

A study on the effect of historical volatility using two option pricing models

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Abstract: This paper analyses the Black-Scholes' and Heston Option Pricing Model. We discuss the concept of historical volatility in the two Models. We compare the two models for the parameter –'Volatility'. A mathematical tool, UMBRAE (Unscaled Mean Bounded Relative Absolute Error) is used to compare the two models for historical volatility while pricing European call options. Real data from NSE (National Stock Exchange) is considered for three different sectors like- Banking, Automobiles and, Pharmaceuticals for comparison through Moneyiness (which is defined as the percentage difference of stock price and strike price) and Time-To-Maturity. Mathematical software – Matlab is used for all mathematical calculations.

Keywords: European call option, Black-Scholes' model, Heston Model, Moneyiness, Time-to-maturity, Implied Volatility.

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I. Introduction

Financial mathematics is the subject of high interest in recent days. Derivative is the financial instrument and an option is a derivative with a particular payoff function that depends on underlying assets. Options are of many types- American, European e.t.c. European options are most widely used in Indian stock market. European options can be exercised only at expiration. There are different types of models available for pricing such options, Black-Scholes' is one of such type. Black-Scholes' model is used in India for pricing European options to calculate theoretical premium value.

Many mathematicians worked on the valuation of the options since early 60's. It was in 1973; Fischer Black and Myron Scholes developed the option pricing formula for calculating theoretical value of European call options. Historical volatility is used as a parameter with various assumptions. Though it had many assumptions but was considered as the most favourable for calculation of theoretical values. In 1993 (Heston) proposed a stochastic volatility model which used assumption that the asset variance v_t follows a mean reverting Cox-Ingersoll-Ross process.

Black-Scholes' model, is formulated with the assumption that volatility remains constant throughout the option's life which is not practical with the real fluctuating market. Heston model considers stochastic volatility and so is more favourable to the practical financial market. Stochastic volatility also removes excess kurtosis and asymmetry that appears in Black-Scholes' model.

UMBRAE (Unscaled Mean Bounded Relative Absolute Error) [2] is a measure of error calculation in the model, it is helpful in removing symmetric and bounded error during forecasting. Here, we use Naive method as the benchmark for forecasting UMBRAE. We observe the performance of Heston Model and Black-Scholes' Model in three different sectors like; Banking, Pharmaceuticals and Automobiles under parameters like time-to-maturity and moneyiness (which is defined as the percentage difference of stock price and strike price) for comparing the premium values for European call option.

Volatility is the most important parameter of option trading. It plays a vital role in the calculation of theoretical premium value. Volatilities are of two types, mainly Historical and Implied Volatility are used for option pricing.

Historical volatility is the annualized standard deviation of the past stock data. It measures the price change in stock over per year.

Implied volatility is derived from Option model formula. It shows the future probability of the volatile market.

This paper is organised as follows; starting with the basic terminologies and volatilities. We discuss the Black-Scholes' model, Heston Model and, Methodology. At last, we present the result and discussion of comparison of the two models for different Indian stock data.

II. Literature Review

Study in financial markets was started in 1960's, by Sprenkle (1961), Ayes (1963), A. James Boness (1964), Samuelson (1965), Baumol, Malkiel and Quandt (1966), Chen (1970) etc. The options pricing formula for European call Options was provided by Black F. and Scholes M. (1973). This gives the theoretical value of options pricing, which is also helpful in corporate bonds and warrants. This formula uses many assumptions, though it is most widely used in Indian stock market. However, Black-Scholes model considers volatility as constant quantity for calculation, which is not favourable with the practical market conditions.

Further, different mathematicians worked on modified models by changing the different conditions in the initial basic model. Recently, Singh and Gor (2020a) studied the B-S options pricing model and the model where underlying stock returns follow the Gumbel distribution at maturity, and compared the result for actual market data. Singh and Gor (2020b) also compared the B-S model to a different model where stock returns follow truncated Gumbel distribution. Chauhan and Gor (2020b) studied the modified truncated Black-Scholes model and compared the result with the original B-S model.

Modified B-S model works better than the original B-S model. Another such model which uses volatility as stochastic quantity was introduced in 1993. Heston (1993) proposed a stochastic volatility model for European Call options. It considers volatility as the stochastic quantity. The stochastic volatility model works better in the real-world market while removing excess of skewness and kurtosis in the model. It is also favourable with the practical market conditions.

Shinde (2012) explains the basic terminologies of the Black-Scholes (B-S) option pricing model in the most familiar and easy way. Crisostomo (2014) and Yuan Yang (2013) derived the Heston characteristic function and formula for call value and compared the Black-Scholes and Heston option pricing formula for different parameters. They studied the graphical comparison among different parameters of option pricing model. Ziqun (2013) introduced the concept of moneyness and compared the two models for different parameters of moneyness and Time-To-Maturity.

Santra (2017) used Matlab software for calculating theoretical call value of Black-Scholes and Heston models. He also provided a detailed explanation and compared the two model for different option of moneyness. Chen et al. (2017) provided a new accuracy measure for calculating error for forecasting methods. We have used this accuracy measure to compare the two options pricing models in real market data for different options of moneyness and time-to-maturity. Sisodia and Gor (2020) worked on estimating the relevancy of option pricing models for European call-put options. They compared the B-S model and Heston model for selective stocks and analysed the result for different options of moneyness and time-to-maturity.

III. Basic Concepts [7]

a. Option: An option is defined as the right, but not the obligation, to buy (call option) or sell (put option) a specific asset by paying a strike price on or before a specific date.

(i) *Call option:* An option which grants its holder the right to buy the underlying asset at a strike price at some moment in the future.

(ii) *Put option:* An option which grants its holder the right to sell the underlying asset at a strike price at some moment in the future.

b. Expiration Date/ Time-to-maturity: The date on which an option right expires and becomes worthless if not exercised. In European options, an option cannot be exercised until the expiration date.

c. Strike Price: The predetermined price of an underlying asset is called strike price.

d. Stochastic Process: Any variable whose value changes over time in an uncertain way is said to follow a stochastic process.

e. Stochastic Volatility: Volatility is a measure for variation of price of a stock over time. Stochastic in this sense refers to successive values of a random variable that are not independent.

f. Geometric Brownian Motion: A continuous time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion.

g. Moneyness: It is the relative position of the current price of an underlying asset with respect to the strike price of a derivative, most commonly a call/put option.

h. Black-Scholes Inputs / Parameters:

There are six basic parameters used in pricing an option in Black-Scholes model.

They are as follows:

- Underlying stock price
- Strike price
- Time to expiration
- Interest rate
- Volatility

Volatility - It is the standard deviation of the continuously compounded return of the stock. In other words, we can say that volatility reflects fluctuations in the market. It is one of the important variables of options pricing. For both Call and Put options, options' price increase as volatility increases. There are mainly two types of volatility in the market – Historical and Implied.

Historical volatility is calculated from past data in the market. Implied volatility is derived from options' prices or options' pricing model. It is also available on a daily basis on the website of stock exchanges. It is denoted by the symbol σ (sigma), in the model formula.

Implied Volatility: Implied Volatility is a metric that captures the market's view of the likelihood of changes in a given security's price. The option's premium price component changes as the expectation of volatility changes over time. It helps to predict future market fluctuations. It shows the move of the market, but not the direction. It is denoted by the symbol σ (sigma), commonly expressed as standard deviation over time. High implied volatility results in options with higher value, and vice-versa.

IV. The Black-Scholes' Model [7]

This model is based on certain assumptions;

- Stock pays no dividends.
- Option can only be exercised upon expiration.
- Random walk.
- No transaction cost.
- Interest rate remains constant.
- Stock returns are normally distributed, thus the volatility is constant over time.

In 1973, Fischer Black and Myron Scholes proposed a model for European Call option based on Geometric Brownian motion.

$$dS_t = \mu S_t dt + \sigma_t S_t dW_t$$

where, S_t is the price of the asset, μ is the drift (constant), σ_t is the return volatility(constant) and W_t is the Brownian motion. Black [1] uses the risk neutral probability rather than the true probability to evaluate the price of an option.

The risk neutral dynamics on asset is given by;

$$dS_t = r S_t dt + \sigma_t S_t dW_t$$

where, r is the risk free rate.

The solution to the above stochastic differential equation is a Geometric Brownian Motion;

$$S_t = S_0 \exp \left[\sigma W_t + \left(\mu - \frac{\sigma^2}{2} \right) t \right]$$

The log of which is a Geometric Brownian Motion (GBM) model for stock prices.

$$\ln \left(\frac{S_t}{S_0} \right) = \sigma W_t + \left(\mu - \frac{\sigma^2}{2} \right) t$$

where, R.H.S. equation is a normal random variable whose mean is $\left(\mu - \frac{\sigma^2}{2} \right) t$ and variance is $\sigma^2 t$.

The Black-Scholes Formula for European call price is,

$$C = S_0 N(d_1) - Ke^{-rt} N(d_2)$$

$$\text{where, } d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)t}{\sigma\sqrt{t}} \text{ and } d_2 = d_1 - \sigma\sqrt{t}$$

K is the strike price, S_0 today's stock price, t is time to expiration, r riskless interest rate (constant), σ is volatility of stock (constant).

V. The Heston Model [12]

In 1993, Heston proposed a Stochastic Volatility Model. Consider at time t the underlying asset S_t which obeys a diffusion process with volatility being treated as a latent stochastic process of Feller as proposed by Cox, Ingersoll and Ross:

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_t^1$$

$$dV_t = k[\theta - V_t]dt + \sigma\sqrt{V_t}dW_t^2$$

where, W_t^1 and W_t^2 are two correlated Brownian motion with a correlation coefficient given by $\rho > 0$:

$$dW_t^1 dW_t^2 = \rho dt$$

where, S_t is the price of the asset, r is the risk free rate, V_t is the variance at time t , $\theta > 0$ is the long term mean variance, $k > 0$ is variance mean-reversion speed, $\sigma \geq 0$ is the volatility of the variance.

The price of a European call option can be obtained by using the following equation:

$$C = S_0 \Pi_1 - e^{-rt} K \Pi_2$$

where, Π_1 is the delta of the option and Π_2 is the risk-neutral probability of exercise (i.e. when $S_t > K$) For $j=1, 2$ the Heston characteristic function is given as;

$$f_j(x, v, \tau; \emptyset) = e^{C(\tau; \emptyset) + D(\tau; \emptyset)v + i\emptyset x}$$

where,

$$C(\tau; \emptyset) = r\emptyset i\tau + \frac{a}{\sigma^2} \left\{ (b_j - \rho\sigma\emptyset i + d)\tau - 2\ln \left[\frac{1 - ge^{d\tau}}{1 - g} \right] \right\}$$

$$D(\tau; \emptyset) = \frac{b_j - \rho\sigma\emptyset i + d}{\sigma^2} \left[\frac{1 - e^{d\tau}}{1 - ge^{d\tau}} \right]$$

$$g = \frac{b_j - \rho\sigma\emptyset i + d}{b_j - \rho\sigma\emptyset i - d}$$

$$d = \sqrt{(\rho\sigma\emptyset i - b_j)^2 - \sigma^2(2u_j\emptyset i - \emptyset^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = k\theta, b_1 = k - \rho\sigma, b_2 = k$$

The characteristic functions can be inverted to get the required probabilities

$$\Pi_j(x, v, T; \ln[K]) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[\frac{e^{-i\emptyset \ln[K]} f_j(x, v, T; \emptyset)}{i\emptyset} \right] d\emptyset$$

VI. Methodology

Data: The data has been collected only for European call option from various sectors like Banking, Pharmaceuticals and, Automobile from the website of National stock Exchange of India.

For Banking ; Axis Bank, Federal Bank, HDFC Bank and, Kotak Mahindra Bank. Automobile companies; Tata Motors, TVS Motors, Maruti Udyog, Hero Honda Motors and, Mahindra & Mahindra. Pharmaceutical companies; Sun Pharmaceuticals Limited, Lupin Limited, Dr. Reddy's laboratories, Cipla limited, Zydus Cadila Healthcare limited from the website of National stock Exchange of India. The period from November 22 to November 30, 2018 has been considered for calculation purpose.

Parameter: The option moneyness is defined as the percentage difference between the current underlying price and the strike price:

- Moneyness (%) = $S/K + 1$

The result has been divided in terms of moneyness and time-to-maturity.

- ATM - At the money, A call option is at the money if the strike price is the same as the current underlying stock price.
- ITM – In the money, A call option is in the money when the strike price is below the underlying stock price.

- OTM – Out of the money, A call option is out of the money when the strike price is above the underlying stock price.
- UMBRAE (Unscaled Mean Bounded Relative Absolute Error) = $\frac{MBRAE}{1-MBRAE}$

$$MBRAE = \frac{1}{n} \sum_{t=1}^n (BRAE)$$

$$BRAE = \frac{|e_t|}{|e_t| + |e_t^*|}$$

$$e_t = y_t - f_t$$

$$e_t^* = y_t - f_t^*$$

where, y_t is observed model price, f_t is the actual market forecasted value and, f_t^* is the market forecasted value Naive Method.

- We have used Matlab function *bsm_price* and run the model to calculate the European call option value [7];
- Risk –Free Interest rate: It is the rate at which we deposit or borrow cash over the life of the option. Call option value increases as the risk-free rate increases. It takes value 0.05 throughout the function.
- Volatility: It is the standard deviation of the continuously compounded return of the stock. Call option value is higher for higher the volatility.
- We have used Matlab function *heston_chfun* for the Heston characteristic function and *heston_price* for the calculation of European call option value [7];
- Initial Variance: Bounds of 0 and 1 have been used.
- Long-term Variance: Bounds of 0 and 1 have been used.
- Correlation: Correlation between the stochastic processes takes values from -1 to 1.
- Volatility of Variance: It exhibits positive values. Since the volatility of assets may increase in short term, a broad range of 0 to 5 will be used.
- Mean-Reversion Speed: This will be dynamically set using a non-negative constraint (Feller, 1951). The constraint $2k\theta - \sigma^2 > 0$ guarantees that the variance in CIR process is always strictly positive.
- Initial Variance = 0.28087
- Long-term Variance = 0.001001
- Volatility of Variance = 0.1
- Correlation Coefficient = 0.5
- Mean Reversion Speed = 2.931465

VII. Result

Theoretical premium value is calculated for both the models for 14 different stocks. UMBRAE (Unscaled Mean Bounded Relative Absolute Error) is calculated and compared for different option of moneyness and time-to-maturity. Historical volatility is used as volatility parameter in both Black-Scholes' and Heston Option pricing model.

| Table No. 1-Axis Bank | | | |
|------------------------------|--|------------|------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=560 | ATM, K=610 | OTM, K=690 |
| Black-Scholes' | 0.16 | 3.62 | 0.55 |
| Heston | 3.29 | 6.38 | 1.32 |

According to Table-1, Black-Scholes' Model outperforms Heston Model for all ITM, ATM and OTM option of moneyness giving lesser error value.

| Table No. 2-Federal Bank | | | |
|---------------------------------|--|-----------|------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=72.50 | ATM, K=80 | OTM, K=105 |
| Black-Scholes' | 1.24 | 1.74 | 0.36 |
| Heston | 0.95 | 0.89 | 0.1 |

According to Table-2, Heston Model outperforms Black-Scholes' Model for all three option of moneyness giving lesser error value.

| Table No. 3-HDFC Bank | | | |
|------------------------------|--|-------------|-------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=1660 | ATM, K=1860 | OTM, K=2200 |
| Black-Scholes' | 2.73 | 0.82 | 1.13 |
| Heston | 0.21 | 1.05 | 2.76 |

According to Table-3, Black-Scholes' Model outperforms Heston Model for ATM and OTM option of moneyness giving lesser error value while, Heston model outperforms Black-Scholes' for ITM option.

| Table No. 4-Kotak Mahindra Bank | | | |
|--|--|-------------|-------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=1120 | ATM, K=1160 | OTM, K=1300 |
| Black-Scholes' | 0.05 | 1.2 | 1.13 |
| Heston | 2.29 | 1.52 | 1.17 |

According to Table-4, that Black-Scholes' Model outperforms Heston Model for ITM option of moneyness giving lesser error value while in the remaining options no big difference is observed. Thus, the performance of both the models is almost same.

| Table No. 5-Cadila Healthcare limited | | | |
|--|--|------------|------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=350 | ATM, K=360 | OTM, K=440 |
| Black-Scholes' | 1.91 | 1.6 | 1.29 |
| Heston | 6.17 | 2.84 | 0.02 |

According to Table-5, Black-Scholes' Model outperforms Heston Model for ITM and ATM option of moneyness giving lesser error value while, Heston Model outperforms better in OTM option

| Table No. 6-Cipla limited | | | |
|----------------------------------|--|------------|------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=500 | ATM, K=520 | OTM, K=650 |
| Black-Scholes' | 0.46 | 1.63 | 0.5 |
| Heston | 21.32 | 2.35 | 0.48 |

According to Table-6, Black-Scholes' Model outperforms Heston Model for ITM and ATM option of moneyness giving lesser error value while, no difference is observed in OTM option.

| Table No. 7-Lupin Pharmaceutical limited | | | |
|---|--|------------|-------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=800 | ATM, K=840 | OTM, K=1040 |
| Black-Scholes' | 0.03 | 22.15 | 1.01 |
| Heston | 0.66 | 1.65 | 0.48 |

According to Table-7, Black-Scholes' Model outperforms Heston Model only for ITM option of moneyness giving lesser error value while, in ATM and OTM options Heston models results are better.

| Table No. 8-Dr. Reddy's laboratory | | | |
|---|--|-------------|-------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=2450 | ATM, K=2650 | OTM, K=3000 |
| Black-Scholes' | 7.73 | 1.17 | 1.02 |
| Heston | 1.37 | 1.44 | 1.42 |

According to Table-8, Dr. Reddy's laboratory, shows that Black-Scholes' Model outperforms Heston Model for ATM and OTM option of moneyness giving lesser error value while in ITM options, difference is observed where Heston outperforms Black-Scholes' model.

| Table No. 9-Sun Pharmaceutical limited | | | |
|---|--|------------|------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=480 | ATM, K=530 | OTM, K=700 |
| Black-Scholes' | 0.18 | 6.22 | 1 |
| Heston | 0.11 | 2.18 | 1 |

According to Table-9, Heston Model outperforms Black-Scholes' Model for ATM option of moneyness giving lesser error value while, ITM and OTM option show no difference in the two models.

| Table No. 10-TVS Motors | | | |
|--------------------------------|--|------------|------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=520 | ATM, K=530 | OTM, K=620 |
| Black-Scholes' | 0.89 | 4.26 | 1.12 |
| Heston | 0.86 | 1.29 | 9 |

According to Table-10, TVS Motors, show that Black-Scholes' Model outperforms Heston Model only for OTM option of moneyness giving lesser error value while, Heston shows better results in ATM option and in ITM option.

| Table No. 11-TATA Motors | | | |
|---------------------------------|--|------------|------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=150 | ATM, K=185 | OTM, K=320 |
| Black-Scholes' | 0.75 | 11 | 1 |
| Heston | 0.75 | 3.96 | 1 |

According to Table-11, Black-Scholes' Model outperforms Heston Model only for ATM option of moneyness giving lesser error value while, rest two gives same result.

| Table No. 12-Mahindra & Mahindra | | | |
|---|--|------------|------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=730 | ATM, K=750 | OTM, K=900 |
| Black-Scholes' | 0.94 | 1.98 | 1 |
| Heston | 0.8 | 2.35 | 1 |

According to Table-12, Black-Scholes' Model outperforms Heston Model only for ATM option of moneyness giving lesser error value and Heston model for ITM option. OTM option shows same result for both the models.

| Table No. 13-Maruti Udyog limited | | | |
|--|--|-------------|-------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=6500 | ATM, K=7300 | OTM, K=9900 |
| Black-Scholes' | 0.3 | 2.64 | 0.86 |
| Heston | 0.32 | 1.39 | 0.86 |

According to Table-13, Heston Model outperforms Black-Scholes' Model only for ATM option of moneyness giving lesser error value while, remaining two gives same result.

| Table No. 14-Hero Motor Corp | | | |
|-------------------------------------|--|-------------|-------------|
| Models | Error Value at different strike price (K) for option moneyness | | |
| | ITM, K=2750 | ATM, K=2900 | OTM, K=3200 |
| Black-Scholes' | 0.45 | 1.06 | 1.01 |
| Heston | 1.59 | 1.13 | 1.08 |

According to Table-14, Black-Scholes' Model outperforms Heston Model for all three option of moneyness giving lesser error value.

VIII. Discussion

Banking sector:

- In case of In-the-money option, Black-Scholes' outperforms Heston model in Axis and Kotak Mahindra Bank data while, Heston shows better result in Federal and HDFC Bank data.
- In case of At-the-money and Out-of-the-money option Heston model outperforms only in Federal bank data while, Black-Scholes' shows better result in all other Bank data.

Automobile Sector:

- In case of In-the-money option Black-Scholes' model outperforms Heston model in Hero Motor Corp data while, Heston model outperforms Black-Scholes' model in remaining all four companies data.
- In case of At-the-money option Heston model outperforms Heston model in TVS Motors and Maruti Udyog Limited while, Black-Scholes' gives better result in remaining three companies data.
- In case of Out-of-the-money option, Black-Scholes' outperforms Heston model in all the given companies data.

Pharmaceutical sector:

- In case of In-the-money option Black-Scholes' model outperforms Heston model for all the companies data except in Dr. Reddy laboratory data.
- In case of At-the-money option Heston model outperforms Black-Scholes' model only in Lupin limited and Sun Pharmaceuticals Limited while, Black-Scholes' gives better result in remaining three companies' data.
- In case of Out-of-the-money option Black-Scholes' model outperforms Heston model only in Dr. Reddy Laboratory data while, Heston model gives better result in remaining all four companies data.

IX. Conclusion

We have considered 14 different stock data from three different sectors; Banking, Automobile and Pharmaceuticals. We have compared the Black-Scholes' model and Heston model for all the option of moneyness; In-the money, out-of-the-money and At-the-money. We conclude that, out of 42 different cases, Black-Scholes' Model outperforms Heston model in 19 cases while, Heston model gives better result in 13 cases. In the remaining 10 cases, both the models show nearly same result. Thus, Black-Scholes' model is best in case of historical volatility. This study is mostly helpful for the investors working in derivatives market for both short term and long term options. We know that, such mathematical models are always helpful in calculating theoretical premium values and so this kind of quantitative study could be extended in future for a large data for more accurate results and suggestions.

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