On Interaction of Current Elements and Self-action

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Abstract: Interaction of current elements in an unclosed system occurs with violation of Newton’s third law. A thin magnetized disk coupled with the unclosed part of a conductor turns out to have a very large self-force. This force completely compensates for the force of action, and the force of reaction is zero.

Key Words: Self-force, Conservation of momentum, Magnetized body, Unclosed conductor, Force of action

I. Introduction

The question concerning the existence of self-action by means of which a body can mechanically act on itself still remains unanswered. On the one hand, the self force has been experimentally discovered and is fully described by classical electrodynamics [1]. On the other hand, its value is so small that it is noticeable only at currents of several tens amperes flowing in the unclosed part of the conductor whose ends are immersed in an electrically conductive liquid. In order in magnitude, the self-force is \( \mu_0 I^2/4\pi \), where \( I \) is the current flowing in the circuit. The square of the current can be rewritten as a product of currents, one of which is the current flowing in an open conductor, and the second is the current which creates magnetic field. The magnetic field is often created by molecular current flowing on the surface of the magnetized body. In such a form, the self force corresponds to the magnetic interaction of currents, one of which is closed, and another is not [2]. It is clear that all this is about the dynamics of an open system consisting of an unclosed part \( L \) of a conductor and a magnetized body, the role of which is played a magnet \( M \) with significant magnetization \( J \) (Fig. 1). Another body of the closed system is two long rectilinear conductors \( V \) and \( U \) with the same current \( I \).

Fig. 1. Interaction of unclosed system \( M+L \) with linear conductors \( V \) and \( U \).

The noted experimental results [1] are unique, contradictory and dubious. This is primarily due to the use of liquid contacts between two parts of the conductor that make up the closed current. Other results [3,4] have no direct relation to the force of self-action. It is not yet clear how to recover the value of the self–force from in the data on the self-torques [5].
II. Forces of action and reaction

As a first step, it is use to consider the force of action \( F_a \) by means of which the electric current in linear long conductors \( V \) and \( U \) acts on the thin magnet. In accord with Stokes’ theorem [6] the magnetic field created by thin magnetized disk with magnetization \( J \) perpendicular to the surface of the magnet is absolutely equivalent to that produced by the current \( Jh \) flowing in the rim of the disk. Since

\[
\mathbf{r} = r \cos \phi \mathbf{e}_x + r \sin \phi \mathbf{e}_y,
\]

\[
\mathbf{R}_\phi = r - y - p \mathbf{e}_x,
\]

that the Biot-Savart force exerted by the current element \( Jh \mathbf{dr} \) due to the current flowing in the linear conductor \( U \) is

\[
d\mathbf{F}_\phi = \frac{\mu_0 Jh}{4\pi} \frac{[d\mathbf{r} \times (d\mathbf{y} \times \mathbf{R}_\phi)]}{R_\phi^2}.
\]

The same applies to effect of the current flowing in the straight section \( V \) on the magnet \( M \). Taking into account that

\[
d\mathbf{r} = -r \sin \phi \mathbf{e}_y \, d\phi + r \cos \phi \mathbf{e}_x \, d\phi
\]

one obtains

\[
F_a = \frac{\mu_0 Jh}{2\pi} \int_0^{2\pi} r \sin^2 \phi \left( p - r \cos \phi \right) \left( r^2 - 2rp \cos \phi + p^2 \right)^{1/2} \, d\phi,
\]

where \( h \) is thickness of the magnetized disk and \( p \) is the distance from the center of disk to linear currents. Note that for given \( I, J, h \) the force of action depends only on relation \( p/r \). In this sense, the obtained result is universal (fig. 2).

![Fig. 2. Self-force and force of action.](image.png)

As for the reaction force, it is enough to note here that since the magnetic forces \( \mathbf{F}_{MU} \) and \( \mathbf{F}_{MV} \) are transverse, they do not contribute to the net force acting on the rectilinear conductors:

\[
\mathbf{F}_r = 0.
\]

The elementary form of Newton’s third law does not hold in electromagnetism. Conservation of linear momentum can be preserved, however, by saying that the sum of all forces, including the forces of the mutual action of the magnet on the circular current loop \( L \) and the loop on the magnet, must be equal to zero. This statement, constituting the law of conservation of momentum, must be verified. This will allow not only checking the result (1), but also finding the missing force.
III. The self-force

In the previous section, it was shown that thin magnetized disk $M$ and the external unclosed electric circuit interact with forces that are not equal in magnitude and not opposite in direction. Therefore, there are reasons to believe that the same happens when the magnet $M$ interacts with the unclosed circular current flowing in $L$.

The total magnetic force exerted by the loop $L$ is obtained by integrating

$$dF_{pr} = \frac{\mu_0 \mu_n}{4\pi} \frac{|d\mathbf{r} \times [d\mathbf{p} \times \mathbf{R}^{pr}]|}{\mathbf{R}^{pr}} ,$$

over the contours containing the currents $I$ and $Jh$.

Analogously, if a small length of wire $d\mathbf{p}$ carries a current $I$ in direction of $d\mathbf{p}$, that the force $dF_{pr}$ exerted by the current element $Jh d\mathbf{r}$ is

$$dF_{pr} = \frac{\mu_0 \mu_n}{4\pi} \frac{|d\mathbf{r} \times [d\mathbf{p} \times \mathbf{R}^{pr}]|}{\mathbf{R}^{pr}} ,$$

where $\mathbf{R}^{pr}$ is the radius-vector plotted from the current element $Jh d\mathbf{r}$ to the infinitesimal element $d\mathbf{r}$. Since the $Y$-component of the sum of the double vector products is

$$d\mathbf{p}(d\mathbf{r}^{pr}) - d\mathbf{r}(d\mathbf{p}^{pr}) |_{y} = pr \sin(\varphi - \theta) \cos \theta - r \cos \varphi d\varphi d\theta ,$$

and

$$\mathbf{R}^{pr} = \mathbf{r} - \mathbf{p} ,$$

with

$$R^{pr} = (r^2 + p^2 - 2 pr \cos(\varphi - \theta))^{1/2} ,$$

then

$$F_x = F_{ML} + F_{LM} = \frac{\mu_0 \mu_n}{4\pi} \int_{0}^{2\pi} \frac{1}{(r^2 + 2rp \cos \varphi + p^2)^{1/2}} - \frac{1}{(r^2 - 2rp \cos \varphi + p^2)^{1/2}} \cos \varphi d\varphi .$$

This force, being the sum of the force action on the current in the wire $L$ due to molecular current in the rim of thin magnetized disk $F_{ML}$ and the force $F_{LM}$ exerted by the magnet, is nothing more than the force of self-action by means of which the body consisting of the magnetized disk and the conductor $L$ acts on itself.

Vector potential of the magnetized disk at a point $P$ is defined by

$$A_P = \frac{\mu_0}{4\pi} \int \frac{Jh d\mathbf{r}}{R^{pr}} ,$$

with integration over the rim of the magnet. This means that the self-force (5) can be rewritten in a form

$$F_x = I(A_P - A_{\omega}) .$$

In such a form, the self-force is absolutely identical to that of the general solution of the problem [2]. In order to compare the obtained result (5) with the force of action (1), it is enough to integrate (5) by parts, that yields

$$F_x = -F_a .$$

It so happens that the sum of all the forces in closed system equals zero. Numerically this is shown in Fig. 2. Electric current flowing in the circuit $U$-$L$-$V$ should not have a noticeable effect on the weight of the body consisting of the magnetized disk $M$ and the conductor with current $I$. Whatever a current flows in the circuit, the weight will not change from this. The force of action is completely compensated by the force of self-action.

In order to measure the self-force, it is enough to determine the force of action in one way or another. The proof of standard electrodynamics lies in the invariability of weight when a current appears in the circuit.
IV. Measurements

Changing the direction of one of the interacting currents leads to a change in the direction of the force. This can separate the magnetic interaction of currents from other processes occurring in a closed or open electrodynamic system. Neither thermal processes nor processes accompanying the interaction of charges change the direction of weight when the direction of the current changes. This can become the main sign of equality to zero of the total force acting on the magnet and the electric current in section \( L \).

The current in the circuit should not change over time. This is the main purpose of the spring \( T \) through which the weight \( P \) is measured by the balance \( W \). The spring is therefore only a link between the measured body and the scale \( W \), although the relationship between the real body weight and what the scale shows is established and used in processing the measurement results. With a slight compression of the spring, electromechanical contacts \( S \) and \( C \) do not significantly affect the measured results of differences in weight when current passes through them.

![Graph showing change in weight with time at different currents](image)

**Fig. 3. Change of the weight with time at different currents \( I \) in the circuit.**

At first glance, it may seem that the condition \( F_a + F_s = 0 \) is not satisfied. When a current appears in the circuit \( (I>0) \), the weight increases (Fig. 3). Not this way. Shown in Fig. 3 the results were obtained for parallel currents \( I \) and \( Jh \) with magnetization \( J=0.73\cdot10^6 \) A/m, the ratio of radii equal to 1.3, thickness of the magnetized disk \( h=3\text{mm} \). This means that at a current of 1 ampere, the action force should be \( 1.5\cdot10^{-3} \) N.

If the conductive current \( I \) flows in the direction opposite with respect to the molecular current \( Jh \), then the corresponding dependencies of change in weight \( \Delta P(t) \) differ little from shown in Fig.3. In this case, the sign of the change in weight does not change. It makes no sense to give similar dependencies. It would be better to pay attention to the dependencies of the maximum difference in weight \( \Delta P_m \) from the current strength \( I \) (Fig. 4). The difference of these dependencies divided by 2 should be equal to the total force acting on the magnet coupled to the part of current \( L \). This force turned out to be several orders of magnitude less than the force of action. The condition \( F_a + F_s = 0 \) can be considered proven.
Fig. 4. A $\Delta P_m$ plotted against $I$. Points are experimental results. Dashed and solid lines are quadratic dependencies of maximum difference of the weight versus current for parallel and anti-parallel currents, respectively.

V. Conclusion
Deprived of attention was the force of self-action with which the unclosed part of the current L acts on itself. It is extremely small, many orders of magnitude less than the self-force with which a magnetized body and an unclosed current act on themselves. An appearance of one more force satisfying the law of conservation of momentum [7], in no way casts doubt on the main result of this work. The self-force by means of which the part L with current acts on itself, even if it exists, extremely little.

Rather limited information pertains to the origin of severe weight change $\Delta P$. A conductor with current is charged [8]. As a result, individual parts of conductor separated by electromechanical contacts can repealed from each other. This process is outside the purpose of this work. Therefore, it makes sense to treat it as a hypothesis.

References