

# The Challenge of Yeh to Hierarchies Consistency Analysis

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**Abstract:** In 1996, Guh, Lee, and Wang published a paper in *Computers and Mathematics with Applications* to develop Hierarchies Consistency Analysis (HCA) to deal with consistency between two hierarchies. Yeh (2013) claimed that HCA is flawed and then provided his extensions. His extension contains three steps. We run a detailed examination of his three-step extension to find many questionable results. We will suggest researchers neglecting the flawed accusation proposed by Yeh (2013) for HCA. Researchers should work on the proof for the convergence of the original sequences generated by Guh et al. (1996). After the mathematical foundation of Guh et al. (1996) is well-established, other extensions can stand on the shoulder of the giant to develop. Yeh (2013) tried to reproduce HCA in three directions without proper managerial meaning and mathematical verification to support that is not an academic approach.

**Key Word:** Hierarchies Consistency Analysis (HCA); estimated weights; aggregated index

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## I. Introduction

Guh et al. [7] developed a new algorithm, denoted as Hierarchies Consistency Analysis (HCA) to seek a balance between two hierarchies such that the final compromised weights for two hierarchies can imply the same aggregated index. Facing a large-scale decision-making problem, two groups of experts usually will construct two different hierarchies and then the aggregated index from two hierarchies will have different values and then implies different ordering of alternatives. Hence, how to develop a consistent approach to solve the rank reversal problem becoming a critical issue in the decision-making environment.

Guh et al. [7] claimed that following their HCA procedure that will generate convergent sequences and then the aggregated index from two hierarchies will be identical. Consequently, Guh et al. [7] believed that their HCA approach can help researchers to avoid the rank reversal problem.

Up to now, 17 papers had cited Guh et al. [7] in their References. We list them in the following: Guh [8, 9], Guh and Po [10], Guh et al. [11, 12], Chang [3], Yeh [17], Chang and Lin [4], Chang et al. [5], Beg and Rashid [2], Wang and Su [15], Ma et al. [13], Yu et al. [18], Chang et al. [6], Beg and Rashid [1], Wang et al. [16], and Sładkowski et al. [14]. Except for Yeh [17], the other 16 papers only mentioned Guh et al. [7] in their Introduction such that they did not provide any mathematical proof for HCA or present any question for the theoretical background of HCA. Hence, in this paper, we will not discuss those 16 papers.

On the contrary, Yeh [17] claimed that Guh et al. [7] is false such that he will develop a new algorithm, denoted as Chaotic Ordered Hierarchies Consistency Analysis (COH). Therefore, the main purpose of this paper is to present a detailed examination for Yeh [17] to find his accusation of Guh et al. [7] and check his new extensions for HCA.

The organism of this paper is described as follows. In Section 2, we provide a detailed review for Yeh [17] for his most important challenge to Guh et al. (2016). In Section 3, we point out that the most important challenge, that is the first step extension, proposed by Yeh [17] is established without any solid operational meaning. In Section 4, we present a detailed citation for the second and the third step extensions proposed by Yeh [17]. Our comments are listed in Section 5 to indicate severely questionable in his derivation procedure. In Section 6, we summarize the three-step extension proposed by Yeh [17] to indicate his goal and his algorithm. In Section 7, We point out possible directions to further work on the three-step extension raised by Yeh [17]. In section 8, we conclude our examination for Yeh [17], and then we advise researchers to ignore the challenge raised by Yeh [17] for HCA until the mathematical foundation for his three-step extension is fully proved.

## II. A detailed review for Yeh [17]

Yeh [17] claimed that HCA is questionable. Hence, we will provide a detailed literature review of Yeh [17] for those materials related to HCA.

There are several places of Yeh [17] that had discussed HCA. First, we cite his Abstract, "The Hierarchies Consistency Analysis (HCA) is proposed by Guh in-cooperated along with some case study on a Resort to reinforce the weakness of Analytical Hierarchy Process (AHP). Although the results obtained enabled aid for the Decision Maker to make more reasonable and rational verdicts, the HCA itself is flawed. In this paper, our objective is to indicate the problems of HCA, and then propose a revised method called chaotic ordered HCA (COH in short) which can avoid problems. Since the COH is based upon Guh's method, the Decision Maker establishes decisions in a way similar to that of the original method." The above citation points out that Yeh [17] would first point out questionable results in Guh et al. [7] and then provided his new algorithm.

Second, In Section 2.2, Notations, Yeh [17] assumed that " $k_t$ : The combination parameter in the convex combination

$$w_{i,l}^{t+1} = k_t w_{i,l}^t + (1 - k_t) \Phi(w_{i,l}^t) \tag{1 of Yeh [17]}$$

Note that  $k_t = k = .5$  in the traditional HCA and AHP-HCA."

**Remark.** In Yeh [17], he created a new expression as " $\overline{w_{i,l}^t}$ ". We believe that the original notation of " $\Phi(w_{i,l}^t)$ " proposed by Guh et al. [7] should be used. to help ordinary readers understand the argument between Guh et al. [7] and Yeh [17].

In Section 3 of Yeh [17], he claimed that "In practice, however, we are not able to check all possible strategies because of the vast size of weights and  $k$ . Therefore,  $k$  is set to a fixed number, say  $k = .5$ , in all traditional HCA or AHP-HCA [1, 8, 9, 10]. To correct the shortcoming of using fixed  $k$ , the chaotic random number is incorporated in the proposed COH to improve the negotiable space of finding comprised weights." This citation shows the first challenge proposed by Yeh [17].

Yeh [17] mentioned that "The well-known logistic equation is adapted here to replace  $k$  in Eq.(1) such that a slight difference in the initial value of the chaotic random number would result in a significant difference in its long time behavior as follows [18].

$$k_{t+1} = mk_t(1 - k_t) \tag{2 of Yeh [17]}$$

where  $m \neq 4$  is the control parameter,  $0 < k_t < 1$ , and  $k_0 \neq 0.25, 0.5$  and  $0.75$ . The logistic equation has the basic characteristic of chaos, i.e., it reveals the sensitive dependence on initial conditions and includes infinite unstable periodic motions."

### III. Our first comments for Yeh [17]

In the Abstract, Yeh [17] raised a severe criticism on Guh et al. [7] and then we cite his first challenge to Guh et al. [7]. We recall that Guh et al. [7] used

$${}_1w_j^1 = k({}_1w_j^0) + (1 - k)\Phi({}_1w_j^0), \tag{1}$$

and

$${}_2w_j^1 = (1 - k)({}_2w_j^0) + k\Phi({}_2w_j^0), \tag{2}$$

where  $k$  is a combination parameter, with  $0 \leq k \leq 1$  and  ${}_1w_j^0$  (or  ${}_2w_j^0$ ) are the initial weights for criteria in Hierarchy 1 (or Hierarchy 2) and  $\Phi({}_1w_j^0)$  (or  $\Phi({}_2w_j^0)$ ) are the derived weights from the opposite hierarchy of Hierarchy 2 (or Hierarchy 1).

Yeh [17] abstractly treated Equations (1) and (2) as

$${}_i w_j^1 = k_i({}_i w_j^0) + (1 - k_i)\Phi({}_i w_j^0), \tag{3}$$

for  $i = 1, 2$  and  $k_1 = k, k_2 = 1 - k$ .

Moreover, Yeh [17] used a new expression, " $\overline{w_{i,l}^t}$ " to replace  $\Phi(w_{i,l}^t)$  which was used by Guh et al. [7].

At last, Yeh [17] changed  $k_i$  to  $k_t$  such that every iteration will have a different combination parameter.

In Equation (2) of Yeh [17], he claimed that  $k_{t+1} = mk_t(1 - k_t)$  with  $m \neq 4$  and  $k_0 \neq 0.25, 0.5$  and  $0.75$ .

Yeh [17] mentioned that "To correct the shortcoming of using fixed  $k$ , the chaotic random number is incorporated".

However, Yeh [17] did not provide us any reason why using a fixed combination parameter that can be claimed by him to be an error.

In Guh et al. [7], they explained the fixed combination parameter. We refer to their reasoning as follows.

If there are two hierarchies: Hierarchy 1 and Hierarchy 2. The decision-maker will decide his importance between Hierarchy 1 and Hierarchy 2.

For example, if  $k = 0.3$ , that means, the decision-maker wanted the final weights for criteria will be evaluated 30% by the initial weights of criteria in Hierarchy 1 and 70% by the initial weights of criteria in Hierarchy 2. Therefore, the fixed combination parameter has its operational meaning.

In Yeh [17], he did not provide any managerial meaning for his variable  $k_t$ .

To the best of our knowledge, we can predict that he reasons that "The well-known logistic equation, that is  $k_{t+1} = mk_t(1 - k_t)$ , is adapted here to replace the fixed combination parameter  $k$ ".

We assert that to replace a fixed combination parameter "k" with a meaning operational explanation to a sequence  $(k_t)_{k=0,1,2,\dots}$  to create a chaotic environment is an invalid challenge raised by Yeh [17].

In many search algorithms, researchers used mutation or chaos parameters to avoid the searching procedure stuck in a local critical solution. Hence, those variable parameters have their operational meaning. In Yeh [17], for chaos, he adopted a variable parameter to replace a constant parameter that is not sufficient to persuade researchers to accept his adoption.

**IV. Further reviewing for Yeh [17]**

We cite from Yeh [17] "3.2 A limitation to k

According to the characteristic of the convex combination shown in Eq.(1), the newly generated number  $w_{i,l}^{t+1}$  is between  $w_{i,l}^t$  and  $\Phi(w_{i,l}^t)$ . If  $k > 0.5$ , then  $w_{i,l}^{t+1}$  is closer to  $w_{i,l}^t$  than to  $\Phi(w_{i,l}^t)$  and vice versa. Since the priority of each hierarchy is always to protect its own profit in real-life situations. Therefore, to be more reasonable, the  $k$  must not less than .5 in this study, i.e., Eq.(1) is revised as:

$$w_{i,l}^{t+1} = \begin{cases} k_t w_{i,l}^t + (1 - k_t) \Phi(w_{i,l}^t), & \text{if } k_t > 0.5, \\ (1 - k_t) w_{i,l}^t + k_t \Phi(w_{i,l}^t), & \text{otherwise.} \end{cases} \quad (3) \text{ of Yeh [17]}$$

**3.3 The Novel Method to Readjustment Weights**

Experienced DM may not be able to provide a 100% correct value to each weight of criterion. However, they can tell which criterion is more important. That is why Guh's AHP-HCA [1] resulted in failure to apply in real-life. To avoid such a flaw of AHP-HCA discussed in Section 3.4, a novel equation is proposed to replace Eqs. (1) and (3) to readjustment each weight by a new convex combination after sorting the weights under the same parent in decreasing order as follows:

$$\omega_{i,l_j}^{t+1} = \bar{k}_t \omega_{i,l}^t + (1 - \bar{k}_t) \Delta_t, \quad (4) \text{ of Yeh [17]}$$

where

$$w_{i,l_j}^t > w_{i,l_i}^t \text{ if and only if } j < l, \quad (5) \text{ of Yeh [17]}$$

$$\Delta_t = \begin{cases} \min \{ \Phi(w_{i,l_j}^t), \omega_{i,l_{j-1}}^{t+1} \}, & \text{if } w_{i,l_i}^t < \Phi(w_{i,l_i}^t) \\ \max \{ \Phi(w_{i,l_j}^t), w_{i,l_{j+1}}^t \} & \text{otherwise} \end{cases}, \quad (6) \text{ of Yeh [17]}$$

$$\bar{k}_t = \max\{k_t, (1 - k_t)\}. \quad (7) \text{ of Yeh [17]}$$

To satisfy the total weight under the same parent is equal to one, a normalization is implemented to Eq.(4):

$$w_{i,l_i}^t = \frac{\omega_{i,l_i}^t}{\sum_j \omega_{i,l_j}^t}. \quad (8) \text{ of Yeh [17]}$$

The major concept behind Eqs. (4)-(8) is that the order of the importance of criteria can never be changed, i.e. the following important relationship must be satisfied for each new generated  $w_{i,l_j}^{t+1}$ :

$$w_{i,l_a}^t < w_{i,l_b}^{t+1} < w_{i,l_c}^{t+1}, \quad (9) \text{ of Yeh [17]}$$

if

$$w_{i,l_a}^0 < w_{i,l_b}^0 < w_{i,l_c}^0, \quad (10) \text{ of Yeh [17]"}$$

**V. Our second comments for Yeh [17]**

There are many questionable results in Sections 3 and 4 of Yeh [17]. We will do our best to point out his questionable results and then indicate his violations.

In Yeh [17], he never informed readers of the value of "m" corresponding to the control parameter of Equation (2) of Yeh [17].

Moreover, Yeh [17] did not tell us the value of  $k_0$ . All we know that  $k_0 \neq 0.25, 0.5, \text{ and } 0.75$ , and in Table 2 of Yeh [17], he mentioned that "Table 2. The compromised weights under  $k = .0, .1, \dots, 1.0$ . Hence, we do not know the exact value of "m" and " $k_0$ " to repeat the same numerical example examination for Yeh [17].

Moreover, we list the partial results from Tables 1 and 2 of Yeh [17] in the following.

**Table 1.** Reproduction of partial findings of Table 1 of Yeh [17]

$w_{1,1}^t$	$w_{1,2}^t$	$w_{1,11}^t$	$w_{1,12}^t$	$w_{1,21}^t$	$w_{1,22}^t$
0.499	0.501	0.702	0.298	0.537	0.463
$w_{2,1}^t$	$w_{2,2}^t$	$w_{2,11}^t$	$w_{2,21}^t$	$w_{2,12}^t$	$w_{2,22}^t$
0.619	0.381	0.567	0.434	0.389	0.611

**Table 2.** Reproduction of partial findings of Table 2 of Yeh [17]

	$w_{1,1}^t$	$w_{1,2}^t$	$w_{1,11}^t$	$w_{1,12}^t$	$w_{1,21}^t$	$w_{1,22}^t$
	0.60	0.40	0.80	0.20	0.30	0.70
$w_{2,1}^t$		$w_{2,2}^t$	$w_{2,11}^t$	$w_{2,21}^t$	$w_{2,12}^t$	$w_{2,22}^t$
0.60	0.40	0.80	0.20	0.30	0.70	

From Table 1, we find that Yeh [17] obtained

$$w_{2,11}^t + w_{2,12}^t = 0.567 + 0.389, \tag{4}$$

and

$$w_{2,21}^t + w_{2,22}^t = 0.434 + 0.611. \tag{5}$$

From Table 2, we know that Yeh [17] derived

$$w_{2,11}^t + w_{2,12}^t = 0.80 + 0.30, \tag{6}$$

and

$$w_{2,21}^t + w_{2,22}^t = 0.20 + 0.70. \tag{7}$$

The results of Yeh [17] in Equations (4) and (6) violate

$$w_{2,11}^t + w_{2,12}^t = 1. \tag{8}$$

The results of Yeh [17] in Equations (5) and (7) violate

$$w_{2,21}^t + w_{2,22}^t = 1. \tag{9}$$

The above discussion reveals that the results of Yeh [17] in his Tables 1 and 2 contained severe problems.

In Tables 1 and 2 of Yeh [17], among his twenty test runs, we find that  $w_{2,11}^t + w_{2,12}^t \neq 1$  and  $w_{2,21}^t + w_{2,22}^t \neq 1$  such that all his twenty test runs are false. At last, not the least, Yeh [17] never informed us of the values of  $m$  and  $k_0$  such that to repeat his numerical example is impossible for researchers.

We agree with his assumption of Equation (5) of Yeh [17], because of "sorting the weights under the same parent in decreasing order". We must remind readers that recalling Equations (9) and (10), to reserve the ordering of weights for criteria is his goal.

However, Equation (6) of Yeh [17] is in chaos. if we compare  $\Phi(w_{i,l_j}^t)$  and  $\omega_{i,l_{j-1}}^{t+1}$ , then we find there are two notations: (a)  $w$ , and (b)  $\omega$ .

In Equation (4) of Yeh [17], he used  $\omega_{i,l_j}^{t+1}$ . On the other hand, in Equation (5) of Yeh [17], he used  $w_{i,l_j}^t$  such that we can predict that no one can absorb his paper under this confusion.

The expression of  $\bar{k}_t \omega_{i,l}^t$  in Equation (4) of Yeh [17] should be revised to  $\bar{k}_t \omega_{i,l_j}^t$ .

Equation (9) of Yeh [17] should be revised to

$$w_{i,l_a}^{t+1} < w_{i,l_b}^{t+1} < w_{i,l_c}^{t+1}. \tag{10}$$

Equation (8) of Yeh [17] is in chaos. Yeh [17] used both  $w_{i,l_i}^t$  and  $\omega_{i,l_i}^t$  in Equation (8) of Yeh [17]. If Equation (8) is the right expression, and then  $w_{i,l}^t$  appeared in Equations (1), (3), (5), and (6) should be revised to  $\omega_{i,l}^t$ .

## VI. Our final discussion for Yeh [17]

Yeh [17] tried to improve the combination procedure of Guh et al. [7]. There are three steps, and then we list them in the following.

For the first step, the fixed combination parameter "k" is changed to a variable as "k<sub>t</sub>" with  $k_{t+1} = mk_t(1 - k_t)$  such that

$${}_1w_j^{n+1} = k_t({}_1w_j^n) + (1 - k_t)\Phi({}_1w_j^n), \tag{11}$$

and

$${}_2w_j^{n+1} = (1 - k_t)({}_2w_j^n) + k_t\Phi({}_2w_j^n). \tag{12}$$

For the second step, Yeh [17] believed that each hierarchy hopes to keep his previous weight more important and the estimated weights from the opposite Hierarchy less important. Hence, the combination process is revised to

$${}_1w_j^{n+1} = \max\{k_t, 1 - k_t\}({}_1w_j^n) + \min\{k_t, 1 - k_t\}\Phi({}_1w_j^n), \tag{13}$$

and

$${}_2w_j^{n+1} = \max\{k_t, 1 - k_t\}({}_2w_j^n) + \min\{k_t, 1 - k_t\}\Phi({}_2w_j^n). \tag{14}$$

For the third step, Yeh [17] wanted the ordering of weights from the same "father" (upper level) will be reserved after the combination procedure. Consequently, Yeh [17] constructed a complicated process,

if  $w_{i,l_i}^t < \Phi(w_{i,l_i}^t)$ ,

$$w_{i,l_j}^{t+1} = \max\{k_t, 1 - k_t\}w_{i,l}^t + \min\{k_t, 1 - k_t\}\min\{\Phi(w_{i,l_j}^t), w_{i,l_{j-1}}^{t+1}\}, \tag{15}$$

and if  $w_{i,l_i}^t \geq \Phi(w_{i,l_i}^t)$ ,

$$w_{i,l_j}^{t+1} = \max\{k_t, 1 - k_t\}w_{i,l}^t + \min\{k_t, 1 - k_t\}\max\{\Phi(w_{i,l_j}^t), w_{i,l_{j+1}}^t\}. \quad (16)$$

Yeh [17] realized that after his combination the total sum of weights under the same father may not be equal to 1 and then Yeh [17] normalized them as

$$w_{i,l_1}^t = \frac{\omega_{i,l_1}^t}{\sum_j \omega_{i,l_j}^t}. \quad (17)$$

We divide Equation (15) into three cases: Case I,  $w_{i,l_i}^t < \Phi(w_{i,l_i}^t) \leq w_{i,l_{j-1}}^{t+1}$ , Case II,  $w_{i,l_i}^t < w_{i,l_{j-1}}^{t+1} < \Phi(w_{i,l_i}^t)$ , and Case III,  $w_{i,l_{j-1}}^{t+1} \leq w_{i,l_i}^t < \Phi(w_{i,l_i}^t)$ :

For Case I, under  $w_{i,l_i}^t < \Phi(w_{i,l_i}^t) \leq w_{i,l_{j-1}}^{t+1}$ , then

$$w_{i,l_j}^{t+1} = \max\{k_t, 1 - k_t\}w_{i,l}^t + \min\{k_t, 1 - k_t\}\Phi(w_{i,l_j}^t), \quad (18)$$

For Case II, under  $w_{i,l_i}^t < w_{i,l_{j-1}}^{t+1} < \Phi(w_{i,l_i}^t)$ , then

$$w_{i,l_j}^{t+1} = \max\{k_t, 1 - k_t\}w_{i,l}^t + \min\{k_t, 1 - k_t\}w_{i,l_{j-1}}^{t+1}, \quad (19)$$

For Case III, under  $w_{i,l_{j-1}}^{t+1} \leq w_{i,l_i}^t < \Phi(w_{i,l_i}^t)$ , then

$$w_{i,l_j}^{t+1} = \max\{k_t, 1 - k_t\}w_{i,l}^t + \min\{k_t, 1 - k_t\}w_{i,l_{j-1}}^{t+1}, \quad (20)$$

We divide Equation (16) into two cases: Case IV,  $w_{i,l_{j+1}}^t \leq \Phi(w_{i,l_i}^t) \leq w_{i,l_i}^t$ , and Case V,  $\Phi(w_{i,l_i}^t) \leq w_{i,l_{j+1}}^t < w_{i,l_i}^t$ , because  $\Phi(w_{i,l_i}^t) \leq w_{i,l_i}^t \leq w_{i,l_{j+1}}^t$  will not happen.

For Case IV, under  $w_{i,l_{j+1}}^t \leq \Phi(w_{i,l_i}^t) \leq w_{i,l_i}^t$ , then

$$w_{i,l_j}^{t+1} = \max\{k_t, 1 - k_t\}w_{i,l}^t + \min\{k_t, 1 - k_t\}\Phi(w_{i,l_j}^t). \quad (21)$$

For Case V, under  $\Phi(w_{i,l_i}^t) \leq w_{i,l_{j+1}}^t < w_{i,l_i}^t$ , then

$$w_{i,l_j}^{t+1} = \max\{k_t, 1 - k_t\}w_{i,l}^t + \min\{k_t, 1 - k_t\}w_{i,l_{j+1}}^t. \quad (22)$$

From our above discussion, there are five possible outcomes as Equations (18-22) for  $w_{i,l_j}^{t+1}$  such that to prove the convergence of those sequences  $(w_{i,l_j}^t)_{t=0,1,2,\dots}$  becomes a severe difficult problem.

### VII. Direction for future research

We believe that researchers should be worked on the convergence of the original sequences proposed by Guh et al. [17] to provide a strong mathematical foundation for this promising research topic.

After the mathematical foundation of HCA is strongly established, researchers may try the variable combination parameter proposed by Yeh [17] for the convergence problem. However, the operational meaning for the variable combination parameter should be properly motivated. The present explanation, creating a chaotic environment, for changing from a constant parameter to a variable parameter is not enough to persuade researchers.

After the variable combination parameter approach is proved to be valid, we can say that researchers can try the second step proposed by Yeh [17] where the minimum and maximum operations are proposed by Yeh [17]. During the second step, we predict that the normalization procedure will be executed which will imply the proof of convergence becoming very difficult. The original purpose of HCA is to provide a compromised weight algorithm for two different hierarchies. During the second extension, Yeh [17] assumed that each hierarchy tried to reserve its weights for criteria from the previous iteration. The maximum operator indicates that Yeh [17] lost control of the values of  $k_t$  and then used  $\max\{k_t, 1 - k_t\}$  to keep the desired combination parameter. We may predict that without the first-step extension, Yeh [17] directly developed the second-step extension such that (i) maximum operator, (ii) minimum operator, and (iii) normalization all can be avoided, and then the proof for convergence of the generated weights may be possible.

After the mathematical foundation of the second step is well developed, then researchers can begin to consider the third step extension proposed by Yeh [17]. Frankly speaking, we do not believe that the proof for the convergence of the generated weights can be verified under such a complicated procedure. Moreover, In Equations (18) and (21), Yeh [17] used  $\Phi(w_{i,l_j}^t)$  that is reasonable as proposed by HCA. However, in Equations (19), (20), and (22), Yeh [17] used  $w_{i,l_{j-1}}^{t+1}$  or  $w_{i,l_{j+1}}^t$  without a reasonable motivation that will not convince researchers to accept his third-step extension.

### VIII. Conclusion

Yeh [17] raised a challenge to HCA proposed by Guh et al. [7]. However, his variable combination parameter lacks operational meaning for his first step extension. Yeh [17] claimed that Hierarchies wanted to reserve their previous iteration weights that will introduce the normalization for his weight procedure. However,

the normalization operation will convert the convergence proof becoming an extremely difficult issue. His third step extension will imply five different cases such that we cannot believe that the proof of the convergence can be analytically verified. Based on our detailed examination of Yeh [17], we can conclude that his challenge of Guh et al. [7] is invalid.

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