

Entropy of a Black Hole

Dr. Ranjan Prasad

Assistant professor department of Physics S.N.S.R.K.S
College, Saharsa, B.N. Mandal University, Madhepura

Abstract

We present a straightforward method to determine the entropy of a Schwarzschild black hole by analyzing the entropy associated with its Hawking radiation. Specifically, we demonstrate that when a thermodynamic system dissipates energy exclusively through blackbody radiation, the corresponding decrease in its entropy equals three-fourths of the entropy carried away by the emitted radiation. This result allows us to establish a connection between the entropy of an evaporating black hole and that of its Hawking radiation. By explicitly evaluating the total entropy of the radiation released over the complete evaporation process, we recover the Bekenstein–Hawking entropy of the original black hole. Furthermore, this study derives the relationship between black hole area entropy and information theory. The curved spacetime around a black hole can act similarly to an optical lens, forming images of objects. The maximum information that a black hole can contain is constrained by both the Compton wavelength of the infalling object and the black hole’s size. When matter enters a black hole, its detailed information is lost due to the no-hair theorem, leading to a corresponding increase in entropy. Consequently, the area law of black hole entropy suggests that Bekenstein–Hawking entropy should be interpreted as a form of information entropy rather than purely thermodynamic entropy.

Keywords: Camera Lenses; Diameter; Hawking Radiation; Proposition; Maximal Information

I. Introduction

The Bekenstein–Hawking entropy, commonly referred to as black hole entropy, represents the amount of entropy that must be attributed to a black hole so that it remains consistent with the laws of thermodynamics, particularly from the perspective of external observers. This requirement is especially significant in relation to the first and second laws of thermodynamics. Although rooted in geometry, the concept of black hole entropy has profound physical implications, linking together ideas from gravitation, thermodynamics, and quantum mechanics. As a result, it is often considered a key pathway toward understanding the still elusive domain of quantum gravity. A major focus in early-universe research is explaining the observed uniformity, isotropy, and spatial flatness of the universe, as evidenced by measurements of the cosmic microwave background (CMB) from the last scattering surface. However, an equally important and more challenging issue is the explanation of how entropy is distributed in the universe. The difficulty arises because, following the Big Bang where gravity is strongly interacting and quantum fluctuations in both stress-energy and spacetime are significant—it would be natural to expect that entropy should be close to its maximum value and evenly shared between matter-energy and gravitational degrees of freedom. Under such conditions, the Weyl curvature, which is the traceless component of the Riemann curvature tensor and encodes contributions from tidal forces and gravitational waves, would be expected to approach very large values. Contrary to this expectation, observations of the last scattering surface reveal a strikingly different entropy distribution, as highlighted by Roger Penrose. The total entropy is vastly lower than the theoretical maximum and is unevenly divided: it is nearly maximal within the matter-radiation sector due to its proximity to thermal equilibrium while remaining extremely small in the gravitational sector. Moreover, since Weyl curvature vanishes in a perfectly homogeneous and isotropic universe, its observed value is negligibly small rather than extremely large. This entropy problem is conceptually separate from the issue of explaining homogeneity and isotropy, as it is theoretically possible to have a universe that is smooth and isotropic but with a very different entropy configuration. In fact, the theory of inflation, originally proposed to explain the universe’s smoothness and flatness, actually exacerbates the entropy problem. For inflation to begin, the universe must transition from the Big Bang into a state dominated by a nearly uniform scalar field, with all other matter, radiation, and gravitational effects being insignificant. This condition requires an even lower and more finely tuned initial entropy than what is needed to explain observations without invoking inflation. Likewise, alternative models such as bouncing cosmologies with a singular bounce where the universe transitions from contraction to expansion near Planck-scale densities do not resolve the entropy issue. In such scenarios, quantum gravitational effects near the bounce would still cause the Weyl curvature to diverge, leaving the entropy problem unresolved.

Black Hole Entropy

A black hole can be characterized as a distortion or irregularity in spacetime, representing a region of extremely intense curvature. Several arguments have been proposed to support the idea of black hole entropy (Bekenstein, 1972, 1973):

- A black hole typically originates from the gravitational collapse of matter or radiation, both of which inherently possess entropy. However, for an external observer, the interior of the black hole and its contents remain inaccessible. Consequently, any thermodynamic description of this collapse cannot rely on the entropy of the original matter or radiation, since it cannot be directly observed. Assigning entropy to the black hole itself provides a meaningful way to describe its thermodynamic properties.
- A stationary black hole is fully described by a limited set of parameters: mass, electric charge, and angular momentum (and hypothetically, magnetic monopole charge, although its physical existence has not yet been confirmed). For a given set of these parameters, there may exist numerous possible formation histories, implying a wide range of internal configurations. This situation is analogous to thermodynamic systems, where multiple microscopic states correspond to a single macroscopic state. Entropy, in this context, serves as a measure of this multiplicity, suggesting that a similar concept should be applied to black holes.
- The event horizon acts as a barrier that prevents any signals from escaping, thereby blocking the transmission of information from the interior to an external observer. As a result, a black hole effectively conceals information, except for a few observable quantities such as mass, charge, and angular momentum. In classical thermodynamics, entropy is often interpreted as a measure of missing or inaccessible information. Therefore, it is reasonable to associate entropy with a black hole.

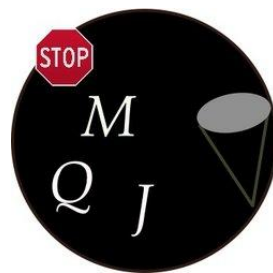


Figure 1

Because of the structure of local light cones, the event horizon prohibits any information-carrying signals from leaving the black hole. An outside observer can detect only a limited set of properties, namely the mass (M), angular momentum (J), and electric charge (Q) of the black hole.

Black hole entropy from the Hawking radiation
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Black Hole Entropy Derived from Hawking Radiation

To build physical intuition, let us first examine an unspecified thermodynamic system about which we have no direct information, except for the black-body radiation it emits. Our goal is to determine both the temperature and entropy of this system based solely on this radiation.

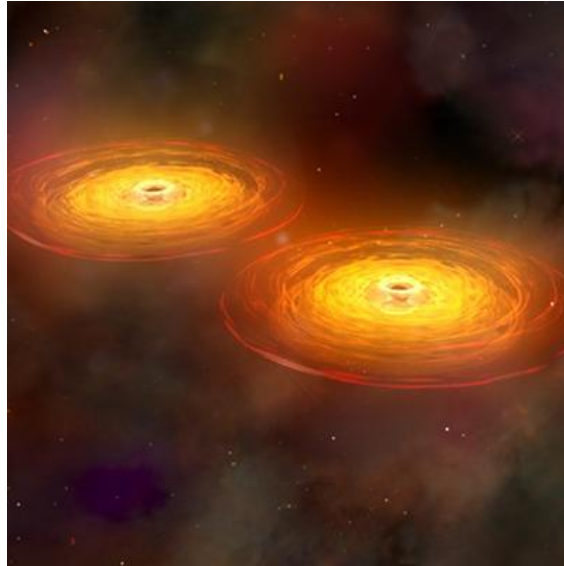


Figure 2

Bekenstein–Hawking Entropy Formula

The entropy of a black hole is expressed by the well-known relation:

$$S = \frac{kA}{4\hbar G} \quad S = 4\hbar GkA$$

Where:

- S = entropy of the black hole
- A = surface area of the event horizon
- k = Boltzmann constant
- \hbar (h-bar) = reduced Planck constant
- G = gravitational constant

Determining the temperature of the system is relatively straightforward, as it can be obtained from the spectrum of its thermal radiation. However, calculating entropy is more involved and requires two key steps:

1. Establishing a relationship between the entropy decrease of the system and the entropy carried away by the emitted radiation.
2. Evaluating the total entropy of the black-body radiation emitted from the beginning until the system has radiated all of its available internal energy.

At the final stage, when no further energy can be emitted, the system may be considered to have reached a state of zero entropy. Thus, by computing the total entropy of the emitted radiation and applying the relation from the first step, the initial entropy of the system can be determined.

Proposition

For a thermodynamic system in equilibrium that loses energy solely through black-body radiation into a vacuum, the decrease in its entropy is approximately three-quarters (3/4) of the entropy carried away by the radiation.

Proof

Let the system have internal energy U, temperature T, and entropy S. Suppose it emits an infinitesimal amount of energy δU , resulting in an entropy change δS .

From the First Law of Thermodynamics, we have:

$$\delta U = T \delta S \quad \delta U = T \delta S$$

Now consider the emitted radiation near the system’s surface as a photon gas in thermal equilibrium. For this radiation, the entropy and energy are related (from statistical mechanics) as:

$$\delta S = \frac{4}{3} \frac{\delta U}{T} \quad \delta S = \frac{4}{3} \frac{\delta U}{T}$$

At equilibrium, the temperatures are equal. By conservation of energy, the energy lost by the

System equals the energy gained by radiation:

$$\delta U_{\text{system}} = -\delta U_{\text{radiation}} \quad \Delta U_{\text{system}} = -\Delta U_{\text{radiation}}$$

Substituting these relations, we obtain:

$$\delta S_{\text{system}} = -34\delta S_{\text{radiation}} \quad \Delta S_{\text{system}} = -\frac{3}{4} \Delta S_{\text{radiation}} \quad \Delta S_{\text{system}} = -43 \Delta S_{\text{radiation}}$$

For a finite radiation process, although the temperature changes gradually, this proportional relationship remains valid. Therefore, for total entropy changes:

$$\Delta S_{\text{system}} = -34\Delta S_{\text{radiation}} \quad \Delta S_{\text{system}} = -\frac{3}{4} \Delta S_{\text{radiation}} \quad \Delta S_{\text{system}} = -43 \Delta S_{\text{radiation}}$$

II. Conclusion

In conventional quantum statistical mechanics, the entropy of a system at a specified energy and set of state variables is defined as the logarithm of the number of accessible microstates within a narrow range around those parameters. This interpretation is generally extended to black holes, suggesting that black hole entropy should likewise be understood as a measure obtained by counting the number of underlying quantum states within a theory of quantum gravity. However, an important conceptual issue arises: where are the degrees of freedom responsible for this entropy located? Various possibilities have been proposed. These degrees of freedom might exist deep within the interior of the black hole, be confined to the event horizon, or be distributed within the surrounding thermal atmosphere. Each of these scenarios presents its own theoretical challenges and unresolved difficulties.

Furthermore, within the framework of quantum general relativity, the notion of degrees of freedom is not necessarily fixed. Instead, such degrees of freedom may emerge or disappear depending on the physical conditions. For instance, in an expanding universe, certain field modes that are considered valid degrees of freedom today would have been sub-Planckian in the early universe and therefore excluded from the effective physical description at that time. In a similar manner, near the singularity inside a black hole, it is plausible that some effective degrees of freedom are lost or cease to exist. Therefore, it is not immediately clear that black hole entropy can be interpreted simply as the counting of microstates in the traditional sense. This indicates that a deeper and more fundamental understanding of entropy within the framework of quantum gravity is still required.

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