

Generation of Rainfall Intensity-Duration-Frequency Relationship for North-Western Region in Bangladesh

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Abstract: The objective of this research is to generate Rainfall IDF relationship for North-Western region of Bangladesh. Two common frequency analysis techniques Gumbel and Log Pearson Type III (LPTIII) distribution were used to develop IDF relationship from rainfall data of this region. Yearly maximum rainfall data for last 41 years (1974-2014) from Bangladesh Meteorological Department (BMD) was used in this study. Indian Meteorological Department (IMD) empirical reduction formula was used to estimate short duration rainfall intensity from yearly maximum rainfall data. Results obtained using Gumbel method are slightly higher than LPT III distribution method. Chi-square goodness of fit test was used to determine the best fit probability distribution. The parameters of the IDF equations and coefficient of correlation for different return periods (2, 5, 10, 25, 50 and 100 years) were calculated by using nonlinear multiple regression method. The results obtained presented that in all the cases the correlation coefficient is very high representing goodness of fit of the formulae to estimate IDF curves in the region of interest. It was found that intensity of rainfall decreases with increase in rainfall duration. Further, a rainfall of any given duration will have a larger intensity if its return period is large.

Keyword: Rainfall Intensity, Rainfall Duration, Rainfall Frequency, Gumbel's Extreme Value Distribution Method, Log Pearson Type III

I. Introduction

Rainfall intensity-duration-frequency (IDF) curves are graphical exemplifications of the amount of water that falls within a given period of time in catchment areas [1]. The rainfall Intensity-Duration-Frequency (IDF) relationship is one of the most important tools in water resource engineering to assess the risk and vulnerability of water resource structure as well as for planning, design and operation. The establishment of such relationship was done as early as 1932 [1, 2]. Since then, many sets of relationships have been constructed for several parts of the globe. However, such relationships have not been accurately constructed in many developing countries [4]. IDF relationship is a mathematical relationship between the rainfall intensity I , the duration d , and the return period T [4, 3]. Indeed IDF curves allow the estimation of the return period of an observed rainfall event or conversely of the rainfall amount corresponding to a given return period for different aggregation times. Further studies were performed on rainfall analysis and regionalization of IDF curves for different regions [5].

In Bangladesh water logging and flood is a common problem during Monsoon period because of inadequate drainage system. In order to solve this problem new drainage design is needed where rainfall data of different duration is needed. But due to instrumental limitation these data were not available. In the present study, annual maximum rainfall series is considered for Rainfall Frequency Analysis (RFA). Rainfall in a region can be characterized if the intensity, duration and frequency of the diverse storms occurring at that place are known [6, 7, and 8]. The frequency-data for rainfalls of various durations, so obtained, can be represented by IDF curves, which give a plot of rainfall intensity versus rainfall duration and recurrence interval. In recent studies, various authors attempted to relate IDF relationship to the synoptic meteorological conditions in the area of hydrometric stations [1, 9]. The IDF curve for North-East region of Bangladesh was developed and observed that the rainfall data in this region follow Gumbel's Extreme Value Distribution [10]. The short duration rainfall IDF curve was developed for Sylhet with return period of 2, 5, 10, 20, 50, and 100 years [11]. The accuracy of IDF was improved curves by using long and short duration separation technique [12]. L-moments and generalized least squares regression method was applied for estimation of design rainfall depths and development of IDF relationships [13]. Pearson Type-III distribution was applied for modelling of short duration rainfall and development of IDF relationships for sylhet city in Bangladesh [14].

Atomic Energy Regulatory Board (AERB) guidelines described that the Order Statistics Approach (OSA) is the most appropriate method for determination of parameters of Gumbel and Frechet distributions [15]. In probability theory, extreme value distributions namely Gumbel, Frechet and Weibull are generally considered for frequency analysis of meteorological variables. In this present study Gumbel's Extreme Value Distribution method is used to develop IDF curves and equations. In this context, an attempt has been made to estimate the rainfall for different return periods for different durations of 'n' such as 10-min, 20-min, 30-min, 60-min, 120-min, 180-min, 360-min, 720-min, 1440-min adopting Gumbel and LPTIII distributions for development of IDF relationships for North-Western region of Bangladesh. Model performance indicators (MPIs) such as correlation coefficient (R) is used to analyze the performance of the developed IDF relationships by Gumbel and LPTIII distributions for estimation of rainfall intensity for the stations under study.

II. Data Collection And Methodology

For this study 24 hr daily rainfall data from year 1974 to 2014 was collected from Bangladesh Meteorological Department (BMD) for North-Western region. From the daily data maximum yearly rainfall data was used in the analysis. In North-Western region there are seven BMD stations (Bogra, Ishurdi, Dinajpur, Rangpur, Rajshahi, Sydpur, Chuadanga) which were taken into consideration to develop IDF curve for this region of Bangladesh.

2.1 Estimation of Short Duration Rainfall

Indian Meteorological Department (IMD) use an empirical reduction formula equation (1) for estimation of various duration like 1-hr, 2-hr, 3-hr, 5-hr, 8-hr rainfall values from annual maximum values. This formula was used to estimate the short duration rainfall from daily rainfall data in Sylhet city and found that this formula give the best estimation of short duration rainfall [11]. In this study this empirical formula equation (1) was used to estimate short duration rainfall of seven stations of North-Western region of Bangladesh.

$$P_t = P_{24} \sqrt[3]{\frac{t}{24}} \dots \dots \dots (1)$$

Where, P_t is the required rainfall depth in mm at t-hr duration, P_{24} is the daily rainfall in mm and t is the duration of rainfall for which the rainfall depth is required in hr.

2.2 Gumbel Theory of Distribution

The Gumbel method calculates the 2, 5, 10, 25, 50 and 100 year return intervals for each duration period and requires several calculations. Frequency precipitation P_T (in mm) for each duration with a specified return period T (in year) is given by the following equation:

$$P_T = P_{ave} + KS \dots \dots \dots (2)$$

Where K is Gumbel frequency factor given by:

$$K = -\frac{\sqrt{6}}{\pi} [0.5772 + \ln[\ln[\frac{T}{T-1}]]] \dots \dots \dots (3)$$

Where P_{ave} is the average of the maximum precipitation corresponding to a specific duration.

In utilizing Gumbel's distribution, the arithmetic average in equation (2) is used:

$$P_{ave} = \frac{1}{n} \sum_{i=1}^n P_i \dots \dots \dots (4)$$

Where P_i is the individual extreme value of rainfall and n is the number of events or years of record. The standard deviation is calculated by equation (5) computed using the following relation:

$$S = [\frac{1}{n-1} \sum_{i=1}^n (P_i - P_{ave})^2]^{1/2} \dots \dots \dots (5)$$

Where S is the standard deviation of P data. The frequency factor (K), which is a function of the return period and sample size, when multiplied by the standard deviation gives the departure of a desired return period rainfall from the average. Then the rainfall intensity, I_T (in mm/hr) for return period T is obtained from:

$$I_T = \frac{P_t}{T_d} \dots \dots \dots (6)$$

Where T_d is duration in hours.

The frequency of the rainfall is usually defined by reference to the annual maximum series, which consists of the largest values observed in each year. An alternative data format for rainfall frequency studies is that based

on the peak-over threshold concept, which consists of all precipitation amounts above certain thresholds selected for different durations. Due to its simpler structure, the annual-maximum-series method is more popular in practice [16].

2.3 Log Pearson type III

The LPT III probability model is used to calculate the rainfall intensity at different rainfall durations and return periods to form the historical IDF curves for each station. LPT III distribution involves logarithms of the measured values. The mean and the standard deviation are determined using the logarithmically transformed data. In the same manner as with Gumbel method, the frequency precipitation is obtained using LPT III method. The simplified expression for this distribution is given as follows:

$$P^* = \log(P_i) \dots\dots\dots (7)$$

$$P_T^* = P_{ave}^* + K_T S^* \dots\dots\dots (8)$$

$$P_{ave}^* = \frac{1}{n} \sum_{i=1}^n P^* \dots\dots\dots (9)$$

$$S^* = \left[\frac{1}{n-1} \sum_{i=1}^n (P^* - P_{ave}^*)^2 \right]^{1/2} \dots\dots\dots (10)$$

Where P_T^* , P_{ave}^* , S^* are desired rainfall peak value for a specific frequency, average of maximum precipitation corresponding to a specific duration, standard deviation of P^* value respectively. K_T is the Pearson frequency factor which depends on return period (T) and skewness coefficient (Cs). The skewness coefficient, C_s , is required to compute the frequency factor for this distribution. The skewness coefficient is computed by equation (11) [2&17]

$$C_s = \frac{n \sum_{i=1}^n (P_i^* - P_{ave}^*)^3}{(n-1)(n-2)(S^*)^3} \dots\dots\dots (11)$$

K_T values can be obtained from tables in many hydrology references; for example reference [2]. By knowing the skewness coefficient and the recurrence interval, the frequency factor, K_T for the LPT III distribution can be extracted. The antilog of the solution in equation (7) will provide the estimated extreme value for the given return period.

2.4 Derivation of IDF equation

To derive an equation for calculating the rainfall intensity (I) for the regions of interest, there are some required steps for establishing an equation to suit the calculation of rainfall intensity for a certain recurrence interval and specific rainfall period which depends mainly on the results obtained from the IDF curves. Two approaches were tried to estimate the equation parameters.

By applying the logarithmic conversion, where it is possible to convert the equation into a linear equation, thus to calculate all the parameters related to the equation. The following steps are followed:

1. Convert the original equation in the form of power-law relation as follows:

$$I = \frac{CT_r^m}{T_d^e} \dots\dots\dots (12)$$

By applying the logarithmic function to get

$$\log I = \log K - e \log T_d \dots\dots\dots (13)$$

Where

$$K = CT_r^m \dots\dots\dots (14)$$

And e represents the slope of the straight line.

2. Calculate the natural logarithm for (K) value found from Gumbel method or from LPTIII method as well as the natural logarithmic for rainfall period T_d .
3. Plot the values of $(\log I)$ on the y-axis and the value of $(\log T_d)$ on the x axis for all the recurrence intervals for the two methods.
4. From the graphs (or mathematically) find the value of (e) for all recurrence intervals. Then it was found out the average value of e value, e_{ave} , by using the following equation:

$$e_{ave} = \frac{\sum e}{n} \dots\dots\dots (15)$$

Where n represents recurrence intervals (years) value noted as Tr .

5. From the graph, it was found $\log K$ values for each recurrence interval where $\log K$ represents the Y-intercept values as per Gumbel method or LPTIII method. Then convert equation (14) into a linear equation by applying the natural logarithm to become:

$$\log K = \log C + m \log Tr \dots\dots\dots (16)$$

6. Plot the values of $\log K$ on the y-axis and the values of $\log Tr$ on the x-axis to find out the values of parameters c and m as per Gumbel method or LPTIII where m represents the slope of the straight line and c represents the (anti log) for the y intercept.

In another approach Estimation of the equation parameters by using nonlinear regression analysis: Using the Solver function of the ubiquitous spreadsheet program Microsoft Excel, which employs an iterative least squares fitting routine to produce the optimal goodness of fit between data and function. The R^2 value calculated is designed to give the user an estimate of goodness of fit of the function to the data.

2.5 Goodness of fit test

The purpose of this test is to decide how good is a fit between the observed frequency of occurrence in a sample and the expected frequencies obtained from the hypothesized distributions. A goodness of fit test between observed and expected frequencies is based on the chi-square quantity, which is expressed as,

$$\chi^2 = \sum_{i=1}^k (O_i - E_i)^2 / E_i \dots\dots\dots (17)$$

where χ^2 is a random variable whose sampling distribution is approximated very closely by the chi-square distribution. The symbols O_i and E_i represent the observed and expected frequencies, respectively, for the i -th class interval in the histogram. The symbol k represents the number of class intervals. If the observed frequencies are close to the corresponding expected frequencies, the χ^2 value will be small, indicating a good fit; otherwise, it is a poor fit. A good fit leads to the acceptance of null hypothesis, whereas a poor fit leads to its rejection. The critical region will, therefore, fall in the right tail of the chi-square distribution. For a level of significance equal to α , the critical value is found from readily available chi-square tables and $\chi^2 >$ constitutes the critical region [19].

III. Result And Discussion

Fig 1-2 show results of the IDF curves obtained by Gumbel and LPT III methods for North-Western region. It was shown that there were small differences between the results obtained from the two methods, where Gumbel method gives slightly higher results than the results obtained by Log Pearson Type III. This is shown also from parameters of the derived equation for calculating the rainfall intensity using the two methods.

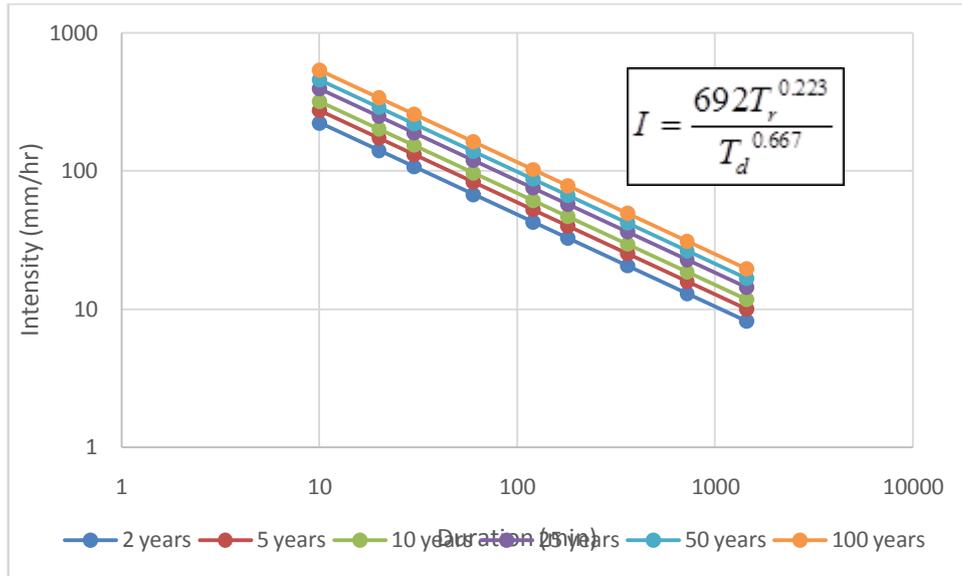


Figure 1: IDF curve by Gumbel method at North-Western region

Parameters of the selected IDF formula were adjusted by the method of minimum squares, where the goodness of fit is judged by the correlation coefficient. The results obtained showed that in all the cases the correlation coefficient is very high, and ranges between 0.998 and 0.987, except few cases where it ranges between 0.986 and 0.978 when using LPT III at 50 and 100 years. This indicates the goodness of fit of the formulae to estimate IDF curves in the region of interest. For each region the results are given as the mean value of the points results. Table 1 shows the parameters values obtained by analyzing the IDF data using the two methods and those are used in deriving formulae for the two regions.

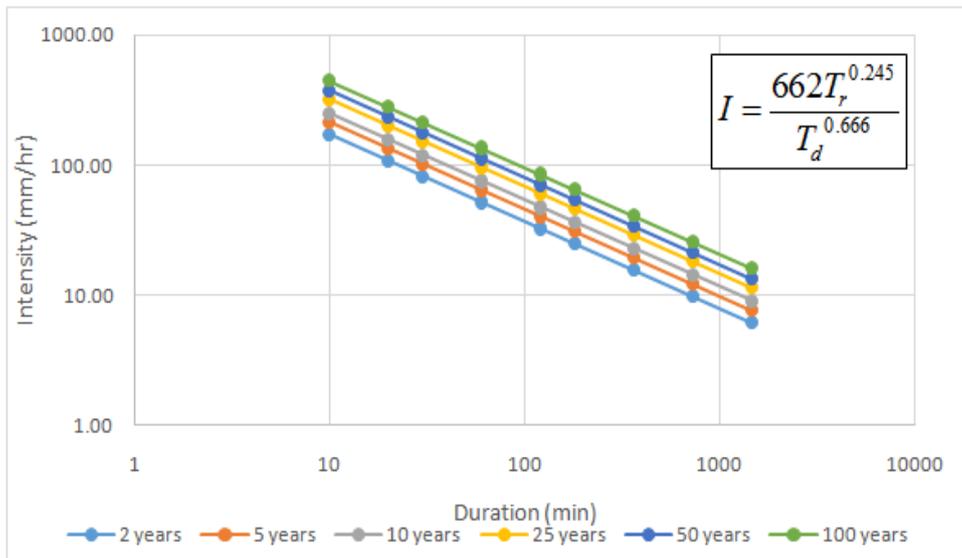


Figure 2: IDF curve by LPTIII method at North-Western Region

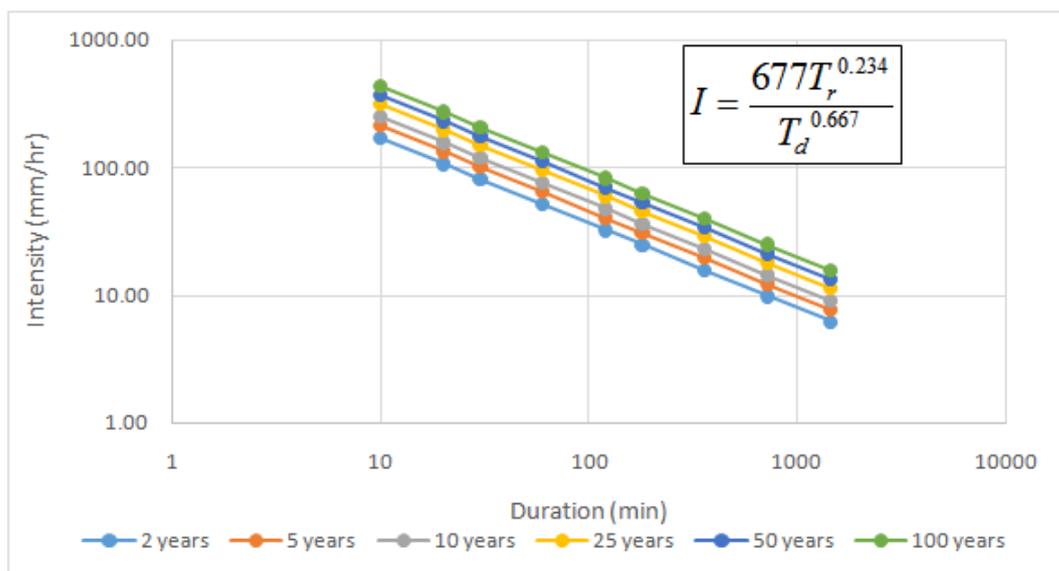


Figure 3: IDF curve by average at North-Western Region

Table 1: IDF parameters values used in deriving formula

| Region | Parameter | Gumbel Method | Log Pearson Type III |
|----------------|-----------|---------------|----------------------|
| North- Western | c | 692 | 662 |
| | m | 0.223 | 0.245 |
| | e | 0.667 | 0.666 |

Also, goodness-of-fit tests were used to choose the best statistical distribution among those techniques. Results of the chi-square goodness of fit test on annual series of rainfall are shown in Table 2. As it is seen most of the data fit the distributions at the level of significance of $\alpha = 0.05$, which yields $\chi_{cal} < 3.84$. Only the data for 10, 20, 30 min do not give good fit using Gumbel method distribution. Also the data for 10, 20 min using LPTIII method do not give good fit.

Table 2: Results of chi-square goodness of fit test on annual maximum rainfall

| Region | Distribution | Duration (min) | | | | | | | | |
|---------------|--------------|----------------|------|------|------|------|------|------|------|------|
| | | 10 | 20 | 30 | 60 | 120 | 180 | 360 | 720 | 1440 |
| North-Western | Gumbel | 9.79 | 6.15 | 4.69 | 2.95 | 1.85 | 1.41 | 0.89 | 0.56 | 0.35 |
| | LPTIII | 7.68 | 4.82 | 3.68 | 2.31 | 1.45 | 1.11 | 0.69 | 0.44 | 0.27 |

IV. Conclusions

Since Bangladesh has different climatic conditions from region to region, a relationship of R-IDF for each region has to be obtained to estimate rainfall intensities for different durations and return periods ranging between 2 and 100 years. It was found that Gumbel method gave some larger rainfall intensity estimates compared to LPT III distribution. In general, the results obtained using the two approaches are very close at most of the return periods and have the same trend. The results obtained from that work are consistent with the results from previous studies done in some parts of the study area. The parameters of the design rainfall intensity for a given period of recurrence interval were estimated for this region. The results showed that in all the cases data fitted the formula with a correlation coefficient greater than 0.97. This indicates the goodness of fit of the formulae to estimate IDF curves in the region of interest for durations varying from 10 to 1440 min and return periods from 2 to 100 years. The Chi-square test was used on one hand to examine the combinations or contingency of the observed and theoretical frequencies, and on the other hand, to decide about the type of distribution which the available data set follows. The results of the chi-square test of goodness of fit showed that in all the durations the null hypothesis that the extreme rainfall series have the Gumbel distribution is acceptable at the 5% level of significance. Only few cases in which the fitting was not good obtained by using the LPT III distribution. Although the Chi-square values are appreciably below the critical region using Gumbel distribution and few values are higher than the critical region using LPT III distribution, it is difficult to say that one distribution is superior to the other. Further studies are recommended whenever there will be more data to verify the results obtained or update the IDF curves.

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