IOSR Journal of Economics and Finance (IOSR-JEF)
e-ISSN: 2321-5933, p-ISSN: 2321-5925.Volume 16, Issue 6 Ser. 1 (Nov. – Dec. 2025), PP87-89
www.iosrjournals.org

Programming Problem Objective Function and Constraints Are Posynomial

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ABSTRACT
In this chapter to explain a nonlinear problem in optimization technique and also explain algorithm and solve the problem. Geometric Programming is a mathematical optimization technique used to minimize a special type of function called a posynomial, which is a sum of monomials products of variables with real exponents and positive coefficients. Although geometric programmingproblems appears nonlinear, it can be transformed into a convex optimization problem through a logarithmic change of variables, making it easier to solve.The paper is describing an older method made by Avriel's team. This method is good at solving a particular kind of optimization problem that uses posynomials

KEY WORDS:*Inequality, recursive, unrestricted, primalfunction, dual function. Posynomial, monomials products.*

Date of Submission: 05-11-2025 Date of Acceptance: 16-11-2025

I. INTRODUCTION

Geometric programming is a type of mathematics method used to solve complex problems where you want to find the best solution, like the smallest cost or highest performance. Since the1960s**,** it has been used in many fields, mechanical and civil engineering to design strong and efficient machines, buildings, and structures chemical engineering to make chemical processes work better and more safely probability and Statistics to solve problems involving chance and data, finance and economics, to make better investment or budgeting decisions, control theory, to control systems like robots, vehicles, or industrial machines smoothly and efficiently, circuit design, to design faster and smaller electronic circuits like in phones and computers. Information theory, Coding, and Signal processing, to improve how data is sent, stored, and understood like in digital communication, wireless networking, to make wireless systems like wi-fi or mobile networks faster and more reliable of monomial terms with positive coefficients and positive variables. Each monomial is a product using this type of formula allows engineers to apply mathematical techniques based on geometric means and other geometric ideas using this type of formula allows engineers to apply mathematical techniques based on geometric means and other geometric concepts.

The main difference is that linear programmingworks with problems where everything both the goal objective and the rules constraints in a straight-line linear form. On hand, geometric programminghandles more complicated problems that aren't linear at first but can be changed into a simpler convex form by using special mathematics tricks, like changing the variables. Geometric Programming was developed to help solve complex engineering design problems. When engineers design something, they often have to choose between manydifferent options. Their goal is to find the best possible balance usually a design that does its job well but also costs the least to do this; engineers adjust certain parts of the design called adjustable parameters while keeping others fixed. Three main benefits of using Geometric Programming, it gives a clear overall view of which design factors are more orless important in this method, all parts of the design are described using special kinds of formulas called generalized positive polynomials, which include only positive values.

GeometricProgramming is a mathematical optimization technique used to minimize a special type of function called a posynomial, which is a sum of monomialsproducts of variables with real exponents and positive coefficients. Although geometric programmingproblems appears nonlinear, it can be transformed into a convex optimization problem through a logarithmic change of variables, making it easier to solve and ensuring a global optimum. Geometric programmingproblems also has a strong duality property, allowing the original primalproblem to be converted into a simpler dual problem with linear constraints, whose solution provides insights into the original problem. This method is widely applied in engineering design fields such as electrical circuit design, structural optimization, and chemical engineering, where costs and resources often follow posynomial relationships

METHODOLOGY OF NONLINEARPROGRAMMING PROBLEM

In an engineering design the total cost$G$ is a sum of component costsu_j. Thus

$$G = u_1 + u_2 + \cdots + u_n$$

Generally, the component costs are expressed as

$$u_j(x) = c_j x_1^{\quad a_{1j}} x_2 a_{2j} \ldots x_m a_{mj}, (j = 1,2,\ldots,n)$$

where $c_j > 0, x_i (i == 1,2,\ldots,m) > 0$ and a_{ij} $(i = 1,2,\ldots,m; j = 1,2,$, n) are unrestricted in sign. The function G is usually referred to as posynomial.

To obtain the geometric-arithmetic mean inequality, we proceed

$$\text{Max. } f(\text{x}) = \left(\frac{C}{n}\right)^n$$

It then follows that Using the method of recursive equation we find that

$$f(x) \leqslant \left(\frac{C}{n}\right)^n$$

$$\prod_{j=1}^{n} x_j \leqslant \left(\sum_{j=1}^{n} x_j/n\right)^*$$

Taking the nth root of each side, the inequality becomes

$$\left(\prod_{j=1}^{n} x_j\right)^{\frac{1}{n}} \leqslant \frac{1}{n}\sum_{j=1}^{n} x_j$$

the geometric-arithmetic mean inequality and is fundamental for the development of geometric programming, so the unconstrained geometric programming problem is defined as

Minimize $f(x) = \sum_{j=1}^{n} r_j \prod_{i=1}^{m} x_i^{a_{ij}}$ where c_j $(j = 1,2,\ldots,n) > 0,$ $x_i (i = 1,2,\ldots,m) > 0$ and a_{ij} (i $= 1,2,\ldots,m; j = 1,\ldots,n)$ are unrestricted in sign. Now using the geometric-arithmetic mean inequality the objective function defined above can be re-written as

$$u_1 + u_2 + \cdots + u_n \geqslant \left(\frac{u_1}{\zeta_1}\right)^{\zeta_1} \left(\frac{u_2}{\zeta_2}\right)^{\zeta_3} \ldots \left(\frac{u_n}{\zeta}\right)^{\zeta_n}$$

where $u_s = u_j(x)$ for $j = 1,2,\ldots,n$ and the weights $\zeta_j (j = 1,2,\ldots,n)$ satisfy the relation

$$\zeta_1 + \zeta_2 + \cdots + \zeta_n = 1$$

The left-hand side of the inequality is known as primal function whereas the right-hand side is called the pre dual function. Since $u_j = c_j \prod_{i=1}^{m} x_i^{\quad a_{ij}}$, the predual function can be written as

$$= \prod_{j=1}^{n} \left(\frac{c_j}{\zeta_j}\right)^{\zeta_j} \prod_{i=1}^{m} x_i^{\sum_{j=1}^{n} a_{ij}\zeta_j}$$

Now, making use of the conditions $\sum_{j=1}^{n} a_{lj}\zeta_j \triangleq 0, (i = 1,2,\ldots,\text{m})$ together with the normality condition $\sum_{j=1}^{n} \zeta_j = 1$, the above expression reduces to

$$u_1 + u_2 + \cdots + u_n \geqslant \left(\frac{c_1}{\zeta_j}\right)^{\zeta_j} \left(\frac{c_2}{\zeta_j}\right)^{\zeta_j} \ldots \left(\frac{c_n}{\zeta_n}\right)^{\zeta_n}$$

$$f(x) \geqslant \phi(\zeta),$$

where ζ is a vector with component $\zeta_1, \zeta_2, \ldots, \zeta_n$.

$$\text{Minimum } f(\text{x}) = \text{ maximum } \phi(\zeta)$$

The original problem of minimization is referred to as pr$i$mal one and the related problem of maximization as its dual. The method of solving unconstrained geometric programming problem is well defined.

Minimize $f(\mathbf{x}) = c_1 x_1 x_2 x_3 + c_3 x_1 x_2^{\quad -1} + c_3 x_2 x_3^{\quad -2} + c_4 x_1^{\quad -3} x_2$
where $c > 0, x_i > 0$;

Let$u_1 = c_1 x_2 x_2 x_3, u_2 = c_3 x_1 x_2^{\quad -1}, u_3 = c_3 x_2 x_3^{\quad -2}$ and

$$u_4 = c_4 x_1^{-3} x_3$$

Substituting the value of u_1, u_2, u_3 , u_4 in the geometric-arithmetic mean inequality then this inequality gives

$$\prod_{j=1}^{4} \left(\frac{c_i}{\zeta_j}\right)^{\zeta_j} x_{1\,(}\,\zeta_1 + \zeta_2 - 3\zeta_3 \Big) \;\; {}_{x_2}(\zeta_1 - \zeta_2 + \zeta_3) x_3^{(\zeta_1 - 2\zeta_2 + \zeta_4)} < f(x)$$

Let us set the powers of x_8 for each i equal to zero. Then the right-hand side of the above inequality will become independent of x_i. For this we require $\zeta_1 + \zeta_2 - 3\zeta_4 = 0, \zeta_1 - \zeta_2 + \zeta_3 = 0$ and $\zeta_1 - 2\zeta_3 + \zeta_4 = 0$

Now, the orthogonality-normality conditions

$$\zeta_1 = \frac{1}{5}, \zeta_3 = \frac{2}{5}, \zeta_3 = \frac{1}{5} \text{ and } \zeta_4 = \frac{1}{5}.$$

then

$$\sum_{j=1}^{4} u_j = N \sum_{j=1}^{4} \zeta_1 = N$$

Minimum

$$f(\mathbf{x}) = f(\mathbf{x}_0) \equiv N$$

Using these values the geometric-arithmetic mean inequality reduces to

$$\left(\frac{c_2}{1/5}\right)^{\frac{1}{5}} \left(\frac{c_2}{2/5}\right)^{\frac{2}{5}} \left(\frac{c_3}{1/5}\right)^{\frac{1}{5}} \left(\frac{c_4}{1/5}\right)^{\frac{1}{5}} < f(x)$$

$$u_j = \zeta_j f(x_0)$$

we shall define new variables y_i by substituting

$$y_i = \log x_i, i = 1,2,3$$

We have

These are four equations in three unknowns and are not linearly independent. Since any three of them are linearly independent, the values of y_i and hence of x_i are uniquely determined. Now substituting $c_1 = 16, c_2 = 4, c_3 = 2$ and $c_8 = 8$,the solution of these equations gives us $x_2 = 1, x_3 = 0.5$ and $x_3 = 0$ which gives us to complete solution for the given problem The original problem of minimization is referred to as primal one and the related problem of maximization as its dual. The method of solving unconstrained geometric programming problem is well defined. They try to find the best settings for these adjustable parts so that the final design is optimal meaning it works well and is as low-cost as possible to help with this Geometric Programming was created as a quick and systematic method for designing cost-effective systems. In this method, all parts of the design are described using special kinds of formulas called generalized positiv**e** polynomials, which include only positive values

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