Virtual Reality And Technologies For Teaching Mathematics: A Systematic Literature Review

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Abstract:
This work is the result of the line of research: Collaborative Environments for Teaching and Teacher Training of the Postgraduate Program in Science Teaching at Cruzeiro do Sul University. In an era of digital natives, the integration of digital technologies in teaching is essential for teachers and students. This scenario offers mathematics, present at all levels of Basic Education, the opportunity to enrich the construction of knowledge in a more meaningful and captivating way. The objective of this study is to carry out a systematic review of the literature to analyze the effective use of Virtual Reality in mathematics teaching, examining its relationships with mathematical learning and identifying potential, challenges and trends. Adopting a systematic literature review methodology with exploratory approaches and qualitative analysis, the studies analyzed highlight that Virtual Reality is driving innovations in mathematics teaching, creating new learning environments and pedagogical interactions. However, challenges persist, such as the lack of access to technological equipment in public schools and deficiencies in teacher training to effectively incorporate the use of technology into their educational practices.

Keywords: Digital teaching; Mathematics education; Teaching and learning technologies.

I. Introduction
When we reflect on technological evolution, we come across significant transformations in society, resulting from globalization, reflections of the industrial revolution and even post-war developments. Education has also been driven in the technological era to promote more innovative, thought-provoking and effective teaching approaches. In this way, Digital Information and Communication Technologies (DIT) are increasingly in vogue, constituting pedagogical possibilities that offer new learning mechanisms and subsidize new teaching methods (OLIVEIRA et al., 2020; OLIVEIRA; BARROCO, 2023).

In a modern society, many changes and technological innovations are available to be used in the school environment, which is in accordance with a society based on communication and information, since, through these means we have the virtual feasibility of having access to various types of information around the world, because the new digital era offers many benefits when it comes to scientific, educational advances, communication and knowledge (LIMA; ARAÚJO, 2020, p. 4).

Among these trends, Reality (VR) and Augmented Reality (AR) are present and gaining ground as active teaching methodologies, resulting in advances at the intersection between education and technology-mediated teaching. In this context, students in the digital era show interest in teaching strategies that incorporate the use of cell phones in activities, the internet, games and other resources that escape traditional and obsolete teaching (CUNHA; MOURAD; JORGE, 2021).

When we think about mathematics as a subject that applies to all levels of education, we come across a series of gaps in students' understanding, many of whom come from initial teacher training when it comes to the use of different resources. To this end, the abstraction of mathematical concepts and the need to rationalize complex, theoretical and practical problems, make teaching mathematics challenging, since the literature indicates that this discipline is feared by a range of students (PIOVESAN; ZANARDINI, 2007).

That said, limitations in teaching mathematics from the Early Years of Elementary School (EF) can lead to students’ lack of interest and mathematical literacy ability in subsequent stages. Thus, studies point to the potential of VR in teaching mathematics by enabling the creation of attractive, immersive, dynamic and interactive virtual environments; outlining playful visualization strategies in approaching logical questions in an innovative manner (SANTOS; OLIVEIRA; OLIVEIRA, 2023).
The development, interest and availability of VR resources in mathematics teaching are directly related to the rise of digital technologies in teaching and learning processes. The expansion of these resources for the use of teachers is intrinsically related to overcoming stereotypes and paradigms in mathematics education, which contributes to offering innovative teaching, making mathematics accessible and engaging. In the meantime, we believe that VR can be an important ally in the application of mathematical knowledge, through meaningful interactive teaching experiences (AURELÍANO; QUEIROZ, 2023).

Therefore, digital technologies are technological resources that are incorporated, providing diverse communication, modifying the education system. And it has been cooperating with a very productive difference, which improves the interaction between teacher and student, improving the way of transmitting and learning. Becoming a technology used to aggregate, contribute and share information, ICT helps towards better development when used for the benefit of education, joining the most used methods such as chalk, blackboards, books and playful pedagogical games, further enhancing learning (LIMA; ARAÚJO, 2020, p. 3).

Starting from this context, we are faced with the following guiding research prerogative: how can Virtual Reality (VR) be effectively incorporated into mathematics teaching, overcoming gaps in student understanding and promoting innovative and engaging teaching? Therefore, this work aims to analyze research on the effective use of VR as a technology in teaching mathematics, its relationships with mathematical learning, portrayals of potential, challenges and trends.

II. Methodology

The method that guides this work is the systematic review of the literature. The choice and development occurred with the aim of carrying out a comprehensive and significant reinterpretation of scientific and academic production regarding the insertion of VR in mathematics teaching. Therefore, we believe that this research is important to promote new dialogues about materials already published by other researchers; adopting new perspectives, which contributes to the systematization of scientific knowledge (SOUZA; OLIVEIRA; ALVES, 2023).

According to Prodanov and Freitas (2013, p. 131):

"The literature review demonstrates that the researcher is updated in the latest discussions in the field of research knowledge. In addition to articles in national and international journals and books already published, monographs, dissertations and theses are excellent sources of consultation. Literature review differs from a collection of summaries or a "patchwork" of citations.

The exploratory nature of the research occurs through the need to know and investigate VR, as it is a new topic that needs to be in the vocabulary of the teaching profession. According to Gil (2008, p. 27) this research focuses on providing a general overview of the topic “especially when the chosen topic is little explored and it becomes difficult to formulate precise and operational hypotheses about it”.

In view of this, a search was carried out on Google Scholar (GA), in order to concentrate studies that fit the scope of the work, using the following descriptors: “digital technologies”; “virtual reality”; “mathematical education”; “teaching mathematics”; “teaching and technologies”; “mathematics teaching and learning” and “mathematics technologies”.

However, the collected studies were selected through title, abstract and keywords, with inclusion and exclusion criteria being: 1) thematic relevance; 2) contributions to the area of mathematics education; 3) quality in structure and scientific-academic writing.

Therefore, the collected and selected studies were analyzed qualitatively, according to Minayo's (2004) thematic analysis. Two categories insurgent to the study were divided and systematized: 1) Applications of Virtual Reality in Mathematics Teaching and its potential; 2) Challenges and Limitations of Implementing Virtual Reality Technologies in Mathematics Teaching. In addition to the primarily selected works (articles), other modalities were incorporated into the discussion to support an expanded and dialogue reading based on works from event annals, book chapters and reports of experiences based on VR in mathematics education.

III. Results And Discussion

Applications of Virtual Reality in Mathematics Teaching and its potential

Considering the constant advances in education in the field of technology, we can say that VR has been a promising highlight as a strengthening tool for mathematics education and even in several other areas of knowledge. In accordance with Varão and Silva (2020), teachers and students are experiencing a scenario of adaptations to virtual, technological and digital teaching, however, “The teacher needs to be connected to these possibilities, to be able to plan and use the appropriate tools in their school practices” (p. 7).

The concept of “technological age” conceals, alongside a reasonable and serious meaning, another, typically ideological, thanks to which those interested seek to intoxicate the consciousness of the masses, making them believe that they have the happiness of living in the best times ever enjoyed. for humanity […] The society
capable of creating stupendous currently existing machines and devices, unknown and never dreamed of by men of the past, cannot fail to be better than any previous one (PINTO, 2005, p. 41).

First of all, it is necessary to conceptualize VR. For Kirner and Siscoutto (2007, p. 19): [...] it is an area of knowledge that has been providing users with better conditions for interacting with computational applications, providing them with natural interactions and enhancing their capabilities. To achieve this, many resources are used, involving hardware, software, peripherals, networks, special technologies, design and evaluation techniques and application development.

In the words of Resende and Santos (2019, p. 3):

VR allows the user to interact with the computer in such a way that the user is transported to a virtual environment and has an experience together with virtual objects. The user must have the impression of acting in a completely virtual environment, manipulating, creating and moving objects in real time.

Mathematics, perceived by many students as challenging, ends up being a hurdle in the construction of knowledge in mathematics. However, as we follow the evolution of technologies and teaching methods, the experiences of several teachers and researchers point to VR as a stimulating approach to concrete, accessible and visual mathematical concepts, breaking a little with the theoretical, abstract and tedious paradigm.

That said, this topic sheds light on the pedagogical applications of VR in mathematics teaching, with an emphasis on its teaching potential. In view of this, we present some resources that result in interactivity provided by VR in active engagement in mathematics learning. However, we emphasize that this resource is for personalizing teaching, that is, it can be adapted and used according to individual and collective needs, according to learning rhythms assessed by the teacher. In this way, students can progress in meaningful learning according to their cognitive abilities and ways of constructing knowledge.

Regarding augmented reality in education, its exponential evolution in recent years has resulted in a range of research produced in the most varied areas of education. Similar to the computational and visual power of virtual worlds in simulations, augmented reality has a significant advantage in enabling its use on mobile devices, which has been one of the main means of spreading its use among students in recent years (HERPICH et al., 2020, p. 16).

We highlight the first point in the application of VR in mathematics teaching, which is the ability to transpose abstract concepts into tangible, clear and attractive visualizations. That said, many students have difficulty constituting and imagining concepts of three-dimensional geometry, algebraic calculations and expressions, therefore, VR allows students to interact with geometric shapes, objectives and formulas, resulting in visual, intuitive and innovative learning.

Furthermore, the use of VR in teaching mathematics goes beyond simple visualization and imagination, allowing students to solve problems and experience real-world situations. This is aligned with the strategies of the National Common Curricular Base (BNCC), which seeks to promote the insertion of students into digital culture and the development of critical thinking. VR, as one of the Digital Information and Communication Technologies (DIT), plays a crucial role in the development of skills and competencies (BRASIL, 2018).

An AR system complements the real world with virtual objects (generated by a computer) that appear to coexist in the same space in the real world. While many researchers expand the definition of AR beyond this view, we define that an AR system must have the following properties: combines real and virtual objects in a real environment, runs interactively and in real time, and records (aligns) real and virtual with each other. (AZUMA et al., 2001, p. 34).

The second point to be emphasized is the positive impact on student motivation when using Virtual Reality (VR) as a teaching tool. VR provides a playful experience, in which the learner is attracted to knowledge, and this can be intensified, including through gamification (Barreto et al., 2021). According to studies by Lima, Cunha and Dinardi (2023), the incorporation of playful elements in teaching and learning processes demonstrates effectiveness in facilitating educational practices, resulting in scientific literacy and meaningful learning.

A survey carried out by Queiroz, Tori and Nascimento (2017) pointed out education as an area of knowledge that weaves research into VR (Figure 1). When reading the article, we identified that this survey was carried out in relation to the production of research groups, showing that the computing area led productivity in VR, however education and health are concentrated in second place, respectively. This distance needs to be overcome, through new trends in software and educational resources aimed at using VR.
Figure 1 - Survey of VR research developed by research groups.


Figure 2 presents a sequence of 3 digital applications for use as VR software, available in conventional smartphone application stores. The first is called as Math VR\(^1\), which offers VR to work on basic mathematical operations, such as: addition, subtraction, multiplication and division; The age range indicated for use is between 5 and 12 years old. The second is the VRMath\(^2\), with three-dimensional resources in spatial geometry, and can be used in different series. The third application is Calculus in Virtual Reality\(^3\), which provides a stunning experience in the development and understanding of more advanced calculations and equations, however, with real-time and three-dimensional visualizations, allowing a practical application of equations and calculations in relation to space (RESENDE; SANTOS, 2019).

Figure 2 - VR possibilities in mathematics.

Source: Adapted from Resende and Santos (2019).

\(^1\) Available in: https://play.google.com/store/apps/details?id=eu.LittleLane.MatematikaVR.


In terms of practical application, VR stands out as a flexible tool, which can be used both in physical environments, such as in the conventional classroom, and in virtual teaching environments. This expands the possibilities for student engagement, promoting a more engaging and motivating approach to the learning process. [...] it can be used in the training of teachers who teach Mathematics to explore its potential in the teaching and learning processes, articulating theory and practice, in a critical and autonomous way, in the construction of knowledge in a meaningful way, so that what is studied has more meaning in the daily life of the person who is learning (OLIVEIRA, 2018, p. 2).

Without a doubt, Virtual Reality (VR) is playing a transformative role in mathematics education, changing the way teachers plan and deliver their classes and how students learn. These applications represent significant advances in mathematics education and are in line with the guidelines of the National Common Curricular Base (BNCC). BNCC recognizes the importance of using digital technologies and applications not only for mathematical investigation, but also for the development of computational thinking, emphasizing the relevance of these modern approaches to education (BRASIL, 2018.). This emphasis on integrating digital technologies, such as VR, into mathematics teaching is aligned with the need to prepare students to understand the digital world and provide them with the tools necessary for a deeper and more meaningful understanding of mathematical concepts.

**Challenges and Limitations of Implementing Virtual Reality Technologies in Mathematics Teaching**

The application of active methodologies, games and practical activities in teaching mathematics, including the use of VR, is not without challenges and limitations. Although in the topic above, we have highlighted its potential, some limitations are found in the approach to teaching mediated by digital technologies and these can be interconnected with several factors, whether obstacles in educational institutions and the handling of this tool by educators (CUNHA; MOURAD; JORGE, 2023).

We begin by pointing out that the first challenging pillar of VR refers to the costs associated with the acquisition of equipment and its maintenance. Some institutions, mainly public schools, may not be provided with a technological and digital apparatus to encourage the use of these technologies in the school environment. Therefore, this barrier may limit the development of mathematics teaching in light of VR (ARRUDA; SILVA; BEZERRA, 2020).

Mathematics teaching must be permeated by different methodologies and focus on argumentation and exploration. In this sense, the error is presented with the perspective of a possibility of learning and, for the teacher, as a way of noticing the gaps and difficulties of students to advance in the construction of their knowledge (CARNEIRO; PASSOS, 2014, p. 1130). The second point in question is teacher training. For VR to be implemented in classroom activities, in problematizations or even as a learning assessment, teachers must have mastery of the use of this technology from a teaching perspective (JUNGER et al., 2023a; JUNGER et al., 2023b). However, we realize that the result of these articulations in Basic Education is the result of this guidance from the initial training in degrees, with approaches and preparation of education professionals for this resource or even, continued training coming from the sponsors to overcome gaps in initial training and promote new updates and pedagogical training (OLIVEIRA; MELO; RODRIGUEZ, 2023).

[...] it is important for both the teacher and the student to have access to the various information offered through the use of ICT during the teaching-learning process, making communication and the scope of school skills more accessible. We can say that ICT, when used responsibly in teaching, can represent a great challenge for teachers, as they require qualification, a redefinition of the methodologies developed in the classroom, as well as an improvement in teaching planning (LIMA; ARAÚJO, 2021, p. 7).

We cannot fail to mention the adequacy and creation of digital educational content, since teachers have different points of view and skills to develop and create activities according to the curricular alignment of mathematics teaching. To this end, it is necessary that teaching materials and software that promote mathematics activities with VR are disseminated to inspire the creation, use and reproduction by Basic Education professionals (SANTOS; PERIN, 2013).

The assumed conception of reality associates it with human perception, worrying about how things in the world are or relate to the subject who interprets them. There is no world separate from the human, but rather the idea of being-human-with-the-world. In this context, what stands out is the human dimension in the spatiality and temporality of the field of lived experiences (DALLA VECCHIA; MALTEMPI, 2012, p. 979).

It is worth mentioning that equity in education resources is a challenge in Brazil. Therefore, public educational policies must encourage the expansion of these resources and ensure that everyone can have access to digital equipment, even if they are students from peripheral regions. This lack of access to equipment represents a gap in several other educational spheres and impedes the development of mathematical education and many other areas of knowledge (LEMOS; FERNANDES, 2022).
We recognize that VR has the potential to attract students to the construction of knowledge, however, its inappropriate use can be harmful by dispersing them throughout the educational process. Having said that, we reinforce that pedagogical planning and mediation is very important so that the pedagogical purposes of its use come to reflect the objectives stipulated by teachers, guaranteeing a space for productive and collaborative learning. Furthermore, Araujo and Padilha (2023), in their research findings, point out that digital inclusion is still limited in pedagogical projects in Higher Education. Therefore, this scenario needs to be transformed to elucidate new perspectives in Basic Education.

For VR and AR applications to be used on a large scale in classrooms, they must meet both pedagogical aspects and requirements such as ease of use, affordable cost and the ability to provide rapid preparation by teachers themselves of the content that will be covered. Furthermore, for technology to add value to the teaching process, it is necessary to create and adopt an effective methodology for inserting it into the educational context (GUIMARÃES; MARTINS, 2013, p. 15).

High School (EM) as the last stage of Basic Education can also be explored using VR, however, as the target audience of students is already more mature, it is important to level the adequacy of activities for a meaningful experience (OLIVEIRA, 2019). According to Silva and Rufino (2021, p. 28):

> It should continue to be employed in EM and applications should mature to take full advantage of AR. There is still a vast field of growth for the use of AR in disciplines that have not yet fully embraced it, such as teaching languages or arts, which have a lot to gain from this tool.

In view of this context, we highlight that the software and applications that promote the VR experience in education are successful, however, to avoid and overcome some obstacles it is necessary to use them carefully. Therefore, we believe in the contributions of VR research and other digital methodologies to incorporate new trends in educational practice.

### IV. Conclusion

In a scenario of constant technological evolution, the integration of TDIC in teaching becomes fundamental for the involvement of students and improvement of educational practices. In this context, VR has provided promising experiences for teaching mathematics at all levels of Basic Education. This systematic literature review study showed that VR is driving innovations in mathematics teaching, providing engaging learning environments and more dynamic pedagogical interactions. The application of VR transposes abstract mathematical concepts into tangible visualizations, facilitating understanding and providing an innovative approach to the discipline.

However, despite the promising potential of VR, significant challenges need to be overcome. The lack of access to technological equipment in public schools represents a barrier to the dissemination of VR in mathematics teaching. Furthermore, teacher training is essential to ensure the effectiveness of integrating VR into their pedagogical practices. It is essential that educators are prepared to use this technology efficiently, personalizing teaching according to students' needs and taking advantage of their cognitive capabilities.

Another challenge is creating appropriate digital educational content, as well as adapting teaching materials to incorporate VR in a meaningful way. Inappropriate use of VR can disperse students, compromising the learning process. Therefore, solid pedagogical planning and appropriate mediation are crucial to ensure that VR achieves educational objectives.

To overcome these challenges, public educational policies must be aimed at providing accessible technological resources at all levels of education. Furthermore, it is important to promote the creation of specific VR software and applications for mathematics education, as well as offering ongoing training for teachers. In this way, VR can play a transformative role in mathematics teaching, making the learning process more engaging, meaningful and preparing students to understand the digital world and mathematical concepts in a deeper way.

In conclusion, this work presented potential/contributions and challenging disparities in light of VR in mathematics education. Therefore, we hope that new studies will add to the literature with new scientific discoveries, creating new software and applications that impact the day-to-day life of the classroom positively with their innovative bias.

### Referências


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