Enhancement of the Performance Characteristics of CIC Decimation Filters for Multirate DSP Applications

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Abstract
Cascaded Integrator Comb (CIC) filters can perform filtering without multiplying operations and this quality makes practicing engineers and researchers find it highly useful especially for high sample rate signals. Regrettably, the monotonic passband characteristic of the filter generates an unavoidable high passband droop and low attenuation in the stopband which are not acceptable in many applications. Several works have been proposed to improve the performance of CIC filters either in the pass band or stop band or both in a manner that will provide either smaller pass band error or greater stop band attenuation. This paper presents a new sharpening technique which depends on Taylor’s expansion of the transfer function of the filter with the assumption that the filter passes dc signal without change. The technique allows the sharpened filter to be re sharpened in order to bring the frequency response of the filter more nearly to an ideal value than the other sharpening techniques without increasing the computational complexity. A design example is presented to compare the passband droop and alias rejection of the original CIC filter, the sharpened CIC filters that use Binomial functions, and the proposed sharpened filter. All computations and response characteristics of the filters are performed using Matlab program. The proposed sharpening technique provides better attenuation of -187.80dB and pass band gain of -0.00171dB which corresponds to 21.4 % and 82.19 % improvement respectively over the best existing one. The proposed technique is used in multirate filtering system where improved filtering is needed.

Keywords: Cascaded Integrator Comb (CIC) filter; Sharpening technique; Taylor’s expansion; Multirate Digital Signal Processing (DSP); Decimation.

I. Introduction
Multi rate DSP enables a digital signal to be converted from one sample rate to another without destroying the required signal component. Owing to rate conversion requirements, multirate filtering becomes an effective tool for developing digital filters with different input and output sampling rates [1], [2]. It presents a simple solution to filtering problems such as when one filter operating at a constant sampling rate is of very high order and suffers from noise due to quantization errors. Multirate filter constructed by combining several low-order sub-filters and sample rate changing devices can be more effective because of its efficiency in computation and enhanced performance. It uses interpolators or decimators to increase or decrease the sampling rate and keep the imaging or aliasing error respectively in the pass band within arbitrary bounds [3]. CIC filters are good choice for constructing interpolators and decimators for anti-imaging and anti-aliasing requirements because of its simplicity, limited coefficient storage system and linear phase response. It can perform filtering without multiplying operations and this quality makes practicing engineers and researchers find it highly useful especially for high sample rate signals. The Integrator filter and Comb filter are the main constituent of a CIC filter which can be constructed as a cascade of either an Integrator section operating at high sampling rate with a Comb section operating at lower sampling rate or vice-versa. It is more efficient in hardware and energy than the traditional Finite Impulse Response (FIR) filters because it uses only summation units and delays. The filter has interpolator or decimator built in its structure as opposed to conventional FIR filters. With an integer factor, it is used as a decimator filter for robust anti-imaging and anti-aliasing filtering. With a non-integer factor, it is implemented with an interpolation filter in order to fine tune the sample rate and to minimize the attenuation and power consumption of the aliasing band. The CIC decimator filter has a K cascaded integrator stages clocked at sampling frequency f_s, followed by a change in the rate by a factor N, and a K cascaded comb stages running at sampling rate \( \frac{f_s}{N} \). Regrettably, the monotonic passband characteristic of the filter generates an unavoidable high passband droop and low attenuation in the stopband which are not acceptable in many applications. The
stop band attenuation increases as the number of filter stages increases leading to an increased droop in the pass band which may distort the decimated signal [4]. This is unacceptable in many applications and needs to be compensated.

II. The Existing Works to Improve the Performance of CIC Filters

Previous works have been proposed to enhance the performance characteristics of CIC filters in the pass band [5] or stop band [6],[7] or both[8] in such a way as to give lower pass band error or higher stop band attenuation.

The authors in [9] worked on realizing an efficient multi-stage comb rotated sinc (RS) decimator filter with the use of recursive and non-recursive architectures. Poly phase decomposition was utilized for the realization of the first section in a non-recursive form whereas the second section was realized in a recursive form with the use of a modified RS compensator. The magnitude response of the proposed filter improved with lower complexity when compared with the original comb filter. The researchers in [10] presented a design of multiplier-less CIC filter to compensate the CIC pass band droop defined by pass band frequency shown in equation (1) and maximum pass band deviation less than 0.4dB.

$$\omega_p = \frac{\pi}{2} N$$ \hspace{2cm} (1)

(where N is the decimation factor). The filter gave better pass band droop compensation in the wideband than the existing methods. Maximally flat error criterion based CIC filter compensator design was proposed in [11]. The filter gave lower pass band droop when compared with sharpened filters of the same order. A technique for enhancing the magnitude characteristics of CIC filters by cascading an FIR filter with inverse magnitude response with the CIC filter was proposed in [8]. The proposed filter performed Decimation was performed efficiently by the proposed filter with additions and subtractions only making it cheaper and attractive for software radio applications.

The authors in [12] presented a paper on how to have control over the amplitude characteristics of the comb decimator filter using a generalized sharpening technique. Amplitude Change Function (ACF) was used to map the amplitude of the original values to the new values through a polynomial function with certain design parameters such as the tangencies of the sharpening polynomial to the line, the compensator parameters and the number of cascaded filters. The generalized sharpening technique showed enhanced pass band characteristic with a little increase in computational complexity when compared to the comb filter sharpened with the traditional method.

A performance enhancement and comparative analysis of the original CIC filter, sharpened CIC filter and modified sharpened CIC filter was presented in [13]. The sharpening technique was employed to improve the pass band and stop band attenuation, the compensation technique improves the characteristic of the passband while improvement in stop band attenuation was done using cosine pre filters. The modified cascaded filter in comparison with the original filter has much pass band droop and better alias rejection. The sharpened CIC filter has better alias rejection while the modified sharpened CIC filter has little pass band droop.

The authors in [14] proposed a sharpening technique that is based on rotated zeros in order to enhance the magnitude response of a comb filter. CIC filters with even length was considered and used Rouche’s theorem was used to verify that the rotated zeros were in the unit circle. The magnitude response of the original comb filter and the sharpened rotated comb filter were compared.

The design of a three-stage CIC decimator filter with better magnitude response characteristic using sharpening technique was presented in [15]. The response of the proposed sharpened filter was compared with existing conventional CIC filter[16] and modified sharpened CIC filter [17]. The results showed that the proposed decimation filter has improved pass droop and better stop band alias rejection. A new CIC compensation filter without multipliers for Software Defined Radio application was presented in [18]. The proposed compensator required eleven adders with an absolute value of maximum pass band deviation less than 0.1dB in the wide pass band. Another paper was also proposed in [19] based on the sine wave magnitude responses multiplierless filters with the aim of achieving better CIC wideband and narrowband compensation with fewer adders. The proposed compensators required a maximum of nine adders with a slight increase of the absolute value of maximum pass band deviation of 0.25dB and 0.05dB for wideband and narrowband SDR applications.

In this research paper, a new simplified and efficient sharpening technique based on Taylor expansion is proposed. This technique involves re-sharpening the sharpened CIC decimation filter. It avoids the use of sub filter without causing any performance degradation leading to a very economical approach. The performances in terms of pass band gain and attenuation rejection ratio of the original CIC filter, two existing sharpening functions proposed by[14] and [20] are compared with the proposed function and the results are evaluated and analyzed.
III. Frequency Characteristics of CIC Filters

The comb-filter has a rectangular window impulse response expressed as,

\[ g[n] = \begin{cases} \frac{1}{N}, & \text{for } 0 \leq n < N-1, \\ 0, & \text{otherwise} \end{cases} \]  

(2)

Where, N is an integer. In a non-recursive form, the transfer function \( G(z) \) of the comb-filter in z-domain representation is given by,

\[ G(z) = \frac{\sum_{k=0}^{N-1} z^{-k}}{\sum_{k=0}^{N'} z^{-k}} \]

(3)

The filter has a low-pass frequency response which can be improved by cascading several identical comb filters. In a recursive form, the transfer function of a multi-stage comb filter composed of K identical single stage comb-filter given by,

\[ G(z) = \left( \frac{1 - z^{-N}}{1 - z^{-m}} \right)^{K} \]

(4)

where N is the decimation factor and K is the order of CIC filter which presents the number of the stages of CIC filters. According to equation (3), the CIC filter at the sampling frequency \( f_s \) can be implemented by cascading the comb section \((1 - z^{-N})\) and the integrator section \((1/1 - z^{-1})\) which give an extremely efficient device that performs filtering operation by only additions of the two sections. The frequency response \( G(z) \) of the filter determined by taking Fourier Transform of equation(3) [3] is given by,

\[ G(e^{j\omega}) = \frac{\sin\left(\frac{\omega M/2}{N}\right)}{\sin\left(\frac{\omega}{N}\right)} e^{j\omega (M-1)/2} \]

(5)

Where \( e \) is a phase shift parameter and it is ignored.

\( G(e^{j\omega}) \) exhibits a linear phase low pass characteristic with a high pass band droop which causes undesirable degradation of the decimated signal. The frequency response of the filter is fully determined by three parameters \( K, M \) (a differential delay), and \( N \) which limit the desired frequency response in the majority of practical applications. Nulls exist at integer multiples of \( 1/M \), and the placement of the null is controlled by \( M \). For CIC filter, aliasing error manifests when the region around every \( M \)th null is folded into the pass band, while imaging errors occurs at a region around the null. Therefore, the design parameters are chosen to provide an acceptable pass band characteristic within the transition band [16].

4 Sharpening Technique

Sharpening technique is used to improve frequency response in a manner that provides both smaller pass band error and greater stop band attenuation. It sharpen the magnitude response of a digital filter by using multiple realizations of a low-order basic filter. A traditional sharpening technique was proposed in [20] to improve performance characteristics of CIC filters in such a way to provide simultaneous pass band droop reduction with respect to increase in rejection of attenuation around the folding band. A family of sharpening technique was introduced based on an Amplitude Change Function (ACF). ACF is a polynomial that establishes a relationship between the amplitude of the sharpened filters and the prototype filter. The polynomial response is given by,

\[ G_{m,n}(f) = G^{n+1}(f) \sum_{k=0}^{m} \left( \frac{n+k}{n+K} \right) [1 - G(f)]^K \]  

(6)

\( G(f) \) is the transfer function of low-order basic filter. The improvement of gain response of the filter depends on the order of tangencies \( m \) and \( n \) of the ACF chosen near the pass band edge \( (G=1) \) or near the stop band edge \( (G=0) \).

Case 1: \( m=1; n=0 \)

For two stage CIC filter the parameter \( K \) is 0 and 1

Equation (6) then becomes

\[ G_{1,0}(f) = G(f) \sum_{k=0}^{1} \left( \frac{(0+k)}{0+1} \right) [1 - G(f)]^K \]  

(7)

\[ G_{1,0}(f) = G(f) \left( \frac{(0+0)}{0+1} [1 - G(f)]^0 \right) + G(f) \left( \frac{(0+1)}{0+1} [1 - G(f)]^1 \right) \]

\[ G_{1,0}(f) = 2G(f) - G(f)^2 \]  

(8)

Taking z domain transform of equation (8)

\[ G_{1,0}(z) = 2G(z) - G^2(z) \]  

(9)

The sharpened CIC filter is obtained by replacing the transfer function of the original filter \( G(z) \) in equation (4) to equation (9),

\[ G(z) = 2 \left( \frac{1 - z^{-N}}{1 - z^{-1}} \right)^K - \left( \frac{1 - z^{-N}}{1 - z^{-1}} \right)^{2K} \]  

(10)

The magnitude response of the sharpened CIC filter is obtained by applying the transfer function of the original filter \( G(z) \) in equation (5) to equation (8).


\[ G(z) = 2 \left( \frac{1}{N} \sin\left(\frac{\omega M}{2}\right) \right)^K - \left( \frac{1}{N} \sin\left(\frac{\omega M}{2}\right) \right)^{2K} \]  

(11)

**Case 2: m=n=1**

Using equation (6), substitute the values for parameters m, n and where k =0 and1 (two stage CIC filter)

\[ G_{1,1}(f) = G^{1+1}(f) \frac{1}{1+t_0} [1 - G(f)]^0 + G^{1+1}(f) \frac{1}{1+t_1} [1 - G(f)]^1 \]

\[ G_{1,1}(f) = G^2(f) [1 + G^2(f)] (12) \]

Taking z domain transform of equation (12),

\[ G_{1,1}(z) = G^2(z)[3 - 2G(z)] \]  

(13)

The group delay of the frequency response G (z) is D samples [21] where D = \( \frac{M-1}{2} \) * K and M is the length of the FIR filter. The term \( z^{-D} \) is therefore introduced to equation (13) to make the frequency components of the input signal experience uniform delay with no distortion.

\[ G_{1,1}(z) = G^2(z)[3z^{-D} - 2G(z)] \]  

(14)

The block diagram of the second-order sharpened CIC filter of equation (14) is in given Figure (1).

The implementation of the sharpened CIC filter requires three copies of G(z), a multipliers of integer values 3, a trivial multiplier of value -2, an adder and a delay line of D samples.

The term in the square bracket is responsible for the pass band droop reduction, whereas \( G^2(z) \) is used to improve the stop band rejection.

The sharpened CIC filter is obtained by substituting the G(z) of the original CIC filter in equation (4) into equation (14),

\[ G_{1,1}(z) = \left( \frac{1 - z^{-N}}{1 - z^{-1}} \right)^{2K} [3z^{-D} - 2 \left( \frac{1 - z^{-N}}{1 - z^{-1}} \right)^K] \]  

(15)

By substituting the transfer function of equation (5) to equation (14), the magnitude response of the sharpened CIC filter with the order K is expressed as,

\[ G_{1,1}(z) = 3 \left( \frac{1}{N} \sin\left(\frac{\omega M}{2}\right) \right)^{2K} - 2 \left( \frac{1}{N} \sin\left(\frac{\omega M}{2}\right) \right)^{3K} \]  

(16)

**IV. The Proposed Sharpening Technique**

The problem associated with the existing functions is that some functions will perform well in pass band and hurt performance in the stop band or vice-versa. A function that improves performance both in pass band and stop band of a low pass filter must satisfy some conditions such as

\[
\begin{align*}
F(0) &= 0 \\
F'(0) &= 0 \\
F(1) &= 1 \\
F'(1) &= 0
\end{align*}
\]

(17)

A polynomial of the order of three or higher has enough coefficients to satisfy all the four conditions. Therefore, a cubic polynomial function \( F(z) \) is considered and the general cubic polynomial is given by,

\[ F(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 \]  

(18)
The four coefficients of the polynomials will be evaluated to determine the function that satisfies all the four conditions.

i. Substitute $F(0) = 0$ from equation (17a) into equation (18), then
$$F(0) = a_o = 0$$
(19)

ii. Differentiate equation (18) and substitute for $F'(0)$;
$$F'(0) = a_1 + 2a_2z + 3a_3z^2$$
$$= a_1 + 2a_2(0) + 3a_3(0)^2$$
$$= a_1$$
(20)

iii. Substitute for $a_o$ and $a_1$ in equation (18), then equation (17c and 17d) become,
$$F(1) = a_2 + a_3 = 1$$
(21)
$$F(1) = 2a_2 + 3a_3 = 0$$
(22)
The coefficients $a_1$ and $a_3$ are resolved simultaneously, $a_2 = 3$ while $a_3 = -2$. Therefore substituting the coefficient $a_o$, $a_1$, $a_2$ and $a_3$ into equation (17), then, we have,
$$F(z) = 3z^2 - 2z^3$$
(23)
The function can be re-presented as,
$$F(z) = 1 - 3(z - 1)^2 - 2(z - 1)^3$$
(24)
A simple sharpening technique based on Taylor’s theorem is proposed to improve filter performance of a two-stage CIC filter. The Taylor’s expansion of the transfer function of a simple low pass filter with transfer function $G(z) = z/(2z - 1)$ is considered. The Taylor expansion of a transfer function $G(z)$ about $z_o = 1$ is expressed as,
$$G(z) = G(z_o) + \frac{z-z_o}{1!} G'(z_o) + \frac{(z-z_o)^2}{2!} G''(z_o) + \ldots + \frac{(z-z_o)^n}{n!} G^{(n)}(z_o) + R_n(z)$$
(26)
The transfer function $G(z)$ when $z = e^{j\Omega}$ is
$$G(z) = 1 + \frac{z-1}{1!} G'(z_o) + \frac{(z-1)^2}{2!} G''(z_o) + \ldots + \frac{(z-z_0)^n}{n!} G^{(n)}(z_o) + R_n(z)$$
(27)
The remainder and the terms of order $\Omega^3$ or higher are ignored.

If $z = e^{j\Omega}$, from equation (26), the Taylor’s expansion of the transfer function is given as,
$$z = 1 + j\Omega - \Omega^2/2$$
(28)

Considering a simple LP filter with a transfer response,
$$G(z) = z/(2z - 1)$$
(29)
then by Taylor expansion about $z = 1$, equation (28) becomes,
$$G(z) = 1 - (z-1)^2$$
(30)
Substituting for $z$ in equation (27) into (29) while ignoring HOT (Higher Order Terms),
$$G(z) = 1 - j\Omega - \Omega^2$$
(31)
The filter is sharpened by substituting the LP filter response into the polynomial function from equation (18), we have F(G(z))
$$F(G(z)) = 3(1 - j\Omega - (\frac{3}{2})\Omega^2)^2 - 2(1 - j\Omega - (\frac{3}{2})\Omega^2)^3$$
(32)
By evaluating equation (31) and eliminating HOT,
$$F(G(z)) = 1 + 3\Omega^2$$
(33)
The function $F(G(z))$ is expected to perform better than $G(z)$ but its magnitude response has indeed increased, therefore, making the magnitude response after sharpening worse. The sharpened filter is re-sharpened, that is $F(F(G(z)))$ and the frequency response of the sharpened filter is brought closer to one, Recall from equation (23),
$$F(z) = 3z^2 - 2z^3$$
The function $F(F(G(z)))$ is evaluated by substituting equation (32) into equation (23), we have,
$$F(F(G(z))) = 3(1 + 3\Omega^2)^2 - 2(1 + 3\Omega^2)^3$$
(34)
Therefore, by evaluating equation (32) and ignoring HOT, the proposed polynomial function for the research work is given by,
$$F(F(G(z))) = 1 - 2\Omega^4$$
(35)
The frequency is re sharpened two times and frequency response of the sharpened filter is closer to one.

V. Design Example and Analysis of Results

Consider the magnitude responses of a CIC decimator filter (with $N = 32$ and $K = 2$) using Binomial function and the proposed Taylor’s function. The existing two sharpening techniques used for improving the magnitude response of a two-stage CIC filter are based on a binomial function. The techniques use amplitude sharpening functions to map the amplitude of the prototype filter into an improved amplitude of the sharpened filter. A Matlab program was written using the steps outlined below which are depicted in the flow chart shown in Fig. 2 to compute the magnitude responses of the decimator filter with the specifications given above.
Design Steps for the magnitude responses of the specified CIC Decimator filter
1. Input design parameters; N, K and M.
2. Compute the amplitude response of the original CIC filter equation (5).
3. Compute the magnitude response of the sharpened CIC filter when m=0 and m=1 equation (11).
4. Compute the magnitude response of the sharpened CIC filter when m=1 and m=1 equation (16).
5. Generate a polynomial function that satisfies the performance response of an ideal low pass filter i.e., F(z) equation (23).
6. Determine the frequency response of the given low pass filter using Taylor’s approximation, G(z) when z=1 while ignoring HOT equation (31).
7. Sharpen the frequency response of the CIC filter, F(G(z)) equation (32).
8. Compare the frequency response F(G(z)) with an ideal low pass frequency response.
9. Re-sharpen the CIC filter response F(F(G(z))) equation (35).
10. Compare the frequency response F(F(G(z))) with the ideal LP response.
11. Compare the responses of the original filter (step 2) response and the binomial sharpened responses (step 3 and step 4) with the proposed Taylor’s approximation sharpened response (step 9).
12. Determine the percentage improvement in pass band gain and alias rejection.

Fig. 2: Flow chart for the computation of the magnitude responses of the specified CIC decimator filter. The magnitude responses of the original CIC filter and the sharpened CIC filters for the specified decimator filter are depicted in Figures 3 and 4.

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Fig. 3: Magnitude response of the original and the sharpened CIC filter with the design specification.

Fig. 4: Magnitude responses of the original CIC filter and the sharpened CIC filter with the design specification.

Figures 3 and 4 show the overall magnitude responses and pass band details of the two sharpened CIC filters and the original CIC filter respectively. In Figure 3, it shows that with design parameters of the two-stage CIC filter, the sharpening function $P_{11}$ gave higher alias rejection in the stop band than the sharpening function $P_{10}$. In Figure 4, a better pass band characteristic is achieved with the filter when sharpened with polynomial function $P_{10}$. The comparisons show that the sharpening function $P_{11}$ improves performance in the stop band but hurts performance in the pass band. The second function $P_{10}$ helps performance in the pass band but hurts it in the stop band. The proposed CIC structure after sharpening gives a more attractive results and the effect is visible in both pass band and stop band as depicted in figures 5 and 6.

The proposed technique is based on Taylor expansion which assumes that the filter passes dc signals without change. The technique allows the filter to be sharpened two times in order to bring the frequency response of the CIC filter more nearly to an ideal value than the other sharpening techniques. The proposed technique after sharpening gives a more attractive result and the effect is visible in both pass band and stop band as depicted in Table 1. The proposed sharpening technique provides better attenuation $\sim$187.80dB and pass band gain $\sim$0.00171dB than the existing methods. The performance of the CIC filter is improved with a better alias rejection stop band and a flat pass band with very sharp transition band.
Table 1 gives the overall magnitude responses of the existing filter sharpening techniques and the proposed technique. It compares the passband droop and alias rejection for the CIC filter, the sharpened CIC filters, and the proposed sharpened filter.

Table (1): The passband droop and alias rejection for the original CIC filter, the sharpened filters, and the proposed sharpened CIC filter.

<table>
<thead>
<tr>
<th>Decimation technique</th>
<th>Specification</th>
<th>Passband droop (dB)</th>
<th>Alias Rejection (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIC filter</td>
<td>N=32; K=2;</td>
<td>-0.2924</td>
<td>-86.60</td>
</tr>
<tr>
<td>CIC filter sharpened with polynomial function P_{11}</td>
<td>N=32; K=2;</td>
<td>-0.1712</td>
<td>-154.70</td>
</tr>
<tr>
<td>CIC filter sharpened with polynomial function P_{10}</td>
<td>N=32; K=2;</td>
<td>-0.0096</td>
<td>-38.05</td>
</tr>
<tr>
<td>The proposed sharpened CIC filter</td>
<td>N=32; K=2;</td>
<td>-0.00171</td>
<td>-187.80</td>
</tr>
<tr>
<td>Percentage improvement</td>
<td></td>
<td>82.19</td>
<td>21.40</td>
</tr>
</tbody>
</table>

P_{10} performs better than P_{11} in the passband region, but there is improvement by the proposed technique. Therefore the percentage improvement can be calculated as:

\[
\text{Pass band gain} = \frac{(\text{Pass Band Gain of the Proposed Technique}) - (\text{Pass Band Gain of P}_{10})}{\text{Pass Band Gain of P}_{10}} \times 100
\]

\[
\% \text{ Pass band gain} = \frac{(-0.00171) - (-0.0096)}{-0.0096} \times 100 = 82.19\%
\]
P₁₁ performs better than P₁₀ in the stop band region, but there is improvement by the proposed technique. Therefore the percentage improvement can be calculated as,

\[
\% \text{ Alias rejection} = \frac{-154.70 - (-187.80)}{-154.70} \times 100 = 21.40\% 
\]

VI. Conclusion

CIC filter is a hardware efficient decimation filter with a high droop pass band which is not acceptable for various DSP applications as most application require flat response pass band to preserve the received original signal. The frequency response of a two-stage CIC filter is improved using the proposed sharpening technique. The proposed technique allows sharpening operation to be performed twice based on Taylor’s expansion of the transfer function of the filter. The proposed sharpened CIC filter has much better improvement in the pass band and higher alias rejection in the stop band than the other sharpened CIC filters with the same decimation factor. The proposed technique is used in multirate filtering system where improved filtering is needed. The proposed technique can be extended to design decimation filter of higher filter sections which can be used for sampling rate equalization in current and future generation communication systems.

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