IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 11, Issue 1 Ver. II (Jan. – Feb. 2019), PP 39-41
www.iosrjournals.org

"The study of System specific coherent states using supersymmetric quantum mechanical approach "

Dr. V.K. Srivastava
Professor, Department of Physics,
K.I.P.M. - College of Engg. & Technology
GIDA, Gorakhpur – 273209, U.P., India
Corresponding Author: Dr. V.K. Srivastava

Abstract: *Supersymmetric quantum mechanics has been developed as an elegant analytical approach to one dimensional problems. It generalizes the ladder operator approach used in the study of the harmonic oscillator. In this treatment, the factorization of a one dimensional Hamiltonian obtained using " charge operators". For 1D harmonic oscillator, lowering and raising charge operators can be used. It not only allow the factorization of 1D Hamiltonian but also form Lie algebraic structure which generates isospectral SUSY partner Hamiltonians. In addition several different approaches have been employed to study generalized and approximate coherent states of systems other than harmonic oscillator. In this paper, algebraic treatments being applied to the extension of coherent states for shape-invariant systems.*

Key words: *Harmonic oscillator, shape invariance systems and charge operators.*

Date of Submission: 01-02-2019 Date of acceptance:18-02-2019

I. Introduction:

The eigenstates of the various partner Hamiltonians are connected by applications of the charge operators. As an analytical approach, SUSY- QM approach has been utilized to study a number of quantum mechanical problems including the Morse oscillator [6] and the radial hydrogen atom equation. It can be used in discovery of new exactly solvable potentials. The harmonic oscillator is fundamental to a wide range of physics including the electromagnetic field, spectroscopy, solid state physics, coherent state theory and SUSYQM. The broad application of the harmonic oscillator stems from lowering and raising ladder operators which can be used to factor the Hamiltonian of the system. For example, cannonical coherent states are defined as the eigenstates of the lowering operator of the harmonic oscillator and they are also minimum uncertainty states which minimize the Heisenberg uncertainty product for position and momentum. The lowering operator of the harmonic oscillator annihilates the ground states and it minimizes the HUP (Heisenberg Uncertainty Product).

Conventional harmonic oscillator coherent states correspond to those states which minimizes the position – momentum uncertainty relation. However, these harmonic oscillator coherent states are also constructed by applying shift operators labelled with points of the discrete phase space to the ground state of the harmonic oscillator, known as " fiducial state "[7]. But, Klauder and Skagerstam choose to define coherent states in broadest sense. Similarly, the charge operator in SUSY-QM annihilates the ground state of the corresponding system. Let us construct system-specific coherent states for any bound quantum system by similarity between treatment of harmonic oscillator and supersymmetric quantum mechanics.

II. System-Specific Coherent States

On the basis of harmonic oscillator coherent state, the analysis of a bound quantum system in terms of the SUSY Heisenberg uncertainty principle suggests that construction of system-specific coherent states based on the SUSY-QM ground state. Similarly, the procedure for creating an over- complete set of such coherent states is to apply the shift operator to the ground state as a fiducial function [7, 8], as

$$\psi_{\propto}(x) = \langle x|\propto\rangle = \langle x|\bar{D}(\propto)|\psi_0\rangle = Ne^{ik_0(x-x_0)}e^{-x_0\left(\frac{d}{dx}\right)}\psi_0(x)$$
$$= Ne^{ik_0(x-x_o)}\psi_0(x-x_o), \qquad \ldots(1)$$

Where N is the normalization constant.

The raising and lowering operators for the shift operator are given by $\bar{a}^+ = (\bar{x} - i\bar{p}_x)/\sqrt{2}$ and $\bar{a} = (\bar{x} + i\bar{p}_x)/\sqrt{2}$ respectively. The quantity $\propto = (x_o + ik_0)/\sqrt{2}$ is a point in the phase space which completely describes the coherent state. Thus, the functions $\psi_{\propto}$ form an overcomplete set of the coherent states in the standard phase space which are specifically associated with the quantum-mechanical system described by the SUSY- displacement $W(x)$.

Let us consider a coordinate transformation given by $x^{'} = (x - x_o)$ for the system-specific coherent states in equation (1). The system-specific coherent state becomes

$$\psi_{\propto}(x^{'}) = e^{ik_0 x^{'}} \psi_0(x^{'}), \qquad ....(2)$$

Where $\psi_{\propto}(x^{'})$ is the normalized real-valued ground state wave function, and thus $\psi_{\propto}(x^{'})$ is also normalized. The momentum operator is invariant under the coordinate transformation (i.e., $\bar{p}_{x'} = \bar{p}_x$). It is clear that

$$\{\bar{W}(x^{'}) + i\bar{p}_{x'}\}|\psi_{\propto}\rangle = ik_0|\psi_{\propto}\rangle \qquad ..(3)$$

The averaged SUSY-displacement for the system-specific coherent state is given by

$$W_{0,\propto} = \langle\psi_{\propto}|W|\psi_{\propto}\rangle = \int_{-\infty}^{\infty} \psi_{\propto}^{*}(x^{'})W(x^{'})\psi_{\propto}(x^{'})dx^{'}$$

$$= -\int_{-\infty}^{\infty} \psi_0(x^{'}) \frac{d\psi_0(x^{'})}{dx^{'}} dx^{'}. \qquad ...(4)$$

Again, it follows from integration by parts that $W_{0,\propto} = 0$ for all system- specific coherent states. Similarly, the averaged momentum for the system-specific coherent state is given by

$$p_{0,\propto} = \langle\psi_{\propto}|\bar{p}_{x'}|\psi_{\propto}\rangle = k_0 - i\int_{-\infty}^{\infty} \psi_0(x^{'}) \frac{d\psi_0(x^{'})}{dx^{'}} dx^{'}. \qquad ....(5)$$

Because the integral is equal to zero, we get $p_{0,\propto} = k_0$. Then equation (3) can be rewritten as

$$\{\bar{W}(x^{'}) + i\bar{p}_x\}|\psi_{\propto}\rangle = (W_{0,\propto} + ip_{0,\propto})|\psi_{\propto}\rangle \qquad ....(6)$$

Analogous to the uncertainty condition for the ground state. This equation implies that the system-specific coherent state $|\psi_{\propto}\rangle$ minimizes the SUSY- displacement-momentum uncertainty product $\Delta\bar{W}\Delta\bar{p}_{x'}$ for the displaced coordinate $x^{'} = x - x_0$.

III. Discretized System-Specific Coherent States

A discretized SUSY-QM coherent state basis can be constructed by discretizing the continuous label $\propto = (q + ik)/\sqrt{2}$ and setting up a Von Neumann lattice in phase space with an appropriate density D. The discretized system-specific coherent state basis is given by

$$\psi_{\propto_i}(x) = \langle x|\propto_i\rangle = Ne^{ik_i(x-q_i)} exp\left[-\int_0^{x-q_i} W(x^{'})dx^{'}\right] \qquad ....(7)$$

Where i=1, …, M and M is the number of basis functions. The phase space grid points are defined in [11] and we can write

$$\{(q_i, k_i)\} = \left\{m\Delta x\sqrt{\frac{2\pi}{D}}, \frac{\pi}{\Delta x}\sqrt{\frac{2\pi}{D}}\right\}, \ m, n \in Z \qquad .....(8)$$

where m and n are an integers, hence i can be thought of as a joint index consisting of m and n. The quantity D is the density of grid points in units of $2\pi\hbar$. As discussed in Klauder and Skagerstam's book [7], generalized coherent states constructed by applying displacement operators to a fiducial state are overcomplete; however, completeness of the discretized system-specific coherent states in equation (7) has not been established here.

Since the ground state solves the time-independent Schrodinger equation for the corresponding Hamiltonian, the system-specific coherent states build in the dynamics of the system under investigation. This property leads to the expectation that these dynamically-adopted and system-specific coherent states will prove more rapidly convergent in calculations of the excited state energies and wave functions for quantum systems using variational methods.

By using the Rayleigh-Ritz variational principle, let us construct a trial wave function in terms of a linear combination of the system-specific coherent states

$$|\psi\rangle = \sum_{i=1}^{M} c_i|\propto_i\rangle, \qquad ......(9)$$

Where c_i are the coefficients.

Because of the non-orthogonality of the system-specific coherent states, the energy eigenvalues and wave functions are determined by solving the generalized eigenvalue problem [12] such that

$$HC = ESC, \qquad ......(10)$$

where $H_{ij} = \langle\propto_i |H| \propto_j\rangle$ is the matrix element of the Hamiltonian, $S_{ij} = \langle\propto_i | \propto_j\rangle$ is the overlap matrix, and C is a vector of linear combination coefficients for the eigenvector. Therefore, solving equation (10) yields the variational approximation to the eigenvalues and eigenvectors of the Hamiltonian operator.

IV. Results And Discussion

The application of SUSY-QM to non relativistic quantum systems generalizes the powerful ladder operator approach used in the treatment of the harmonic oscillator. The lowering operator of the harmonic oscillator annihilates the ground state, while the charge operator annihilates the ground state of corresponding quantum systems. The similarity between the lowering operator of harmonic oscillator and SUSY charge operator implies that the superpotential can be regarded as a system-specific generalized displacement variable. Analogous to the ground state of the harmonic oscillator which minimizes the HUP, the ground state of any bound quantum system was identified as minimizer of SUSY HUP. It was observed that such dynamically adopted coherent states yields significantly more accurate excited state energies and wave functions than were obtained with the same number of the conventional coherent states and from the standard harmonic oscillator basis.

V. Conclusion

The ladder operator approach of the harmonic oscillator and SUSY-QM formulation share strong similarity. This observation suggests that connection of the SUSY-QM with Heisenberg minimum uncertainty (μ^{-}) wavelets should be explored. The SUSY-displacement with the SUSY HUP can lead to the construction of the SUSY minimum uncertainty wavelets and the SUSY distributed approximating functions. These new functions and their potential applications in mathematics and physics are in progress.

Acknowledgement

The author gratefully acknowledges Head of the Department of Physics, Deen Dayal Upadhyay Gorakhpur University, Gorakhpur for encouragement and Prof H.C. Prasad for giving necessary suggestions.

References

[1]. Aleixo ANF and Balantekin A B, J.Phys-A: Math.Gen.,37(2004)8513.
[2]. Benedict MG and Molna B, Phys. Rev. A, 60 (1999) 1737.
[3]. Cooper I L , J. Phys. A; Math. Gen., 25 (1992) 1671.
[4]. Fukui T and Aizawa N, Phys. Lett. A, 180 (1993) 308.
[5]. Chou CC, Biamonte MT, Bodmann BG and Kouri DG (2012), "New System-Specific coherent states".
[6]. Jafarpour M and Afshar D, J. Phys. A; Math. Gen., 35 (2002) 87.
[7]. Klauder JR and Skagerstam BS [1985], Coherent states: Application in Physics and Mathematical physics, World Scientific, Singapore.
[8]. Kouri DJ, Papadakis M, Kakadiaris I and Hoffman DK, J. Phys. Chem A, 37(2003) 7318.
[9]. Merzbacher E, Quantum Mechanics [1997], Willey, New York.
[10]. Liu Y, Lei Y and Zeng J, Phys. Lett. A, 231 (1997) 9.
[11]. Anderson LM, J. Chem. Phys., 115(2001) 1158.
[12]. Varga K and Driscoll, JA [2011], Computational Nanoscience, Cambridge University Press, New York.

IOSR Journal of Applied Physics (IOSR-JAP) is UGC approved Journal with Sl. No. 5010, Journal no. 49054.

Dr. V.K. Srivastava" ."The study of System specific coherent states using supersymmetric quantum mechanical approach "." IOSR Journal of Applied Physics (IOSR-JAP) , vol. 11, no. 1, 2019, pp. 39-41.