IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 4 Ver. III (Jul - Aug. 2015), PP 37-42
www.iosrjournals.org

Numerical Solution of Multi-Echelon Inventory Model Using Matlab

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Abstract: *In this paper we consider an integrated inventory control system that consists of a Manufacturer (MF), single Warehouse (WH), one Distribution Centre (DC) and n identical Retailers. The Demands occurring during the stock out periods are backlogged up to a specified quantity 'b' at DC. The optimization criterion is to minimize the total cost rate incurred at all the location subject to the performance level constrains. The steady state probability distribution and the operating characteristics are obtained explicitly. The required algorithm is designed and it is executed using MATLAB. Numerical example with sensitivity analysis is provided to illustrate the proposed model.*

Keywords: *Inventory control, Optimization, Markov process, Multi-echelon system, Supply Chain.*

I. Introduction

The study of supply chain management (SCM) started in the late 1980s and has gained a growing level of interest from both companies and researchers over the past three decades. There are many definitions of supply chain management. Hau Lee, the head of the Stanford Global Supply Chain Management Forum (1999), gives a simple and straight forward definition at the forum website as follows: 'Supply chain management deals with the management of materials, information and financial flows in a network consisting of suppliers, manufacturers, distributors, and customers'. From this definition, we can see that SCM is not only an important issue to manufacturing companies, but is also relevant to service and financial firms. A supply chain may be defined as an integrated process wherein a number of various business entities (suppliers, distributors and retailers) work together in an effort to (1) acquire raw materials (2) process them and then produce valuable products and (3)transport these final product to retailers. The process and delivery of goods through this network needs efficient maintains of inventory, communication and transportation system. The supply chain is traditionally characterized by a forward flow of materials and products and backward flow of information, money.

One of the most important aspects of supply chain management is inventory control. Inventory control models are almost invariably stochastic optimization problems with objective being either expected costs or expected profits or risks. In practice, a retailer may want an optimal decision which achieves a minimal expected cost or a maximal expected profit with low risk of deviating from the objective

A complete review was provided by Benita M. Beamon (1998) [4]. However, there has been increasing attention placed on performance, design and analysis of the supply chain as a whole. HP's (Hawlett Packard) Strategic Planning and Modeling (SPaM) group initiated this kind of research in 1977. From practical stand point, the supply chain concept arose from a number of changes in the manufacturing environment, including the rising costs of manufacturing, the shrinking resources of manufacturing bases, shortened product life cycles, the leveling of planning field within manufacturing, inventory driven costs (IDC) involved in distribution (2005) and the globalization of market economics. With-in manufacturing research, the supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic work of Clark and Scarf (1960)[6]. Hadley and Whitin (1963)[9], among others, present the methods to find the optimal or near optimal solution to minimize the inventory costs at a single stocking point with stochastic demand, based on the continuous review (r,Q), periodic review (R,T), and one-for-one polices. A complete review on this development was recorded by Federgruen (1993) [8]. Axsäter, S (1990)[2] proposed an approximate model of inventory structure in SC. This model presumed that the demand of distribution centre and retailers obeyed the Normal distribution. This method reduced the algorithmic complexity and it was applicable for large-scale inventory. But when the individual retail demand is small, using the Normal distribution to approximate demand would have quite a part of negative value, so it is un-reasonable in some degree. Recent developments in two-echelon models may be found in Q. M. He, and E. M. Jewkes (2000) [12], and Antony Svoronos & Paul Zipkin(1991) [1]. A continuous review perishable inventory system at service facilities was studied by Elango.C 2000[7]. A continuous review (s, S) policy with positive lead times in two- echelon Supply Chain was considered by Krishnan. K and Elango.

C. 2007 [10]. A Modified (Q*, r) Policy for Stochastic Inventory Control Systems in Supply Chain with lost sale model was considered by Bakthavachalam. R, Navaneethakrishnan. S, Elango.C,(2012)[3].

In this paper, an integrated inventory control system that consists of a Manufacturer (MF), single Warehouse (WH), one Distribution Centre (DC) and n identical retailers are considered.

The rest of the paper is organized as follows; the model formulation is described in section 2. In section 3, the steady state analyses are done. Section 4 deals the operating characteristics of the system and section 5, deals with the cost analysis for the operation. Numerical example and sensitivity analysis are provided in the section 6 and in the last section 7 concludes the paper.

II. The Model Description

The inventory control system in supply chain considered in this model is defined as follows.

We considered a Supply Chain System consisting of a manufacturer (MF), warehousing facility (WH), single distribution centre (DC) and n identical retailers(R_i) dealing with a single finished product. These finished products move from the manufacturer through the network that consists of WH, DC, Retailers then the final customer.

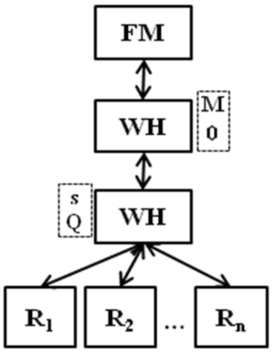

Figure 1 Multi-echelon Inventory System

A finished product is supplied from MF to WH which adopts (0, M) replenishment policy then the product is supplied to DC which adopts (s, Q) policy. Then demand at retailers node follows an independent Poisson distribution with rateλi (i = 1, 2, 3, …, n). Scanners collect sales data at retailer nodes and Electronic Data Interchange (EDI) allows these data to be shared to DC. With the strong communication network and transport facility, a unit of item is transferred to the corresponding retailer with negligible lead time. That is how all the inventory transactions are managed by DC. Supply to the Manufacturer in packets of Q items is administrated with exponential lead time having parameter μ (>0).The replenishment of items in terms of pockets is made from Manufacturer to WH instantaneously. The Demands occurring during the stock out periods at DC are backlogged up to a specified quantity ‘b’. The maximum inventory level at DC node S is fixed and the reorder point is s and the ordering quantity is Q(=S-s) items. The maximum inventory level at Manufacturer is M (M=nQ).

The optimization criterion is to minimize the total cost rate incurred at all the location subject to the performance level constrains. According to the assumptions the on hand inventory levels at all the nodes follows a random process.

Notations:

$[R]_{ij}$	:	The element /sub matrix at $(i,j)^{th}$ position of R.
0	:	Zero matrix.
I	:	Identity matrix.
e	:	A column vector of ones of appropriate dimension.
k_0	:	Fixed ordering cost, regardless of order size at DC.
k_1	:	The average setup cost for at WH
h_0	:	The holding cost per unit of item per unit time at DC.
h_1	:	The average holding cost per unit of item per unit time at WH
g	:	The unit shortage cost at DC.

$$\sum_{i=1}^{k} a_i = a_1 + a_2 + \ldots + a_k. \qquad \sum_{i=0}^{nQ}{}^{*}\, i = 0 + Q + 2Q + \ldots + nQ.$$

III. Analysis

Let I_0 (t) and I_1 (t) denote the on-hand inventory levels at Distribution Centre and Warehouse respectively at time t+. From the assumptions on the input and output processes,

$I(t) = \{ (I_0(t), I_1(t) : t \geq 0 \}$ is a Markov process with state space

$$E = \left\{ \begin{array}{l} (j,q) \ / \ j = S, (S-1), \ \dots \ , s, (s-1), \ \dots \ , 2, 1, 0. \ -1, -2, \dots -b, \ \text{and} \\ \qquad q \ = \ nQ, (n-1)Q, \ \dots \ , Q, 0. \end{array} \right\}.$$

We use negative sign for backlogging quantity.

Since E is finite and all its states are recurrent non-null, $\{I(t), t \geq 0\}$ is an irreducible Markov process with state space E and it is an ergodic process. Hence the limiting distribution exists and is independent of the initial state.

The infinitesimal generator of this process $R = (a(j, q : k, r))_{(j,q),(k,r) \in E}$ can be obtained from the following arguments.

The arrival of a demand for an item at Distribution Centre makes a state transition in the Markov process from (j, q) to (j-1, q) with intensity of transition λ (where $\lambda = \lambda i$, i = 1,2,3… n.)

Replenishment of inventory at Distribution Centre makes a state transition from (j, nQ) to (j + Q, (n-1) Q) with rate of transition μ (> 0).

The infinitesimal generator R is given by

$$R = \begin{pmatrix} A & B & 0 & \cdots & 0 & 0 \\ 0 & A & B & \cdots & 0 & 0 \\ 0 & 0 & A & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & B \\ B & 0 & 0 & \cdots & 0 & A \end{pmatrix}$$

The entries of the block partition matrix R can be written as

$$[R]_{q \times r} = \begin{cases} A & \text{if} \quad q = r, \qquad r = nQ, (n-1)Q, (n-2)Q, \dots Q. \\ B & \text{if} \quad q = r + Q \qquad r = (n-1)Q, (n-2)Q \dots 2Q. \\ B & \text{if} \quad q = nQ \\ 0 & \text{otherwise} \end{cases}$$

The sub matrices A and B are given by

$$[A]_{j \times q} = \begin{cases} \lambda & \text{if} \quad q = j-1, \quad j = S, \ S-1, S-2, \ \dots \ 1, 0, -1, \dots -(b+1) \\ -\lambda & \text{if} \quad q = j \qquad j = S, \ S-1, S-2, \ \dots \ (s+1) \\ -(\lambda + \mu) & \text{if} \quad q = j \qquad j = s, s-1, s-2, \dots, \ 1 \ , 0, -1, \dots -(b+1) \\ -\mu & \text{if} \quad q = j \qquad j = \ 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$[B]_{j \times q} = \begin{cases} \mu & \text{if} \quad q = j + Q \qquad j = s, s-1, s-2, \dots, 0. \\ 0 & \text{otherwise} \end{cases}$$

3.1 Transient analysis

Define the transition probability function

$$p_{j,q}(k, r ; t) = \Pr\{ (I_0(t), I_1(t) = (k, r) \mid (I_0(0), \ I_1(0)) = (j, \ q) \}.$$

The corresponding transient matrix function is given by $P(t)=(p_{j,q}(k,r\,;t))_{(j,q),(k,r)\in E}$ vector which satisfies the Kolmogorov-forward equation $P'(t)=P(t)R$, where R is the infinitesimal generator of the process $\{I(t), t\geq 0\}$. The above equation, together with initial condition $P(0)=I$, the solution can be expressed in the form $P(t)=P(0)e^{Rt}=e^{Rt}$, where the matrix expansion in power series form is $e^{Rt}=I+\sum_{n=1}^{\infty}\frac{R^n t^n}{n!}$.

3.2 Steady state analysis:

The structure of the infinitesimal matrix R reveals that the state space E of the Markov process $\{I(t), t\geq 0\}$ is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$\nu_j^q=\lim_{t\to\infty}pr\{(I_0(t), I_1(t)=(j,\,q)\}_{(j,q)\in E}$, where ν_j^q is the steady state probability that the system be in state (j, q) , (Cinlar [5]).

Let $\nu=(\nu^{nQ},\nu^{(n-1)Q},\nu^{(n-2)Q},\ \ldots\ ,\nu^{Q},\nu^{0})$ denote the steady state probability distribution where $\nu^q=\left(\nu_S^q,\nu_{S-1}^q,\ldots,\nu_s^q,\nu_{s-1}^q,\ldots,\nu_0^q,\nu_{-1}^q,\ \nu_{-2}^q\ldots\nu_{-b}^q\right)$ for the system under consideration. For each (j, q), ν_j^q can be obtained by solving the matrix equation $\nu A=0$ together with normalizing condition $\sum_{(j,q)\in E}\nu_j^q=1$

Assuming ν_Q^Q = a, we obtain the steady state probabilities of the system states as

${}^{iQ}\nu=(-1)^k\ a\,(BA)^k$, i = 1,2, … n ; k = n-i+1,

Where a = $e^1\left[\sum_{i=0}^{n-1}(-1)^i\ \ (BA^{-1})^i\right]^{-1}$.

3.3 Algorithm

The algorithm to compute the long run expected inventory cost (for all the echelons) is designed.

Step – 1 : Determine the rate matrices A and B.
Step – 2 : Generate the **R** Matrix.
Step – 3 : Solve the system Π R = 0 with normalizing condition ΣΠ= 1.
Step – 4 : Compute the operating characteristics
Step-5 : Compute the long run expected inventory cost.

IV. Operating Characteristics

In this section, we derive some important system performance measures.

4.1. Mean Reorder Rates:

The Mean reorder rate at Distribution Centre (β_0) and Warehouse (β_1) are given by

$$\beta_0=\lambda\sum_{q=Q}^{nQ}{}^{*}\nu_{s+1}^q \qquad \ldots\ 4.1$$

$$\beta_1=\mu\sum_{j=0}^{s}\nu_j^Q \qquad \ldots\ 4.2$$

4.2. Mean Inventory Levels:

The mean inventory level in the steady state at Distribution Centre (I_0) and Warehouse (I_1) are given by

$$I_0=\sum_{q=Q}^{nQ}{}^{*}\left(\sum_{j=0}^{S}j.\nu_j^q\right) \qquad \ldots\ 4.3$$

$$I_1 = \sum_{j=0}^{S}\left(\sum_{q=Q}^{nQ}{}^{*} q.\nu_j^q\right). \qquad \ldots 4.4$$

4.3.Mean Shortage Rate:

The unsatisfied demands has been backlogged at DC up to ‘b’ numbers, hence the shortage rate (α_0) at DC is given by

$$\alpha_0 = \lambda \sum_{q=Q}^{nQ}{}^{*} \nu_{-b}^{q} \qquad \ldots 4.5$$

V. Cost Analysis

In this section, a cost structure for the proposed model is imposed and it is analyzed by the criteria of minimization of long run total expected cost per unit time.

The long run expected cost rate C(s, Q) is given by

$$C(s,Q) = h_0 I_0 + h_1 I_1 + k_0\beta_0 + k_1\beta_1 + g\alpha_0 \qquad \ldots 4.6$$

Substituting 4.1 to 4.5 in 4.6 we get

$$C(s,Q) = h_0\left(\sum_{q=Q}^{nQ}{}^{*}\left(\sum_{j=0}^{S} j.\nu_j^q\right)\right) + h_1\left(\sum_{j=0}^{S}\left(\sum_{q=Q}^{nQ}{}^{*} q.\nu_j^q\right)\right) +$$

$$k_0\left(\lambda\left(\sum_{q=Q}^{nQ}{}^{*}\nu_{s+1}^{q}\right)\right) + k_1\left(\mu_0\sum_{j=0}^{s}\nu_j^Q\right) + g\left(\lambda\sum_{q=Q}^{nQ}{}^{*}\nu_{-b}^{q}\right) \qquad \ldots 4.7$$

Although we have not proved analytically the convexity of the cost function C(s,Q), the experience with considerable number of numerical examples indicates that C(s,Q) for fixed Q appears to be convex in s. In some cases, it turned out to be an increasing function of s. For large number of causes C(s, Q) revealed a locally convex structure. Hence, the numerical search procedure was adopted to determine the optimal values s^*.

VI. Numerical Example And Sensitivity Analysis

In this section, the problem of minimizing the steady state expected cost rate under the following cost structure is discussed. It is assumed that We assume $k_1 \geq k_0$, since the setup cost which includes the freight charges could be higher for the larger size order (pockets) compared to that of the small one initiated at retailer nodes. Regarding the holding cost, it is assumed that $h_1 \leq h_0$, since the holding cost at distribution node is less than that of the retailer node as the rental charge may be high at retailer node. The results obtained in the steady state case may be illustrated through the following numerical example.

For the following example, it is assumed that the average demand rate for 5 retailers is $\lambda = 0.4$, and the other parameters are fixed as S = 12, M = 4 (packets), $\mu = 0.5$, $h_0 = 0.5$, $h_1 = 0.3$, $k_0 = 0.5$, $k_1 = 4$, and g = 0.3. The total cost rate for expected feasible reorder levels are given by

Table – 1: The total cost rate expected as a function of C(s, Q).

s	C(s, Q)
1	10.54397
2*	**10.45190***
3	10.68504
4	10.74023
5	10.91414

The optimal reorder level and optimal cost rate are indicated by the symbol ‘*’.

The graphical representation of the long run expected cost rate C(s,Q) is given in figure -2

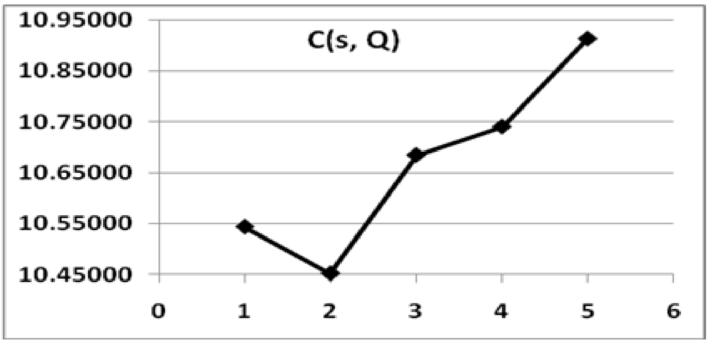

Figure – 2: The graphical representation of C(s,Q)

Sensitivity Analysis : Table -2 presents a numerical study to exhibit the sensitivity of the system on the effect of varying holding cost, h_0 and h_1 with fixed reorder level s = 2.

Table – 2: The total expected cost deviation based on various h_0 and h_1

h0 / h1	0.5	1.5	2.5	3.5	4.5
0.1	6.71116	15.526	24.341	33.155	17.642
0.2	8.58153	17.396	26.211	35.026	18.428
0.3	10.4519	19.267	28.081	36.896	19.214
0.4	12.3223	21.137	29.952	38.766	20

It is observed that when the holding costs, (h_0 and h_1) of the items are increasing then the total expected cost rate C(s,Q) also monotonically increase. Hence, the holding costs are key parameters for the proposed inventory control system in supply chain.

VII. Conclusion

In this paper, we analyzed a continuous review inventory control system to Multi-echelon system with partial backlogging. The structure of the chain allows vertical movement of goods from warehouse to distribution center then to retailer. The model dealing with the supply from warehouse to distribution center then to retailer is in the terms of pockets. We are also proceeding in this multi-echelon stochastic inventory system with perishable products. This model deals with only tandem network (basic structure of supply chain). This structure can be extended to tree structure and to be more general.

Acknowledgements

I acknowledge Prof. Balasubramanian Sathyamurthy, Associate Professor, Department of Biochemistry, REVA University for his intensive help for pursuing and publishing this research article

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