d$

67. OK = $\frac{OH}{2} = \left(\frac{\sqrt{6}}{4}\right)d$

68. KI = $\sqrt{(OI)^2 - (OK)^2} = \left(\frac{3\sqrt{2}}{4}\right)d$

69. Radius of the third circle

= GI = 2 x KI = $\left(\frac{3\sqrt{2}}{2}\right)d$

70. Area of the GHI triangle = $\frac{1}{2} \times GI \times HK$

$HK = \frac{OH}{2} = \left(\frac{\sqrt{6}}{4}\right)d$

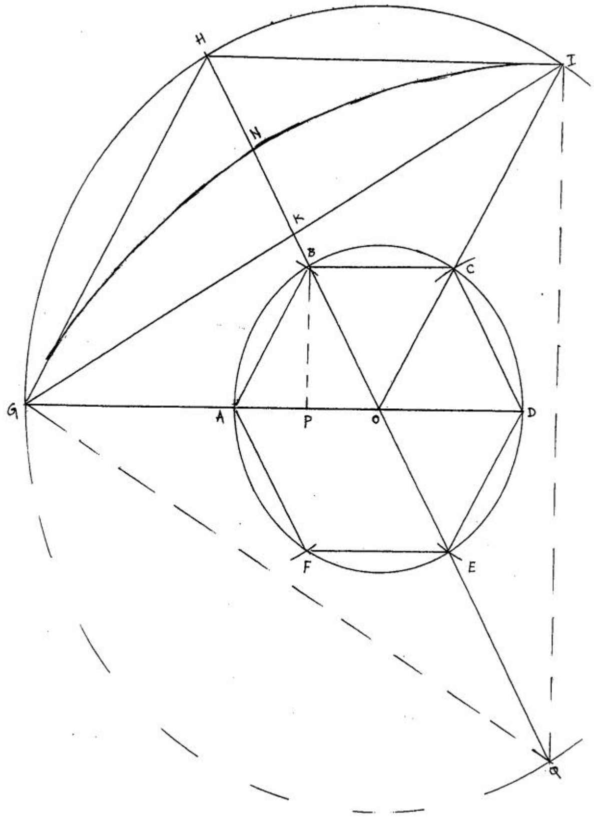
= $\frac{1}{2} \times \left(\frac{3\sqrt{2}}{2}\right)d \times \left(\frac{\sqrt{6}}{4}\right)d = \left(\frac{3\sqrt{3}}{8}\right)d^2$

71. Area of the AOB triangle

OA = AB = $\frac{d}{2}$; AP = $\frac{OA}{2} = \frac{d}{4}$

PB = $\sqrt{(AB)^2 - (AP)^2} = \left(\frac{\sqrt{3}}{4}\right)d$;

Area = $\frac{1}{2} \times OA \times PB = \frac{1}{2} \times \frac{d}{2} \times \left(\frac{\sqrt{3}}{4}\right)d = \left(\frac{\sqrt{3}}{16}\right)d^2$



72. Area of the hexagon = Area of the triangle AOB x 6

= $\left(\frac{\sqrt{3}}{16}\right)d^2 \times 6 = \left(\frac{6\sqrt{3}}{16}\right)d^2 = \left(\frac{3\sqrt{3}}{8}\right)d^2$

73. Area of the smaller circle = $\frac{\pi d^2}{4} = \left(\frac{\pi}{4}\right)d^2$

74. Area of the segment GH = Segment HI

75. Area of the larger circle = $\frac{\pi d^2}{4}$

Where d = $\left(\frac{\sqrt{6}}{2}\right)d \times 2 = \sqrt{6}d = \pi \times \sqrt{6}d \times \sqrt{6}d \times \frac{1}{4} = \left(\frac{6\pi}{4}\right)d^2$

76. Area of the larger circle is divided into 6 sectors = $\left(\frac{6\pi}{4}\right)d^2 \times \frac{1}{6} = \left(\frac{\pi}{4}\right)d^2$

77. Area of the triangle OGH = OHI = GHI = $\left(\frac{3\sqrt{3}}{8}\right)d^2$

(S.No. 70)

78. Area of the GH segment = HI segment

= Sector – Triangle (OGH) = $\left(\frac{\pi}{4}\right)d^2 - \left(\frac{3\sqrt{3}}{8}\right)d^2 = \left(\frac{\pi}{4} - \frac{3\sqrt{3}}{8}\right)d^2$

(S.No. 76) (S.No. 77)

There are two segments GH and HI = $2\left(\frac{\pi}{4} - \frac{3\sqrt{3}}{8}\right)d^2 = \left(\frac{2\pi - 3\sqrt{3}}{4}\right)d^2$

79. Similarly, GNIK is also another segment which is the part of the sector GNIQ. It consists of the triangle GIQ and GNIK segment.

80. To find out the area of the sector GNIQ, let us **first** find out the area of the circle whose diameter is equal to that of the **third circle**.

Diameter of the third circle = riadus x 2 = $\left(\frac{3\sqrt{2}}{2}\right)d \times 2 = \left(3\sqrt{2}\right)d$

(S.No. 69)

Area = $\frac{\pi d^2}{4} = \pi \times \left(3\sqrt{2}\right)d \times \left(3\sqrt{2}d\right) \times \frac{1}{4} = \left(\pi \times 18 \times \frac{1}{4}\right)d^2 = \left(\frac{9\pi}{2}\right)d^2$

81. Then let us find out the area of the sector = $\left(\frac{1}{6}\text{th}\right) = \left\{\left(\frac{9\pi}{2}\right)d^2\right\}\frac{1}{6} = \left(\frac{9\pi}{12}\right)d^2 = \left(\frac{3\pi}{4}\right)d^2$

82. Now let us find out the area of the triangle GIQ = $\frac{1}{2} \times GI \times KQ$

Where KI = $\frac{GI}{2} = \left(\frac{3\sqrt{2}}{2}\right)d \times \frac{1}{2} = \left(\frac{3\sqrt{2}}{4}\right)d$

GI = QI = GQ = Radius of the third circle.

$$KQ = \sqrt{(QI)^2 - (KI)^2} = \sqrt{\left\{\left(\frac{3\sqrt{2}}{2}\right)d\right\}^2 - \left\{\left(\frac{3\sqrt{2}}{4}\right)d\right\}^2} = \left(\frac{3\sqrt{6}}{4}\right)d$$

Area = $\frac{1}{2} \times GI \times KQ = \frac{1}{2} \times \left(\frac{3\sqrt{2}}{2}\right)d \times \left(\frac{3\sqrt{6}}{4}\right)d = \left(\frac{9\sqrt{3}}{8}\right)d^2$

83. Area of the segment GNIK = Sector – Triangle

(S.No. 81) (S.No. 82)

$= \left(\frac{3\pi}{4}\right)d^2 - \left(\frac{9\sqrt{3}}{8}\right)d^2 = \left(\frac{6\pi - 9\sqrt{3}}{8}\right)d^2$

84. Now it has become possible to calculate the area of GHIN segment

= Triangle GHI – Segment GNIK

(S.No. 70) (S.No. 83)

$= \left(\frac{3\sqrt{3}}{8}\right)d^2 - \left(\frac{6\pi - 9\sqrt{3}}{8}\right)d^2 = \left(\frac{6\sqrt{3} - 3\pi}{4}\right)d^2$

Area = $\left(\frac{6\sqrt{3} - 3\pi}{4}\right)d^2$

85. Area of the lune GHIN

Segments + Segments + Circle =

GH & HI GHIN

S.No. 78 S.No. 84 S.No. 73

$= \left(\frac{2\pi - 3\sqrt{3}}{4}\right)d^2 + \left(\frac{6\sqrt{3} - 3\pi}{4}\right)d^2 + \left(\frac{\pi}{4}\right)d^2 = \left(\frac{3\sqrt{3}}{4}\right)d^2$

86. Area of the triangle GHI + Area of the hexagon ABCDEF

(S.No. 70) (S.No. 72)

$$\left(\frac{3\sqrt{3}}{8}\right)d^2 + \left(\frac{3\sqrt{3}}{8}\right)d^2 = \left(\frac{3\sqrt{3}}{4}\right)d^2$$

87. Area of lune + Circle = $\left(\frac{3\sqrt{3}}{4}\right)d^2$ = Area of triangle+ hexagon = $\left(\frac{3\sqrt{3}}{4}\right)d^2$

(S.No. 85) (S.No. 86)

So, the sum of the areas of lune and circle is equal to the sum of the areas of triangle and hexagon.

V. Post Script

The following are the points on which some thinking is necessary:

1. 3.14159265358… is accepted as π value.
2. 3.14159265358… is a transcendental number.
3. As this polygon's value is accepted as π of the circle, circle and its π value have become transcendental entities.
4. The concept of transcendental number vehemently opposes squaring of circle.

Latest developments

5. $\frac{14-\sqrt{2}}{4}$ = 3.14644660942… is the new π value.
6. $\frac{14-\sqrt{2}}{4}$ is the exact value.
7. This number is an algebraic number, being the root of $x^2 - 56x + 97 = 0$
8. Squaring of circle is done with this number.

Conclusion

9. Hippocrates did square the circle.
10. 3.14159265358… is a transcendental number – it is correct.
11. 3.14159265358… can not square a circle, - is also correct.

Final verdict

12. As Hippocrates did the squaring a circle, it amounts to confirming that circle and its π value are algebraic entities. It implies that as 3.14159265358… is a borrowed number from polygon and attributed to circle, called a transcendental number, said squaring a circle an unsolved geometrical problem, the final verdict is, all are correct, except one, i.e. attributing 3.14159265358 of polygon to circle. Hence 3.14159265358… is not a π value at all.

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