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Partial Derivatives Involving Generalized I–Function of Two Variables

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Abstract: *The aim of this paper is to derive partial derivatives involving generalized I–function of two variables.*

I. Introduction

The generalized I–function of two variables introduced by Goyal and Agrawal [1], will be defined and represented as follows:

$$I\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right] = I^{m,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\middle| \begin{matrix}[(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}]\\ [(b_j:\beta_j,B_j)_{1,n}],[(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}]\end{matrix}\right.$$
$$\left.\begin{matrix}:[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_{i'}}];[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}]\end{matrix}\right]$$
$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\,x^{\xi}y^{\eta}d\xi d\eta, \qquad (1)$$

where

$$\phi_1(\xi,\eta) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi-B_j\eta)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi+A_j\eta)}{\sum_{i=1}^{r}[\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi-A_{ji}\eta)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi+B_{ji}\eta)},$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_1}\Gamma(d_j-\delta_j\xi)\prod_{j=1}^{n_1}\Gamma(1-c_j+\gamma_j\xi)}{\sum_{i'=1}^{r'}[\prod_{j=m_1+1}^{q_{i'}}\Gamma(1-d_{ji'}+\delta_{ji'}\xi)\prod_{j=n_1+1}^{p_{i'}}\Gamma(c_{ji'}-\gamma_{ji'}\xi)},$$

$$\theta_3(\eta) = \frac{\prod_{j=1}^{m_2}\Gamma(f_j-F_j\eta)\prod_{j=1}^{n_2}\Gamma(1-e_j+E_j\eta)}{\sum_{i''=1}^{r''}[\prod_{j=m_2+1}^{q_{i''}}\Gamma(1-f_{ji''}+F_{ji''}\eta)\prod_{j=n_2+1}^{p_{i''}}\Gamma(e_{ji''}-E_{ji''}\eta)},$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, $p_{i'}$, $p_{i''}$, q_i, $q_{i'}$, $q_{i''}$, m, n, n_1, n_2, n_j and m_k are non negative integers such that $p_i \geq n \geq 0$, $p_{i'} \geq n_1 \geq 0$, $p_{i''} \geq n_2 \geq 0$, $q_i \geq m > 0$, $q_{i'} \geq 0$, $q_{i''} \geq 0$, (i = 1, …, r; i′ = 1, …, r′; i′′ = 1, …, r′′; k = 1, 2) also all the A's, α's, B's, β's, γ's, δ's, E's and F's are assumed to be positive quantities for standardization purpose; the definition of I-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the ξ–plane and runs from – ω∞ to + ω∞, with loops, if necessary, to ensure that the poles of $\Gamma(d_j–\delta_j\xi)$ (j = 1, ..., m_1) lie to the right, and the poles of $\Gamma(1 - c_j + \gamma_j\xi)$ (j = 1, ..., n_1), $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ (j = 1, ..., n) to the left of the contour.

The contour L_2 is in the η–plane and runs from – ω∞ to + ω∞, with loops, if necessary, to ensure that the poles of $\Gamma(f_j - F_j\eta)$ (j=1,....., n_2) lie to the right, and the poles of $\Gamma(1 - e_j + E_j\eta)$ (j = 1, ..., m_2), $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ (j = 1, ..., n) to the left of the contour. Also

$$R' = \sum_{j=1}^{p_i}\alpha_{ji} + \sum_{j=1}^{p_{i'}}\gamma_{ji'} - \sum_{j=1}^{q_i}\beta_{ji} - \sum_{j=1}^{q_{i'}}\delta_{ji'} < 0,$$

$$S' = \sum_{j=1}^{p_i}A_{ji} + \sum_{j=1}^{p_{i''}}E_{ji''} - \sum_{j=1}^{q_i}B_{ji} - \sum_{j=1}^{q_{i''}}F\delta_{ji'} < 0,$$

$$U = \sum_{j=n+1}^{p_i}\alpha_{ji} - \sum_{j=m+1}^{q_i}\beta_{ji} + \sum_{j=1}^{m_1}\delta_j - \sum_{j=m_1+1}^{q_{i'}}\delta_{ji'} + \sum_{j=1}^{n_1}\gamma_j - \sum_{j=n_1+1}^{p_{i'}}\gamma_{ji'} > 0, \qquad (2)$$

$$V = -\sum_{j=n+1}^{p_i}A_{ji} - \sum_{j=m+1}^{q_i}B_{ji} - \sum_{j=1}^{m_2}F_j - \sum_{j=m_{21}+1}^{q_{i''}}F_{ji''} + \sum_{j=1}^{n_2}E_j - \sum_{j=n_2+1}^{p_{i''}}E_{ji''} > 0, \qquad (3)$$

and |arg x| < ½ Uπ, |arg y| < ½ Vπ.

II. Result Required

The following result are required in our present investigation:
From Rainville [2]:

$$z\Gamma(z) = \Gamma(z+1). \tag{4}$$

III. Main Result

In this paper we will establish the following partial derivatives:

$$\frac{\partial}{\partial x}\left\{I^{m,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]\right\}$$

$$= x^{-1}I^{m,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\middle|\begin{matrix}[(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}]\\ [(b_j:\beta_j,B_j)_{1,n}],[(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}]\end{matrix}\right.$$
$$\left.\begin{matrix}:(0,1),[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_{i'}}],(1,1);[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}]\end{matrix}\right], \tag{5}$$

where | arg x | < ½ Uπ, | arg y | < ½ Vπ, where U and V are given in (2) and (3) respectively.

$$\frac{\partial}{\partial y}\left\{I^{m,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]\right\}$$

$$= y^{-1}I^{m,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\middle|\begin{matrix}[(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}]\\ [(b_j:\beta_j,B_j)_{1,n}],[(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}]\end{matrix}\right.$$
$$\left.\begin{matrix}:[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];(0,1),[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_{i'}}];[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}],(1,1)\end{matrix}\right], \tag{6}$$

where | arg x | < ½ Uπ, | arg y | < ½ Vπ, where U and V are given in (2) and (3) respectively.

$$\frac{\partial^2}{\partial x\partial y}\left\{I^{m,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]\right\}$$

$$= (xy)^{-1}I^{m,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\middle|\begin{matrix}[(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}]\\ [b_j:\beta_j,B_j)_{1,n}],[(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}]\end{matrix}\right.$$
$$\left.\begin{matrix}:(0,1),[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];(0,1),[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_{i'}}],(1,1);[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}],(1,1)\end{matrix}\right], \tag{7}$$

where | arg x | < ½ Uπ, | arg y | < ½ Vπ, where U and V are given in (2) and (3) respectively.

Proof:

To establish (5), we use for the generalized I-function of two variables Mellin-Barnes types of contour integral as given in (1), on the left-hand side of (5), change the order of integration and derivative (which is justified under the conditions given with (5)), we then obtain

Left-hand side of (5) $= \frac{(-1)}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\left[\frac{\partial}{\partial x}x^{\xi}\right] y^{\eta}d\xi d\eta$

$= \frac{(-1)}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)[\xi x^{\xi-1}]\, y^{\eta}d\xi d\eta$

$= \frac{(-1)}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\left[\frac{\xi\Gamma(\xi)}{\Gamma(\xi)}x^{\xi-1}\right] y^{\eta}d\xi d\eta$

Now using the result (4) and interpreting the resulting contour integral as the generalized I-function of two variables, we once get the right-hand side of (5). Proceeding on the similar way, the results (6) and (7) can be obtained.

IV. Special Cases

On choosing m = 0 main results, we get following partial derivatives in terms of I-function of two variables:

$$\frac{\partial}{\partial x}\left\{I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]\right\}$$

$$= x^{-1}I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\middle|\begin{matrix}[(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}]\\ [(b_j:\beta_j,B_j)_{1,n}],[(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}]\end{matrix}\right.$$
$$\left.\begin{matrix}:(0,1),[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_{i'}}],(1,1);[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}]\end{matrix}\right]; \tag{8}$$

$$\frac{\partial}{\partial y}\left\{I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\begin{bmatrix}x\\y\end{bmatrix}\right\}$$
$$= y^{-1} I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\begin{bmatrix}x\\y\end{bmatrix}\Bigg|^{[(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}]}_{[(b_j:\beta_j,B_j)_{1,n}],[(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}]}$$
$$\left.\begin{matrix}:[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];(0,1),[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_{i'}}];[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}],(1,1)\end{matrix}\right]; \qquad (9)$$

$$\frac{\partial^2}{\partial x\partial y}\left\{I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\begin{bmatrix}x\\y\end{bmatrix}\right\}$$
$$= (xy)^{-1} I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\begin{bmatrix}x\\y\end{bmatrix}\Bigg|^{[(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}]}_{[(b_j:\beta_j,B_j)_{1,n}],[(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}]}$$
$$\left.\begin{matrix}:(0,1),[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];(0,1),[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_{i'}}],(1,1);[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}],(1,1)\end{matrix}\right], \qquad (10)$$

where | arg x | < ½ U'π, | arg y | < ½ V'π, where U' and V' are given as follows respectively:

$$U' = \sum_{j=n+1}^{p_i}\alpha_{ji} - \sum_{j=1}^{q_i}\beta_{ji} + \sum_{j=1}^{m_1}\delta_j - \sum_{j=m_1+1}^{q_{i'}}\delta_{ji'} + \sum_{j=1}^{n_1}\gamma_j - \sum_{j=n_1+1}^{p_{i'}}\gamma_{ji'} > 0,$$

$$V' = -\sum_{j=n+1}^{p_i}A_{ji} - \sum_{j=1}^{q_i}B_{ji} - \sum_{j=1}^{m_2}F_j - \sum_{j=m_{21}+1}^{q_{i''}}F_{ji''} + \sum_{j=1}^{n_2}E_j - \sum_{j=n_2+1}^{p_{i''}}E_{ji''} > 0,$$

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