

Effect of Concrete Manipulative Approach on Attitude, Retention And Performance in Geometry Among Junior Secondary School Students in Benue State, Nigeria

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Abstract: This study investigated the Effect of Concrete Manipulative Approach on Attitude, Retention and Performance in Geometry among Junior Secondary School Students in Benue State, Nigeria. Quasi-experimental research design was used for the study. The total population of the study consisted of all the Junior Secondary School Two Students in Benue State, Nigeria. A sample of 211 students was drawn from the population for the study using a stratified proportionate random sampling technique. The instruments used to collect data were: Students' Geometry Attitude Questionnaire, Geometry Performance Test and the Instructional Instrument (Lesson Plans). The Questionnaire was validated by three experts and had a reliability of 0.90. The test instrument was also validated by three experts and had a reliability of 0.71 while the lesson plans were given a face and content validity by the three experts. Five research questions were raised and five hypotheses were formulated to guide the study. The research questions were answered using the mean ranks, means and standard deviation while the hypotheses were tested at 0.05 level of significance using Mann-Whitney U-test and t-test statistical tools. The results of this study indicated that (1) Students' attitude was rated positive in the Geometry concepts taught due to the use of Concrete Manipulative Approach. (2) Concrete Manipulative Approach was not gender bias in attitude formation. (3) The Concrete Manipulative Approach improved the performance of students taught Geometry concepts in Mathematics. (4) Female students exposed to the Concrete Manipulative Approach, performed equally with their males counterparts. (5) The retention ability of students exposed to Concrete Manipulative Approach was found to be better than that of the students' exposed to the Conventional Lecture Method. Based on the findings from this study, it was recommended that: (1) The Concrete Manipulative Approach be used by Students of Geometry in learning as it helped to form positive attitude in learning Geometry. (2) It was also recommended that the Concrete Manipulative Approach be used by Teachers of Mathematics, as the use of it aid students to retain much of what was learnt. (3) It was recommended that the Concrete Manipulative Approach be used to teach Junior Secondary School Students in order to clear the beliefs that the study of Geometry is masculine. (4) It was also recommended that Concrete Manipulative Approach be incorporated in the junior secondary school curricular.

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Aratation: I, Jacob Tertsea AKAAZUA (P16EDSC9077) hereby declare that the work in this thesis titled "Effect of Concrete Manipulative Approach on Attitude, Retention and Performance in Geometry among Junior Secondary School Students in Benue State, Nigeria" was performed by me in the Science Education Department, under the Supervision of Prof. C. Bolaji, Prof. Y. K. Kajuru and Prof. M. Musa. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this work has been presented for another degree or diploma at any institution.

Dedication

This thesis is dedicated to my late mother Mrs. Tabitha Mernan Akaazua (of blessed memory) for laying a solid foundation of my life.

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I. Introduction

Geometry is the study of space and spatial relationship. As one of the longest-established branches of Mathematics, it is widely applied in various applications, such as Computer Aided Design (CAD), Geometric modeling, robotics, medical imaging, computer animation and visual presentation (Chan, Tsai & Huang, 2006; Hannafin, Truxaw, Vermillion & Liu, 2008 and Rafi, Samsudin & Said, 2008). Other areas where Geometry problems arise are in Computer Graphics, Architecture, Chemistry, Material Physics, Biology, Geographic Information Systems (GIS), and most fields of engineering (Chang, Sung & Lin, 2007). Its recalling powers has made policy makers to consider it as an important and essential branch of Mathematics curriculum at all grades levels in Nigeria (Federal Ministry of Education, 2006). The ability to apply geometric concepts and reasoning is the life wire of many occupations. For example, in satellite dish building, acquiring the practical knowledge in deriving the area of a square ($A = 4 \times 4 = 16$ or in general $A = X \times X = X^2$), a foundation is been laid for the satellite dish builder as this function is the same with that of a parabolic function $Y = X^2/A$ when placed on the Cartesian plane; while that of a rectangle ($A = xy$) is a good example of a utility function for consumers. Furthermore, the study of Geometry provides the learner with a vehicle for enhancing logical reasoning and deductive thinking for modeling abstract problems. It helps to develop the mind in determining differences especially at early stages of our life as in games where children are made to place different shapes (squares, rectangles, triangles among others) in the right positions or slots. These help the toddlers to make deductions that expand their mind and this served as their first exposure to Geometry.

The importance of geometry lies also within the fact that geometry is not only relating to Mathematical courses, but also concerned with the development of students' cognitive, affective and psychomotor skills, such as investigating, researching, criticizing, creative thinking, illustrating what students have learnt and self-expression (Erdogan, Akkaya & Akkaya, 2009). Geometry is necessary to our life and various professions because shapes and objects learnt from geometry are also available in our world (Cherney, 2008). Nevertheless, geometry looks like Mathematics, which may be difficult to most people (Andersson, 2008).

In view of this wide use, research in geometry is becoming an important and essential area in current research all over the world for better performance in geometry (Güven & Kosa, 2008; Walker, Winner, Hetland, Simmons & Goldsmith, 2011; Meng & Idris, 2012 and Darwish, 2014). With a desire to continue the teaching and learning of Geometry and the development of spatial abilities (understanding relations visually, making changes on shapes, rearranging and interpreting them) as well as acquisition of practical skills that are expected to be of use to the learner and the society, the Federal and State Government of Nigeria and other bodies like National Mathematics Centre, Abuja, Mathematics Association of Nigeria (MAN), National Teachers Institute (NTI), National Council of Teachers of Mathematics (NCTM) and Science Teachers Association of Nigeria (STAN) among others have invested huge amounts of money and time in training and retraining of teachers who are expected to bring about the desired educational change. However, no significant changes seem to have been recorded (Emaikwu & Nworgu, 2005); Salau, (2009); Obanya, (2010); Emaikwu, (2012); Onah, (2012) and Benue State Examination Board (BSEB), (2003-2014). Students still continue to record poor performance in schools as shown in Appendix J. From this appendix, it could be observed that, the percentage pass of the students that participated in the Basic Education Certificate Examination (BECE) Mathematics paper over the period is not encouraging. This implies that only a mean of 37.23% of students had credit pass against a mean failure rate of 62.77% over the period of twelve years. This fall in standard of performance at post primary school level is attributable to pedagogical approaches adopted by teachers in schools (Bayram, 2004; Eze, 2011). The BSEB chief examiner's report (2007-2014) also stressed that students had problems in geometry areas of Mathematics and such problems have been traced to lack of visualization, inability to conserve concepts, exercise their transitive abilities, reason logically or show concrete manipulative skills which require a special instructional strategy. Development of these skills by learners may take place through the use of two or more combinations of the three senses (touching, hearing, seeing among others); that is, a combination of the cognitive, affective and the psychomotor domains (Akakabota, 2005). He believes that manipulative skill development requires a blending of the mind and muscle. He further explains that manipulative acts are guided by thought with a direct relationship existing between the quality of thought and the quality of manipulative actions. Singh (2006) reasons that since human learning follow some processes which are sequentially built up: acquisition, retention and recall, this sequence must be passed before learning takes place. In other words, some information must be acquired as the first stage in the learning process. It could be that the students in the first place do not retain or achieve any information in their class work before these examinations and if the necessary information is merely acquired it may sooner or later lost and not be available for future reference or use. Retention in Geometry therefore means recalling pieces of knowledge, processes and skills that were learned earlier in time. The teacher's method of teaching may be responsible to guide students in the process of learning (Steyn, 2004).

The role attitude plays in teaching and learning of Geometry cannot be over emphasized. Bayram (2004); Etukudo (2006); Odeleye, Oluwatimilehin and Okereke (2009) believed that positive attitude of learner towards a subject and the teaching strategy employed by the teacher enhanced meaningful learning and thus better concept formation. It is therefore pertinent to search for ways of encouraging positive attitude in learners towards learning in order to improve the learning of Mathematics generally and for the study of Geometry in particular. In another development, gender based studies like Becker, (2005); Finn, (2008); Erickson, (2009); Glenn & Ashley, (2009) and Kajuru & Kauru, (2010) show that there are under-representation and inconsistencies as to the nature, extent and sources of the differences in the performance of males and females in geometry. The Second International Mathematics Study (SIMS, 1982-1983) shows that gender differences do not appear in mathematical learning except in the poorly taught areas, such as geometry and measurement. In these areas, prior out-of-class experience is significant. In many societies, girls often do not play games that enhance their visual and spatial knowledge. According to Hyde & Mertz, (2009) girls are therefore disadvantaged when these topics are taught in class. Can the use of concrete manipulatives in the mathematics class provide opportunities for all children to develop their skills in measurement and geometry? This give reasons why a large body of literature reported that gender should be included as a variable in analysis, even if it is not the main focus of the study (Santos, Ursini, Ramirez & Sanchez, 2006). With this inconsistencies and significant methodological flaws observed, more empirical researches are needed to investigate the existence of gender differences in the classrooms.

Sequel to this, there is a wide spread concern among parents and considerable public about the methods used in teaching at the secondary school level especially in teaching geometry in Nigeria. The inadequacy of the heavily criticized Conventional Lecture Method (CLM) to improve students' attitude, retention and performance in geometry has become a source of concern to many educators in Nigeria. Tom (2011) was of the view that students-centered approaches be used in teaching school subjects if students are to learn maximally.

Instructional strategies adopted by teachers influence the affective, cognitive and psychomotor educational outcomes (Micklich, 2012). The call for departure from the Conventional Lecture Method (CLM) of teaching has been sounded by many mathematics educators in Nigeria with the intention of obtaining admirable

results (Abakpa, 2010). The method in any teaching and learning situation is very important because the way a teacher presents subject matter to learners may make the students' like or dislike the subject. It has also been reported by Mtsem, (2011) that a teaching method affects the responses of students and determines whether they are interested, motivated and involved in the lesson in such a way as to engage in a good learning and that what constitutes good teaching and learning of school subjects is the use of appropriate methods of teaching.

The Concrete Manipulative Approach is a student's centered method of teaching that draws its inspiration from the concrete operational stage as well as part of the formal operational stage of the Piaget's learning theory of cognitive development. These two stages are concretized to cover ten to fifteen year old students in this study. This was because the approach needs reference to familiar actions using concrete manipulatives or observable properties and simple relationships in learning geometry. In addition to this, learners will need a practical interaction with concrete manipulatives before the meaning of a concept and a phenomenon is understood. The approach helps students acquire qualities like conservation, orderliness and irreversibility properties that helps students form positive attitude and creates' geometric images that will remain more permanently in students for good performance and retention in geometry. It is based on this background that the researcher investigated the Effect of Concrete Manipulative Approach on Attitude, Retention and Performance in Geometry among Junior Secondary School Students in Benue State, Nigeria.

1.1 Statement of the Problem

The alarming rate of students' poor performance in Basic Education Certificate Examinations (BECE) in Mathematics and the Benue State Examination Board (BSEB) chief examiners report on students' weakness in Geometry is of great concern to mathematics educators, teachers, parents, students, non-governmental organizations and in fact, the entire nation. BSEB Chief Examiners Report in Mathematics of 2007-2014 theory papers reviewed that many of the students avoided geometry questions and the few that attempted did not perform well. Also students did not have the ability to present their work orderly and logically. This by implication means the students did not retain any knowledge while learning geometry or did have negative attitude towards geometry. This led them to poor performance. At least students should have touched, seen and manipulated concrete objects to have images that would help them recall while in the examination halls.

The researcher observed that geometry concepts are very abstract and when the concepts are not understood it makes students to perform poorly. This observation, apart from the problem of pedagogy, forms the problems of this present study. It is based on these problems that the researcher carried out the study on the Effect of Concrete Manipulative Approach on Attitude, Retention and Performance in Geometry among Junior Secondary School Students in Benue State, Nigeria.

1.2 Objectives of the Study

The objective of this study is to determine the effect of concrete manipulative approach on attitude, retention and performance in Geometry among Junior Secondary School Students in Benue State, Nigeria. The Specific objectives of the study were to:

1. investigate the altitudinal change of students towards the learning of geometry.
2. ascertain whether there was gender difference on the attitude of Junior Secondary School Students towards the learning of geometry
3. investigate whether Junior Secondary School students improved in their academic performance in the learning of geometry
4. ascertain whether there was any gender difference in the Junior Secondary School students academic performance in the learning of geometry
5. determine whether Junior Secondary School students improved in their retention of the geometry concepts taught.

1.4 Research Questions

This Study sought to answer the following research questions:

1. What are the effects of attitude ratings on Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method?
2. What are the effects of attitude ratings on Junior Secondary School male and female students' taught geometry using Concrete Manipulative Approach?
3. What are the effects of mean academic performance scores on Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method?
4. What are the effects of mean academic performance scores on Junior Secondary School male and female students taught geometry using Concrete Manipulative Approach?
5. What are the effects of mean retention scores of Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method?

1.5 Null Hypotheses

The following null hypotheses were formulated and tested at $P \leq 0.05$ level of significance

1. There is no significant difference between the attitude ratings of Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method.
2. There is no significant difference in the attitude ratings of Junior Secondary School male and female students taught geometry using Concrete Manipulative Approach.
3. There is no significant difference between the mean academic performance scores of Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method.
4. There is no significant difference in mean academic performance scores of Junior Secondary School male and female students taught geometry using Concrete Manipulative Approach.
5. There is no significant difference between the mean retention scores of Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method.

1.6 Significance of the Study

It is hoped that the findings from this study will help students, secondary school teachers, Mathematics educators, curriculum planners and higher institution lecturers and various associations like the Science Teachers Association of Nigeria (STAN), Mathematics Association of Nigeria (MAN), and National Council of Teachers of Mathematics (NCTM) among others to articulate the Concrete Manipulative Approach. The students will be able to learn meaningfully using concrete manipulatives and be able to read geometric shapes and bring out their relationships and properties for decision making. The Concrete Manipulative Approach as a student centered approach can be adopted by students for learning geometry concepts in the classrooms. This is significant in the sense that students will use manipulatives that will assist them see the need to apply their cognitive, affective and psychomotor factors in their learning. The students will exercise most especially their affective and psychomotor skills which will in turn improve their attitude and academic performance. It is also hoped that this research will serve as a powerful tool in tackling the gender differences that have lingered in mathematics for decades. Subjecting both male and female students' to familiar actions and manipulatives that will make them observe directly geometric properties in a practical approach may likely sort out the gender difference problem. Teachers can now identify the steps of learning to serve as a guide for selecting appropriate instructional plans.

Therefore, this study is significant based on the following implications:

- (1) Learning will be faster as students associate it with concrete manipulatives and reason from unscientific to scientific ways of doing geometry.
- (2) Students taught Geometry using Concrete Manipulative Approach will be more involved in the learning and as such will help the students' develop good attitude in what they learn.
- (3) The approach encourages the interplay of thoughts and actions between the three educational domains. Students become more alert and can respond to difficult problems on their own, and would be able to solve geometric problems with confidence.
- (4) The implication for teachers is that, the teacher will now make available and relevant manipulatives in teaching geometry at Junior Secondary Schools level to enhance performance.

The applications for Mathematics educators, curriculum planners, and researchers are obvious. Institutions responsible for teachers' professional development like National Teachers Institute (NTI), Nigerian Educational Research and Development Council (NERDC) would also find the results of this study useful, thereby incorporating the findings into their curriculum design processes and instruction innovation programmes in mathematics and create more awareness. The findings of this present study will provoke further research in order to make a wakeup call for the depressed state of Mathematics learning in particular and education in general in Nigerian Secondary Schools. This present study will also add new information to all existing literatures when published.

1.7 Scope/Delimitation of the Study

The study was specifically focused on the JSSII students who were being prepared for the Basic Education Certificate Examination in Benue State, Nigeria. This class was preferred because it was not an examination class and has already acquired substantial knowledge at JSSI. The schools that were covered for this study comprised both the Federal, State, Private and Missionary secondary schools in the state. The geographical scope of this study was Benue State. She is one of the 36 States in the Federal Republic of Nigeria with Makurdi as her capital. She is made up of the following tribes: Tiv, Idoma, Igede and Utulo (Benue State Government, 2005). The State lies between longitude 19.2 and 19.9 east and latitude 17.0 and 16.8 north of the equator. She shares the administrative boundaries with the Republic of Cameroon on the east, Taraba State on

the north-east Cross River and Ebonyi State on the south, Enugu on the south-west, Kogi State on the north-west and Nassarawa State on the north (Benue State Government, 2005). Benue State has rivers Benue and Katsina Ala as her main rivers. The State has a land mass of 30,955 square kilometers and a population of 5,780 million people (Federal Republic of Nigeria, 2006). This area was chosen for this study because her poor performance in Mathematics at Senior School Certificate Examination. Sixty Two point seventy seven percent (62.77%) of students that enrolled for Mathematics in Basic Education Certificate Examination in the period 2003-2014 in Benue State failed.

The content scope include: the properties of polygons, perimeter of a square and a rectangle. The area of a square, a rectangle, a parallelogram, a trapezium, a triangle and a circle were also considered. Others are the circumference of a circle, angle properties of parallel lines, properties of an angle at a point of intersecting straight lines and at a point and construction of triangles. These areas are considered because they are part of the fundamental concepts of Mathematics in JSS II syllabus. The delimitation was that the researcher within the twelve weeks period of the research carried out geometry activities that enhanced good attitude, retention and high academic performance.

1.8 Basic Assumptions

The following are the basic assumptions for the study:

1. The Concrete Manipulative Approach was within the reach of the students for the topics in Geometry. The selected topics are appropriate for the level of the subjects used for the study.
2. The students in the study sample have acquired basic JSSII geometry concepts through Conventional Lecture Method but not exposed to Concrete Manipulative Approach.
3. The selected concepts for the study are appropriate for the level of the students targeted for the study.
4. The schools used for the study are true representations of secondary schools in Benue State.

II. Review Of Related Literature

2.1 Introduction

This study investigated the effect of Concrete Manipulative Approach on attitude, retention and performance in Geometry among Junior Secondary School students in Benue State, Nigeria. Various researches conducted in Science and Mathematics Education within and outside Nigeria relevant to the present study were reviewed. In this chapter the literature reviewed is presented. The theoretical framework consists of three theories: the Operant Conditioning Theory of Attitude Change, Decay Theory of Forgetting and the Piaget Learning Theory of Cognitive Development. The conceptual framework dealt with the Concept of Attitude Formation of Students in Geometry, Academic Performance in Geometry, Concrete Manipulative Approach and Students Performance and Conventional Lecture Method and Students Performance. Others include Retention of Learned Concepts in Geometry and Gender and Academic Performance in Geometry. The overview of related studies and the implications of literature reviewed to this study were also presented in this chapter.

2.2 Theoretical Framework

2.2.1 Operant Conditioning Theory of Attitude Change

The operant conditioning theory of attitude change was propounded by Thorndike, Pavlov and Skinner in 1914. It states that attitudes are formed utilizing rewards (reinforcers). Thorndike (1914) found out that animals learn not only by reasoning or by means of instinct but by application of responses to given stimulus. If these stimuli which he called physical (psychomotor), mental will (cognitive) and attitude (affective), are well blended, will produce positive attitude. Thorndike used a small chamber that he called a puzzle box. He would place an animal in the puzzle box, and if it performed the correct response (such as pulling a rope, pressing a lever, or stepping on a platform), the door would swing open and the animal would be rewarded with some food located just outside the cage. The first time an animal entered the puzzle box, it usually took a long time to make the response required to open the door. Eventually, however, it would make the appropriate response by accident and receive its reward: escape and food. As Thorndike placed the same animal in the puzzle box again and again, it would make the correct response more and more quickly. Soon it would take the animal just a few seconds to earn its reward (Thorndike, 1914).

Based on these experiments, Thorndike developed a principle he called the Law of effect. This law states that behaviors that are followed by pleasant consequences will be strengthened, and will be more likely to occur in the future. Conversely, behaviors that are followed by unpleasant consequences will be weakened, and will be less likely to be repeated in the future. Thorndike's law of effect is another way of describing what modern psychologists now call operant conditioning (Thorndike, 1914).

A major idea of this theory is its emphasis on the stimulus characteristics of the communication situation. The implication of this theory to this study is that good attitude cannot adequately be formed without making available a good stimulus (instructional strategy and the learning objects). Teachers have to make good

instructional strategies and learning objects (reinforcement) contingent (close in time) upon the knowledge they want the learner to gain in geometry. This will help in bringing about the desired attitudinal change in the learning of geometry.

2.2.2 Decay Theory of Forgetting

The Decay theory of forgetting propounded by Thorndike, (1914) states that if a person does not access and use the memory trace representation he/she has formed, the memory trace will fade or decay over time. The theory suggests that the short term memory only holds information for between fifteen and thirty (15 and 30) seconds unless it is rehearsed. After this time the information decays or fades away. This explanation of forgetting in short term, assumes that memories leave a trace in the brain. A trace is some form of physical and/or chemical change in the nervous system. "Trace" is therefore often added to qualify the theory as "Trace Decay Theory of Forgetting". The theory focuses on time and the limited duration of short term memory.

No one disputes the fact that memory tends to get worse the longer the delay between learning and recall, but there is disagreement about the explanation for this effect. According to this decay theory of forgetting, the events between learning and recall have no effects whatsoever on recall. It is the length of time the information has to be retained that is important. The longer the time, the more the memory trace decays and as a consequence more information is forgotten (McLeod, 2008). McLeod (2008) further explains that there are problems facing the Trace Decay Theory of Forgetting. One of the major problems is controlling the events that occur between learning and recall. Clearly in any real-life situation, the time between learning something and recalling it will be filled with all kinds of different events. This makes it very difficult to be sure that any forgetting which takes place is the result of decay rather than a consequence of the intervening events.

Moreover, it is more or less impossible to test the Decay Theory mechanically. In practice, it is not possible to create a situation in which there is a blank period of time between presentation of material and recall. Having presented information, participants will practice it. If you prevent practice by introducing a distracter task, it results in interference (McLeod, 2008). The Decay theory of forgetting can be applied to this work because Thorndike, (1914) believed that children who are taught using concrete materials end up having long term memory of what they have been taught. The physical and/or chemical change in the nervous system leaves a trace that last long.

2.2.3 Piaget Learning Theory of Cognitive Development

Piaget's, (1966) learning theory of Cognitive Development states that a child's mental development is in stages. Piaget's learning theory is sometimes described as 'genetic epistemology.' 'Genetic' because he believed that the stages we go through and the structures and processes we use, are inbuilt and true for all of us regardless of culture. 'Epistemology' actually means the study of knowledge. The Piaget learning theory has the Sensory Motor Stage (0-2 Years). This is where the child has not gone to school even though children go to school and call it daycare or kindergarten. The child finger-grasps any hand stretched forth towards him; he is sensitive to light and sound. If the place is dark, he may cry and if you make noise or knock the child can respond. The child moves from simple reflex activities to reach realization of his environment. That is, he moves from body-centered activities to object-centered activities like playing with objects.

The second stage, Pre-Operational Stage (2-7 years) is the stage where vocabulary is acquired and the use of symbols begins. The child plans his actions mentally before he carries out the action. He communicates in the language of the immediate environment and his mental activities at this stage are irreversible. The child at this stage also does symbol play, drills and imitation. He is self centered and does not do trans-deductive reasoning (reasoning across more than two things). He has difficulty in seeing more than one object at a time. The third stage, Concrete Operational Stage (7 to 11 years) is the stage the child is now able to carry out operations on its environment and develops logical thought. However, it still requires the use of concrete reasoning patterns, being unable to think in abstract terms. Less importance is attached to information from our senses as we use thought and imagination more. The concrete reasoning patterns are applied to concrete manipulatives or directly observable properties and simple relationships. The major features of this stage are reversibility which refers to the ability to mentally picture and retain an action being carried out permanently. This is essential for conservation, e.g. imagining the water being poured back into the original beaker. Conservation is made possible by the ability to de-centre at this stage. Transitivity is only possible with concrete examples. For example 'Jackie is fairer than Sarah, Jackie is darker than Nicola. Who is the darkest?' The concrete operational child would not be able to work this one out mentally; it would require dolls or pictures of the three girls. Similarly $A > B > C$. This would not be possible since it requires abstract thought rather than concrete examples. The stage is a stage for concrete environment and not abstract concepts.

The fourth and the last stage of the Piaget theory is the Formal Operational stage (11 years onwards). Piaget used the term 'formal' since children in this stage can concentrate on the form of an argument without being distracted by the content. For example if x is greater than y but less than z . The child can now work this

out without needing to know what x, y and z refer to. For example: 'All green birds have two heads. I have a green bird called Charlie. How many heads does Charlie have?' A child in the earlier stages would be bogged down by the content, that is, birds have one head. Formal thinkers can concentrate on the structure (or form) of the question in this context. Piaget maintained that everyone would reach this stage eventually, even if it took us until 20. However, there is plenty of evidence to suggest that this is not the case and that certainly it tends to occur later than Piaget predicted.

However, Bello (2013) citing Shayer & Adey's clearly showed that Piaget learning theory of cognitive development stages do not correspond neatly with the ages of physical maturity. That, from their result, only about one third of the secondary school population by the age sixteen had reached the formal stage of the cognitive development theory. This study therefore merged the concrete operational and part of formal operational stage as concrete operational in teaching and learning the geometry concepts in our secondary schools. In addition to the concretizing of both concrete and formal concepts the researcher used ages ten to fifteen years as found in the intact classes.

2.2.3.1 Concrete Manipulative Approach (CMA)

The Concrete Manipulative Approach is a student's centered learning approach adapted from the Piaget's (1966) learning theory of cognitive development. It draws its inspiration from the use of concrete objects of Jean Piagets' work. Piaget (1966) postulated four developmental stages as levels in learning. The stages were defined in terms of quantitatively different modes of intellectual and moral reasoning at different ages and specified as sensory-motor, pre-operational, concrete operational and formal operational stages. Since each stage does not arise full-blown but arises gradually from the integration and incorporation of earlier stages the researcher focused on the concrete stage. This is because it is at these stage and age bracket (10-15 years) that these Junior Secondary School students were considered for this study. The concrete operational stage is preferred to other stages because it is the theoretically proved stage for the JSSII class and it uses observable properties and simple relationships in learning geometry. It also puts into consideration the use of concrete manipulatives which educationists believe is the best way to make sense of what is seen, heard and touched.

2.3 Conceptual Framework

2.3.1 Concept of Attitude Formation of Students in Geometry

The operant conditioning theory of attitude change using a Skinnerian approach is a mode of attitude formation which utilizes rewards (reinforcers). A major idea of this theory is its emphasis on the stimulus characteristics of the communication situation. The implication of this theory to this study is that good attitude cannot adequately be formed without making available a good stimulus (instructional strategy and the learning objects). According to Obodo, (2004), the more exposure one has towards a given object, the more positive one's attitude is likely to be. Teachers have to make good instructional strategies and learning objects (reinforcement) contingent (close in time) upon the knowledge they want the learner to gain in geometry. This will help in bringing about the desired attitudinal change in the learning of geometry.

Attitudes are also formed through direct experience. Bayram (2004), in support of Thorndike is of the view that attitudes were learned as a result of using concrete reasoning patterns with the environment. Bayram proposed that since the person trying to form or change attitude usually lack's direct knowledge of the internal stimuli available to the learner, it was necessary to rely on external cues in order to understand the concept or not. It was the combination of external cues and observable behaviors that produced changes in attitude. It is thus not surprising that when a learner performs very well in geometry, the tendency for developing a positive attitude towards it will be there. Geometric attitude are formed or changed in the students, if a reward in form of a reinforcer is put in place. It could be a student-centered learning strategy that creates interest and images for students' quicker recall like the Concrete Manipulative Approach.

In view of this, Lawal, (2008) citing Alao, revealed that the learners' attitude to a subject is influenced by two factors, which she described as internal and external factors. The internal factor according to Lawal, (2008), include individuals practical skills and certain amount of relevant concrete images knowledge that the learner already possess about the concept to which the new attitude will be directed. The external conditions, he declared, are mainly the home attitude, success in the subject and the positive or negative attitude of the teacher. Lawal, (2008) observed that both the internal and external factors must be satisfied before attitudes are formed or changed. Farooq and Shah, (2008) researched on students' attitude towards Mathematics. The researchers used a total of 112 Mathematics students' for data collection. They found among others that most students exhibited negative attitude towards the subject, while a positive relationship was expressed between the students' attitude and their performance in the test. This finding is interesting as one would expect students with negative attitude to a subject to achieve very little. The present study deemed it necessary therefore to find out whether or not Concrete Manipulative Approach would have any effect on the development of positive attitude in the learner and the process of learning Geometry.

2.3.2 Academic Performance in Geometry

There are various definitions and concepts of academic performance. According to Obodo (2004) and Emaikwu (2012), academic performance refers to an individual's academic attainment after a specified course of instruction. It is an individual's status in a specific content area or specified course of instruction after the individual or group has undergone tutoring in a programme. A performance test measures an individual's performance after a specified course of instruction. It is supposed to test mastery of a laid-down curriculum after an instructional programme. Performance test can either be teacher-made or standardized test. The conventional test that teachers give to students at the end of each term or semester after series of lessons/lectures are given is described as teacher-made or un-standardized test. The standardized ones are those with distinct characteristic features like norms, expert touch, manual and good psychometric properties, for example, like those constructed by the Benue State Examination Board. In this study, the teacher made type would be used. This is because it is content specific, unlike the standardized test that covers a lot of content that may not be relevant to the researcher's topic of interest.

Performance test is used for evaluation of learning outcomes that determines whether students have learnt what they are supposed to learn (Odilli, 2006 & Emaikwu, 2011). It should also be used to evaluate the entire curriculum. Curriculum evaluation is a continuous process in academic and to ascertain if any particular method is still serving its purpose. The evaluated result will reveal to us if the said method needs improvement, modification or total elimination. Based on these, many researchers have done various investigations on academic performance in geometry. Nnaobi, (2007) view academic performance as attainment of set objectives of instructions in the learning process for instance, if a learner accomplishes a task successfully and attains the specified goals for a particular learning experience, he is said to have performed. The attainment of the goals of mathematics education is a major concern of education policy makers and one of such goals is the inculcation of scientific reasoning (Federal Ministry of Education, 2006).

Despite the importance of geometry to students and the society at large, the general academic performance in geometry has been affected by students' poor performance in geometry of public examinations over the years. This ugly trend is attributed to pedagogical approaches adopted by teachers, poor attitude of students towards the subject and lack of application of spatial abilities. Moreover the persistent students' poor performance in Mathematics could be one of the reasons for the shortage of qualified candidate for admission into courses where a minimum of credit level pass in Mathematics is required. Consequently, the 60:40 ratio of science differential weighting for University and 80:20 Polytechnic admission policies in favour of Science are not being achieved over the years in Nigeria (Agbo, 2012). He went further to say that these Mathematics failures have made Nigerian universities to ridiculously reduce the cut-off point for admissions at variance to the standard of excellence already set by some universities from 400 to a mere 180 points just to woo failed students as their potential candidates for admission. This might have a serious repercussion on the country's effort towards becoming a scientifically and technologically oriented workforce by the year 2020. This is because in order to achieve this vision, it would require nothing short of good performance in Mathematics and geometry in particular at all levels of schooling.

One of the challenges teachers are facing therefore is how to improve students' performance in Mathematics as its pass rates in BECE are low. As Mathematics is one of the important subjects in all fields of natural sciences, with the current failure rates, the country would not be able to produce sufficient number of natural scientist to meet the demand of the country's socio economic development. Odubunmi, (2006) reported that students perceive geometric content of Mathematics as being difficult to learn. Therefore, when students find the learning of concepts difficult in a subject, they develop hatred for such subjects. Collaborating the perceived difficulty in geometry content, the BSEB Chief Examiner's Report in (2007-2014) noted that students have poor grasp of the derivation and application of two dimensional shapes formulas in the learning process of Mathematics in Nigerian Secondary Schools. The poor performance of students in geometry could be attributed to the following: First, poor quality of Mathematics teachers whose method of teaching centre mainly on excessive talking, copying of notes, rote learning of textbooks materials which tend to inhibit interest and attitude (Eze, 2011). Secondly, the prevalent expository methods of instruction with very little involvement of students in experimentation are worse than lecture methods. Thirdly, the lack of laboratory facilities and equipments necessary for practical work has contributed immensely to the poor performance of students in geometry (Obioma, 2005; Aburime, 2007 and Galadima & Okogbenin, 2012).

Consequently, there arises a need to seek for other strategies to bring improved performance of students in geometry. Other reasons according to Micklich, (2012) and White, (2012) include the learners, environmental factors and instructional strategies employed. Learners lack visualization and spatial skills which can only be developed by the use of a combination of the cognitive, affective and psychomotor skills. Students' inability to accurately observe objects, measure, relate, manipulate, draw and even re-arrange objects form reasons for poor academic performance in geometry (Eze, 2011). Meng & Idris, (2012) in their study revealed that a child introduced to any knowledge using concrete manipulatives receives a permanent and irreversible knowledge and

ends up having long term memory of what they have been taught. Walker, et.al (2011)'s study on Visual Thinking: Arts Students have an Advantage in Geometric Reasoning, revealed that Concrete Manipulatives Approach raised the level of concentration, develops positive attitude and enhanced learning. In addition to this, Yara, (2009) and Yagci, (2010) further observed that students who learned with concrete manipulative were more attentive, engaged in learning, and participated more actively in the classroom than those who did not. It is based on this that the researcher investigated the effect of concrete manipulative approach on attitude, retention and performance in Geometry among Junior Secondary School students in Benue State, Nigeria.

2.3.3 Concrete Manipulative Approach and Students' Performance

Swan and Marshall (2010); Kosko and Wilkins (2010); Puchner, O'Donnell and Fick (2008) and Belenky and Nokes (2009) studies used solely concrete manipulatives in their research. Swan and Marshall (2010) defined a manipulative as "concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students." This definition pertains to all of the literature that focused on concrete manipulatives. Throughout the literature that focuses on concrete manipulatives, there were trends and themes that appeared within the research. Research has demonstrated that manipulatives generally have a positive effect on student learning and mathematical performance than compared to the more traditional way of mathematics instruction, which often involves talking and computational fluency.

Kosko and Wilkins (2010)'s secondary data were collected to examine whether a relationship between students' manipulative use and communication in geometry learning exists. Correlational analyses found a significant relationship between students' verbal and written communication and manipulative use. Data from a subsample of these students were collected by the students' mathematics teachers who completed a questionnaire related to classroom practice and information on each individual student in their classroom. Studies were performed to see if concrete manipulatives has an adverse effect on student's performance. Swan and Marshall (2010) based their findings on a previous research study. These researchers found that manipulatives benefit the learning and teaching of mathematics. The research also found that the use of concrete manipulatives links strongly with concept formation in student learning. Puchner, Taylor, O'Donnell and Fick (2008) conducted a collective case study that analyzed the use of manipulatives in Mathematics lessons developed and taught by 4 groups of elementary teachers involved in lesson study as part of a professional development program. The researchers found that in three of four lessons studied, use of manipulative use was turned into an end in and of itself, rather than a tool and that in the fourth lesson use of manipulative hindered rather than helped student learning. These problems with manipulative use by teachers in the lessons provide helpful guidance for successful implementation of manipulatives for both teachers and professional developers. Most importantly, they stressed the need to emphasize the link between pedagogy and content, not the specific use of manipulatives.

Belenky and Nokes (2009) investigated the nature of students' learning of geometry concepts when using manipulatives. This study examined how the type of manipulative (concrete, abstract, none) and problem-solving-prompts (metacognitive or problem-focused) affect students' learning, engagement, and knowledge. Students who were given concrete manipulatives with thinking (metacognitive) prompts showed better transfer of a procedural skill than students given abstract manipulatives or those given concrete manipulatives with problem-focused prompts.

2.3.4 Conventional Lecture Method and Students Performance

Conventional lecture method is a collection of teaching methods other than the concrete manipulative approach. The main characteristics are verbal presentation of ideas about the topics. It is a teaching procedure in which there is a one – way channel of communication where the teacher makes an oral presentation of the subject matter content and students react by silently listening and taking notes. In this method also the teacher gives out all the facts he wants the students to know and master, caring very little if at all whether or not, the students are actively participating and contributing to the success of the lesson (Cohen & Manion, 2006). The method reduces learners to mere note – takers and passive listeners. Learner's perception and assimilation of the subject matter is slow. Conventional Lecture Method often inhibits active participation of students in the classroom and teacher dependence on the part of students (Ransdem, 2009).

Many students turn out to be very miserable and inattentive in a geometry class after being taught a topic and discover they could not memorize or recall such a concept with ease. The reason for this difficulty may vary but this could sometimes be related to the teaching method being used to explain such topics. Abakpa, (2010) blamed poor performance of students in mathematics on poor methods and approaches to teaching which has reduced the level of motivation. Harbor-Peters (2001) asserted that the issue of poor performance in mathematics examinations was due to problem of teaching methods. There has also been an increasing awareness by those concerned with mathematics education that the conventional methods of teaching mathematics, has not been very successful. For effective teaching to take place, the skillful mathematics teacher

needs to use many different methods and techniques at his disposal. A carefully designed teaching method can make teaching and learning effective (Chianson, 2008).

2.3.5 Retention of Learned Concepts in Geometry

People have a tendency to forget lots of things. We often forget appointments, have difficulty remembering someone's name, and can't remember the precise details of books we read last week. There are two basic explanations to why we forget. Firstly, the information has disappeared. It is no longer available. Secondly, the information is still stored in the memory system but for some reasons, it cannot be retrieved. Human learning follows some processes which are sequentially built up; acquisition, retention and recall. These stages must be passed before learning takes place (Sedig, 2008). In other words, some information must be acquired as the first stage in the learning process; but if the necessary concepts are merely acquired it may sooner or later be lost. Such apparently acquired materials may not be available to us for future reference or use. Thus, there is much need for retaining what has been acquired. This leads to the second stage of the process of learning known as retention. Having acquired and retained information materials in the process of learning may not after all be enough guarantee that learning has taken place. The only way to claim that information material has been effectively acquired and retained is if such information can be expressed, otherwise an outsider may not know precisely what have been acquired and retained. Thus, if even some body of information materials have been acquired and retained in the process of learning, such information could not be said to have been learnt. There is a need to express this acquisition and retention by the third stage in the process of learning which is known as recall. This is the third and last stage in the process of human learning. In other words, learning consists of three important stages or processes, acquisition, retention and recall.

Instructional strategies that enhance meaningful learning will equally bring about acquisition, retention and recall of information materials to be learnt. Steyn (2004) argues that knowledge retention is related to the way the concept is taught to the learners and that the teacher's method of teaching may be responsible to guide students in the process of learning. Knowledge retention should be a focus of every academic institution and it should be evaluated to assure the quality of programmes. Knowledge retention which serves as a prerequisite to good performance is therefore considered as having the power of recalling geometric concepts (Clair, 2004). It is also the ability of the working memory of an individual to retrieve stored information from long-term-memory for processing. Clair (2004) has observed that long term memory retention is a significant goal of education.

Retention in geometry therefore means recalling pieces of knowledge, processes and skills that were learned earlier in time. The existence of academic institutions is based, in large part, on the belief that students remember what they learn (Connie, David, Heather, Hilda & Paulson, 2006). Students may pass an examination following an intense study session but fail to retain much of these materials in long-term memory. This may be the experience of the Nigerian candidates who become qualified for Basic Education Certificate Examination at the end of JSS3 programmes.

Connie et.al. (2006) suggested techniques to help students retain information in the long-term memory. These are:-

- (1) In learning of geometric processes and skills that have to do with concrete objects, students must obtain a high degree of performance in the first stage of the learning process; since it is obvious that the most important factor in retention is the degree of initial proficiency.
- (2) Begin all lessons with concrete manipulatives and advanced organizers. That is create interest through the use of questions on concrete objects of all previous topics relating to what is being taught presently. This brings students to a psychological feeling that they need to know more and that they need to become more able.
- (3) Retention is enhanced by an internalization of the geometric skill and its process.
- (4) Learners should also practice the processes and skill as soon as possible after the demonstration is given. Continued practice is related to improved retention. This concept is known as overloading.
- (5) Relevant information and processes performed in the beginning and ending of a learning task are better retained, implying that more emphasis during teaching should be placed upon the processes occurring during the middle stages of the tasks; since events occurring between the demonstration and practice session tend to reduce retention.
- (6) Students should distribute the practice of their work since practice make perfect and increases retention.
- (7) Students provide a demonstration of the geometric process and skill. Involving students mentally and physically throughout the demonstration will increase its effectiveness. Steps to be performed should be in a proper sequence.
- (8) Teacher has students practice the skill.
- (9) Teacher provide alternating sessions of practice and evaluation until the desired ability level is reached.

Connie et.al. (2006) is of the strong opinion that when a student notes these and adhere to them strictly in the process of learning, he is able to retain a lot of what is taught. Thus, because of the importance of retention in the learning of geometry, this study would measure how much of the materials taught will be retained by the students' after using the Concrete Manipulative Approach which is structured to improve students' performance and attitude towards geometry concepts.

2.3.6 Gender and Academic Performance in Geometry

A large body of literature reports that there are gender differences in the learning of geometry in favour of male students (Halat, 2006; Guven & Kosa, 2008 and Yang & Chen, 2010). Strategy use, as a reflection of different patterns in learning geometry between male and female, is found to be related to psychomotor abilities, together with psychological characteristics and mediated by experience and education. Many complex variables including biological, psychological, socio-cultural and environmental variables are revealed to contribute to gender differences in the learning of geometry. The combined influence of all cognitive, affective and psychomotor variables may account for the gender differences in geometric reasoning patterns (Zheng, 2007).

Review of studies show inconsistency in results of male and female students' performance in Mathematics Performance Tests. Studies by Halpern, Wai, and Saw, (2005); Caplan & Caplan, (2005) show no significant differences in Mathematics performance between boys and girls. Zheng, (2007) and Yang & Chen, (2010) on the other hand indicate sex differences in favour of boys. Halat (2006) examined gender differences in the acquisition of geometry knowledge and the result showed that gender was not a factor in geometric learning. The researcher used a 46-item questionnaire for data collection and a sample of 300 students. He found out that there was no difference between male and female adult students' attitude towards the study of geometry and that they have positive attitude towards the study of the subject. Abubaka, (2012) investigated the attitude of Mathematics students towards the effect of using probability instructional materials for performance. The researcher used 298 Mathematics students, and he found out that their attitude towards Mathematics was negative while gender and class level did not significantly influence students' attitude towards Mathematics.

Ajewole, (2006) found in his study that most of the factors discouraging female students from choosing Mathematics related career courses are socio-cultural. That 93% of the respondents see mathematics as a masculine course and 98% frown at women in science in general. Suggestions have been proffered by Mathematics educators to help redress the issue of learning geometry of both male and female students in Mathematics. Some of this included the consideration of spatial abilities as a critical skill in the learning of geometry, attitudinal changes on the part of parents, teachers, girls, guidance and counselors at early ages of education and use of concrete environmental objects rather than memorizing (Caplan & Caplan, 2005, Chipman, 2005, Sherry, 2010 and Yang & Chen, 2010). For instance, Sherry (2010), investigated how to use digital games to improve spatial abilities. However, not every learner favours this kind of support. To this end, Achor, Imoko & Ajai, (2010) examined how human factors affect learners' reactions to the use of a computer simulation technique to support geometric learning.

In their study, Yang & Chen, (2010) developed a digital pentominoes game and examined the effects of two essential human factors, gender differences and spatial abilities, on students' performance. The results demonstrated that students' spatial abilities were significantly improved after they took the digital pentominoes game. The study also demonstrated that the digital game can reasonably reduce the differences between boys and girls. Moreover, the major gender differences of the study lie within mental rotation among the three types of spatial ability.

Finally, the findings are applied to develop a framework that can be used to enhance the understanding of gender differences and spatial abilities within the digital pentominoes game. Pentomino is a kind of polyomino, which is a geometric shape formed by joining five congruent squares connected orthogonally along their edges (Yang & Chen, 2010). In general, a standard pentomino puzzle is designed to tile a rectangular box (game board) using 12 different pieces. More specifically, learners need to select, rotate, translate, flip, mirror and land these pieces onto the game board as well as remove pieces from the board for completing a pentomino puzzle (Yang & Chen, 2010). Manipulating these pieces of pentominoes is an enjoyable exercise to learners and can help them improve geometric learning, such as spatial reasoning, problem solving and visual skills (Akakabota, 2005; Prescott & Ugbooduma, 2008; Njoku, 2010 and Shafi & Areelu, 2010). Thus, manipulating pentominoes could be a useful way for geometric learning and it is often used to illustrate geometric concepts (Akakabota, 2005).

2.4 Overview of Related Studies

In Science and Mathematics education, a great deal of related research on concrete manipulative approach, attitude, retention and performance had been carried out and summaries of some of such research works are highlighted. Bayram (2004) studied the effects of concrete models on eighth grade students' geometry performance and attitudes toward geometry in the Middle East Technical University, Ankara-Turkey. The

research design was a matching-only pre-test- post-test control group design. The population of the study comprised all the eighth grade students enrolled in one of the private schools in Ankara-Turkey. A sample size of 106 students was used for the study. The following measuring instruments were used to collect data: the Geometry Attitude Scale (GAS), Geometry Performance Test (GPT) and open ended questions. The data of the study were analyzed using Analysis of Co-Variance and two-way Analysis of Variance. The results of the study indicated that: (1) There was a statistically significant mean difference between students that received instruction with concrete models and those who received instruction with traditional method; (2) there was no statistically significant mean difference between girls and boys; (3) there was no statistically significant interaction between treatment and gender on subjects score obtained from GPT; (4) there was no statistically significant mean difference between students who received instruction with concrete models and those who received instruction with traditional method in terms of subjects score obtained from GAS; (5) there was no statistically significant mean difference between girls and boys in terms of subjects score obtained from GAS; and (6) there was no statistically significant interaction between treatment and gender on subjects score obtained from GAS.

This study is similar to the present one. The only difference is in the use of a matching-only pre-test-post-test control group design instead of a randomized pretest-posttest-post posttest experimental control group research design. The consideration of the entire school as sample without randomization created some sort of bias as no room was created for the experimental group which would have clearly defined what happened before and after treatments. Again the use of ANOVA statistical tool instead of t-test also made a clear difference between this study and the present one. t-test would have made the analysis more straight forward but because it was missed it created this gap. The present study therefore investigated the effect of concrete manipulative approach on attitude, retention and performance in Geometry among Junior Secondary School students in Benue State, Nigeria.

Yagci, (2010) worked on the effect of instruction with concrete models on eighth grade students probability achievement and attitudes towards probability. The researcher applied the quantitative and qualitative research design. The population of the study was all eighth grade students in Central Anatolia Region. The researcher used Pre-requisite Knowledge and Skills Test (PKT), Probability Achievement Test (PAT), Probability Attitude Scale (PAS) and Interview as the instrument of data collection. In order to analyze the data, Friedman and Wilcoxon test were used. Also interviews were carried out with 11 students to determine their views about the instruments. It was found that there was a statistically significant change in probability achievement of the students in the instruction with concrete models. Similarities are observed more in this study than the differences. The major difference was in the use of Friedman and Wilcoxon test as the statistical tools to analyze data. These tools are lower in rank as far as statistics are concerned and cannot adequately solve the problem of the research well. Mann-Whitney U-test would have served better in that case.

Kurumeh, Onah and Mohammed (2012) study, designed to determine the effect of the ethnomathematics teaching approach on Junior Secondary three (JS3) students' retention in statistics was aimed at determining whether any of the sexes (male and female) would retain statistics concepts more than the other from the teaching. The study was carried out in Obi and Oju educational areas of zone C in Benue state of Nigeria using a sample size of 248 Junior Secondary three (JS3) students. The study employed quasi-experimental design of non-equivalent but culturally homogenous group. Intact classes were used for both the experimental and control groups. The experimental group was taught using the ethnomathematics approach while control group was taught using conventional approach. Two research questions and two research hypotheses were formulated to guide this study. Statistics Retention Test (SRT) instrument with the reliability coefficient of 0.80 was used as pre, post and retention tests though reshuffled each time for data collection. Mean and Standard deviation were used to answer the research questions while an ANCOVA was used to test the null hypotheses at a 0.05 level of significance. The results revealed among others that the ethnomathematics teaching approach was more effective in facilitating and improving students' retention in statistics than the conventional approach. The ethnomathematics teaching approach did not significantly differentiate between the sexes (male and female) retention scores in statistics. These findings have implications for all Mathematics teachers and stakeholders in mathematics education. Based on the findings, it was recommended among others that the ethnomathematics teaching approach be adopted in schools particularly in our junior secondary school education, while teaching statistics since it has proved to be a viable option in promoting meaningful learning and affected students' retention rate positively. Again, the Ministry of Education and professional bodies such as Mathematical Association of Nigeria (MAN) and Science Teachers Association of Nigeria (STAN) should be involved in promoting this method through conferences, seminars and workshops so as to expose to teachers an ethno method in teaching mathematics.

This study is similar to the present one except in the ANOVA Statistical tool used. The present study applied a simple t-test and Mann-Whitney statistical tool because of their simple way of simplifying data. As for

the use of ethnomathematics, it is the practical way of doing geometry in individual cultures even before the arrival of western education so it similar to what the present study set to do.

White (2012) study on the effect of an instructional model utilizing hands-on learning and use of manipulatives on standardized test scores of middle school students in Georgia used a quasi-experimental non-equivalent control-group design. The population of the study comprised all seventh-grade students from a North Georgia middle school. The sample of the study was 145 students. The instrument for data collection was a teacher-made, curriculum based, unit test. The Mann-Whitney test and ANCOVA statistical tool were used to examine the questions and test the null hypotheses. This study is similar to the present study by its use of manipulatives to ascertain the effects of instructional models on students. The major difference of the study with the present one is in its descriptive statistics. The present study used means and standard deviations, mean ranks and mean ranks difference to answer research questions.

Bello, (2013) investigated the effect of learning cycle teaching strategy on students' acquisition of formal reasoning ability and academic performance in genetics. The quasi-experimental research design was used for the study. The population of the study consisted of all the Junior Secondary three Biology students in Sabon Gari Local Government of Area, Kaduna State. Two instruments were used for data collection: the Group Assessment of Logical Thinking (GALT) and Genetic Academic Achievement Test (GAAT). GALT with reestablished reliability coefficient of 0.79 was used to measure the reasoning ability of the subjects while GAAT was used to measure the academic achievement of the subjects in genetic. The reliability coefficient of GAAT was found to be 0.81. The data collected were analyzed using t-test statistical tool. One of the findings of this study indicated that, the control group showed no appreciable gain in formal reasoning ability after instruction. Formal reasoning ability used in Bello, (2013)'s study was in the last stage of Piaget theory. The present study considered the third and the fourth stages of the Piaget learning theory that revolves around the idea and use of concrete objects while Bello, (2013)'s study handled only the fourth stage which was concerned with more or less abstract thought without concretizing the stage. That is, practical operations on plane shapes at ages 10-15 years where not carried out. The present study combined and concretized the last two stages instead of considering the last stage of Piagets' theory on an abstract basis only. Again the used of lower statistical tool like t-test than ANCOVA for this present work is sufficient since the two groups were considered equivalent at start of treatment.

Agbo-Egwu, (2014) investigated the effect of metalinguistic learning approach on students' achievement, interest and retention in Statistics in Secondary Schools in Makurdi Metropolis. The study employed a quasi-experimental research design. The population of the study consisted of 2665 students and the sample of the study was 350 students. A 50-item options A-D multiple choice test and a 16-item Statistics Interest Inventory were used as instruments for data collection while ANCOVA statistical tool was used for data analysis. The result of testing one of the hypotheses indicated that the use of MLA resulted in higher mean achievement scores than the Conventional Teaching Method. The use of Metalinguistic Learning Approach which is not concrete manipulative based but symbol based is almost the same as the Conventional Teaching Method (a story telling teaching approach) since students did not manipulate the symbols or practically derived the statistical knowledge themselves. For example the use of \cap to denote intersection of two sets in set theory can only be represented and not manipulated. Moreover language learning can only be more cognitive than psychomotor. This researchers' application of the Concrete Operational Stage which is more of concrete manipulative may tend not to help the statistics students especially for retention purposes. The present researcher therefore investigated the effect of concrete manipulative approach on attitude, retention and performance in Geometry among Junior Secondary School students in Benue State.

Darwish (2014) studied the abstract thinking levels of the science-education students in Gaza Universities Palestine. The population of the study was all the science students attending the first and fourth year at two Palestinian universities (Al-Aqsa and Al-Azhar). The sample consisted of 133 students from Science Education Departments. The instrument used to measure abstract thinking, was one of the Science Reasoning Tasks developed by the CSMS in UK in 1994. Data was analyzed using percentages and correlation statistical tools. The results showed a positive correlation between the scores of the students in the abstract thinking test with the performance level in the universities. The study generally concluded that, the current systems in Science-Education Departments had not succeeded in assisting the students to reach their potential in terms of concrete abilities through four years of their preparation programme. It was possible that concretizing the formal operational stage can assist the students to reach their concrete ability potentials fully. This is also to avoid contradicting the Piaget (1966) learning theory. Therefore the present study worked on the concrete abilities to see if students can understand Geometry very well.

In general, the overview of related studies revealed that the previous researchers left gaps that need to be filled. There is a mix up of intellectual formation. The present study therefore investigated the effect of Concrete Manipulative Approach on attitude, retention and performance in Geometry among Junior Secondary School students in Benue State Nigeria.

2.5 Implications of Literature Reviewed to the Present Study

This chapter critically examined literatures that are related to this study. A careful examination of the reviews shows that, students' attitude, retention and performance in geometry can be improved using the Concrete Manipulative Approach. White, (2012) demonstrated that students who had higher understanding of concrete manipulative showed very high rate of reasoning from unscientific to scientific ways of doing geometry, while the students who had low understanding of concrete manipulatives showed very little improvement.

Another implication of literature reviewed to this study is that, with Concrete Manipulatives, students should be given activities for logical and concrete reasoning before abstract thought. That is, the students should be guided to put into action their entire educational trait to enable them develop concrete abilities for better geometry performance. The class activities should be able to develop physical and mental actions between a person and his/her environment. The child should be able to adapt to the environment by assimilation and accommodation.

In addition to this, the learning of geometry based on the Concrete Manipulatives should emphasize the perception of learners. The topics should be schemed to help learners see significant relationships and organized their experiences into functional and effective learning. Also, the Concrete Manipulative Approach should emphasize on the fact that children learn faster when they associate learning with concrete manipulatives. Prior knowledge determines what further learning may take place and the use of concrete manipulatives in teaching can retain knowledge better. This can be achieved through movement from touching to thinking, simple to complex. This by implication means that concrete manipulatives have a role to play in the teaching and learning process and so teachers need to make available relevant manipulatives in teaching geometry at Junior Secondary Schools level to enhance performance. Teachers equally need to identify the steps of learning to serve as a guide for selecting appropriate instructional plans.

Piaget, (1966) believes that children who are taught using concrete manipulatives end up having long term memory of what they have been taught. The implication of this to the present study is that, memory of what is learnt becomes retentive with the use of relevant manipulatives and consequently, performance is enhanced. When manipulatives are not involved, learning becomes difficult and boring for the learners. It is based on these implications that the present study investigated the Effect of Concrete Manipulative Approach on Attitude, Retention and Performance in Geometry among Junior Secondary School Students in Benue State, Nigeria.

III. Research Methodology

3.1 Introduction

In this chapter, the research design and procedure for data collection, as well as the instruments used for data collection are described. The procedure for analysis of data was also described. The chapter is presented under the following subheadings: Research Design, Population of the Study, Sample and Sampling Technique, Instrumentation and Validation of Instruments. Others subheadings include, Pilot Testing, Reliability of Instruments, Administration of Treatment and the Procedure for Data Collection and Data Analysis.

3.2 Research Design

The research design was quasi-experimental in nature. Precisely, a pretest-posttest-post posttest experimental control group research design was employed for the study. This was because it was not practicable to randomize students into experimental and control groups. This was also because the design has the following advantages:

- i). the mean scores obtained from the pretest was used to statistically find out any difference between the groups at the start of the study. That was to ensure equality of the two groups academically at the start of the treatments.
- ii) the mean scores of posttest and post-posttest results were used to determine the level of retention of the learned geometry concepts.
- iii). the design allowed for the calculation of individual as well as group mean pretest and posttest scores. With this, one was able to determine the superiority of one teaching strategy relative to the other.
- iv). there was also the assumption that any difference in the performance of the groups can only be attributed to the treatment given, since the treatment administered was the only difference between the two. The research design is as illustrated in figure 3.1

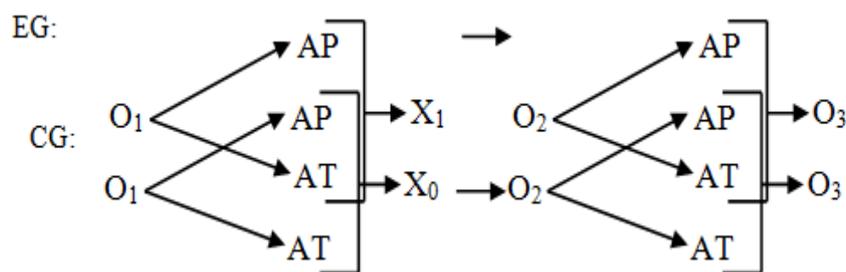


Figure 3.1 Research Design

Where
 EG = Experimental Group
 CG = Control Group
 O₁ = Pretest.
 O₂ = Posttest.
 O₃ = Post-Posttest.
 X₁ = Concrete Manipulative Approach (CMA)
 X₀ = Conventional Lecture Method (CLM)
 AT = Attitude
 AP = Academic Performance

Two groups of students were used for data collection; the experimental and the control groups. A pretest (O₁) was administered to the two groups in order to determine equivalence of the two groups. The experimental group (EG) was taught using Concrete Manipulative Approach (X₁). The control group (CG) was taught using the conventional lecture method (X₀). At the end of the treatment period, a posttest (O₂) and post posttest (O₃) were then administered to both groups of students in order to evaluate the effectiveness of the treatment on students’ attitude, retention and performance in geometry.

3.3 Population of the Study

Despite the existence of more private secondary schools in the state than others, the choice of Federal Government, State Government and Missionary owned secondary schools was made in order to have students with homogeneous background in terms of teachers’ experiences, retraining and teaching materials. The JSII students selected for the study have relative stability in terms of subject coverage more than JSI, and were not committed to any Final examination like JSSIII. The total number of the JSSII students that served as population sample was 6,230 students out of which 4,604 were males and 1,626 were females in these zones with the average age of 13 years. The entire sampled schools were public and singled secondary schools (Teaching Service Board, 2014). The total population of the study is as presented in Table 3.01 and (Appendix A).

Table 3.01 Population of the Study

Educational Zone	Total no of School	Pop. Of Male Students	Pop. Of Female Students	Total Pop. Of Students
A	11	1352	724	2076
B	9	1551	617	2168
C	10	1701	285	1986
TOTAL	30	4604	1626	6230

Source: Teaching Service Board Makurdi (2014) Annual Report

3.4 Sample and Sampling Technique

The study used a Stratified Proportionate random sampling technique for school selection. According to Sambo 2004, a researcher can stratify a population in terms of one or more variables of interest, such as location, tribe, religion, gender and so on. The stratum of interest for this study is students’ gender. This is because all public schools in Benue State are mixed and single sex schools. The study sampled four schools from the population of 30 schools using draw-from-hat method. The school selection was based on gender. This yielded four (2 male and 2 female) schools. The students in these schools were subjected to pre-test using Geometry Performance Test and Students Geometry Attitude Questionnaire. Scores obtained were subjected to one-way ANOVA and Scheffe’s test for the equivalence. The outcome of this test reviewed that Federal Government College Vandekya (FGCV), Divine Love Caholic Girls College, Katsina-Ala (DLCGCK), Federal

Government Girls College, Gboko (FGGCG) and Government College Makurdi (GCM) were equivalent and were selected for the study. The students' sample consists of 211 JSSII students drawn from the four selected schools. Intact classes were randomly selected by balloting from each school as sample, taking into consideration of the recommended sample population accepted to be sufficient for experimental research by Kerlinger 2005. For the experimental group, the sample size (N = 103) comprising of 42 males students and 61 females students from 93 males and 118 females students while for the control group, the sample size (N = 108) comprising of 51 males and 57 females from 93 males and 118 females respectively. The summary of the sample is presented in Table 3.02.

Table 3.02 Sample of the Study

School	Group	No of Male Student	No of Female Students	Total Students
FGGCG	Experimental	-	61	61
GCM	Experimental	42	-	42
FGCV	Control	51	-	51
DLCGCK	Control	-	57	57
Total		93	118	211

The sampling of the schools in two different educational zones A and B was to avoid interaction between the groups during the period of treatment thus eliminating the effect of student interaction. The subjects in the experimental and control groups were matched using the results obtained from the pre-test. The two groups were found to be equivalent academically at start of treatments as there was no significant difference recorded between the mean scores of the two groups at pre-test see appendix J. The result of no significant difference in the pretest of the experimental and control groups allowed the researcher to use the subjects in the two groups for the study (t-cal = 0.928; t-crit. = 1.96; P-value = 0.355 at P > 0.05 level of significance, the mean scores of both the experimental and control groups was 45.00 respectively).

3.5 Selection of Topics

The topics chosen for the study were the properties of polygons, perimeter of a square and a rectangle. Considered also are the area of a square, a rectangle, a parallelograms, a trapezium, a triangle and a circle. Others are the circumference of a circle, angle properties of parallel lines, properties of an angle at a point of intersecting straight lines and at a point and construction of triangles. The choice of these topics was because they are part of the fundamental concepts of Mathematics in JSS II syllabus.

3.6 Instrumentations

Three instruments were used for the study (two instruments for data collection and one instrument for teaching the subjects) as follows:

- (i) Geometry Performance Test (GPT)
- (ii) Students Geometry Attitude Questionnaire (SGAQ)
- (iii) The Instructional Instrument (Lesson Plans).

3.6.1 Geometry Performance Test (GPT)

The Geometry Performance Test is a 50-items multiple choice (objective) test, with four options lettered A-D, one correct option and three distracters. This comprised two sections (A and B). Section A sought for the students' personal information, name and gender. Section B contained a 50-items multiple choice test as shown in appendix B. The items were subjected to a pilot test to enable the selection of items "good" with difficulty and discrimination indices. The selection of the items was based on the table of specification. See Table 3.03

Table 3.03 Table of Specification on Geometry Performance Test

Content Area	Weight %	Knowledge 38%	Comprehension 22%	Application 16%	Analysis 8%	Synthesis 8%	Evaluation 8%	Total 100
Area of parallelogram, trapezium and triangles	15%	3	2	1	1	1	1	9
Properties of polygons	10%	2	1	1	0	0	0	4
Area of a Circle	10%	2	1	1	0	0	0	4
Properties of Angle at a point	5%	1	1	0	0	0	0	2
Properties of an angle at a point of intersecting straight lines.	15%	3	2	1	1	1	1	9

Area of a square and a rectangle	15%	3	2	1	0	0	0	6
The Circumference of a Circle	5%	1	1	1	0	0	0	3
Angle Properties of Parallel Lines.	5%	1	1	0	0	0	0	2
Construction of Triangles	15%	3	2	1	1	1	1	9
Perimeter of a square and a rectangle	5%	1	1	0	0	0	0	2
Total	100%	20	14	7	3	3	3	50

3.6.2 Students' Geometry Attitude Questionnaire (SGAQ)

Students' Geometry Attitude Questionnaire (SGAQ) was used to determine any attitudinal change towards Geometry. It was adapted from the modified Fennema-Sherma, (1970) mathematics attitude scale cited in Lawal, (2008) to suit the present study and thus used for data collection. The Fennema-Sherma attitude scale was used to test for a change in attitude after teaching mathematics. The likert scale attitude questionnaire was adapted to a four-point rating scale of 40 items that provides four response options anchored on a continuum of Strongly Agree (SA), Agree (A), Disagree (D) and Strongly Disagree (SD) respectively. Using this graduated response options; each respondent is to indicate his or her level of agreement or disagreement with the statement. The positively skewed items has response of Strongly Agree merits 4 marks, Agree merits 3 marks, Disagree merits 2 marks and Strongly Disagree merits 1 mark. On the other hand negatively skewed items are assigned values in the reverse order. The likert scale attitude questionnaire was adapted to a four-point rating scale because the undecided response was omitted in the scale for the obvious reason that it lies half way between the positive and negative responses to an item and because the stand of such respondent is not well defined or clear on the issue. Another reason is that whenever the undecided response option is removed from the five point likert scale, it is called a four point rating scale instead of a five point likert scale. The questionnaire was administered to both the experimental and control groups for the sole aim of determining whether the students have a positive or negative attitude towards Geometry. The same attitude questionnaire was administered to the control and experimental groups before and after instruction in order to determine the change if any in the attitude of the students towards Geometry (see Appendix D).

3.6.3 The Instructional Instrument (Lesson Plans)

Ten lesson plans based on the Concrete Manipulative Approach and ten for the Conventional Lecture Methods were prepared by the researcher. The experimental group was taught using the Concrete Manipulative Approach lesson plans while the control group was taught using the Conventional Lecture Methods Lesson Plans. Experimental group lesson plans presented a gradual progression of consciousness or problem solving in form of students' actions while that of the control groups had no activities as it was teacher centered.

3.7 Validation of Instruments

The face and content validity were carried out on the following instruments: Geometry Performance Test (GPT), Students' Geometry Attitude Questionnaire (SGAQ) and The Instructional Instrument (Lesson Plans) respectively.

3.7.1 Geometry Performance Test (GPT)

The Geometry Performance Test items initially comprising fifty five items along with the marking scheme were validated by three experts made up of one expert in Mathematics education, one Measurement and Evaluation expert and one Mathematics Teacher. The experts checked whether the test items conformed to the table of specification and were clearly stated and are appropriate in terms of the reading difficulty level of the students' understanding. The experts also confirmed that the test items were appropriate to reveal concrete manipulative ability levels of the students. They also identified the item difficulty, item discrimination and item distracter indices of the test items. They were to check whether the test items included ambiguous statements that can confuse or be misinterpreted by the examinees and are arranged in order of difficulty from simple to complex. Finally, the experts checked whether the test did not have any identifiable pattern of answers at the time of testing.

To ensure that a proper content validity was done for the GPT, an empirical item analysis and a table of specification showing an equal distribution of the test items over the concepts taught were handed over to experts for their validation. The item analysis was carried out on the scores obtained from the pilot-test to determine the item difficulty, item discrimination and item distracter indices of the items in GPT.

Facility Index (Item Difficulty Index)

The facility index (item difficulty index) of a test according to Cohen & Manion, (2006) was the percentage of candidates that got an item right. The discrimination index was the difference between the proportions of the candidates scoring an item correct in the upper group and those scoring the item correct in the lower group. This calculation was done using also scores of the upper one-third of eighty seven students and the lower one-third of the eighty seven students which was 23 papers both ways. The discriminatory indices of a range between 0.30 to 0.70 are regarded as moderately positive and were accepted for the present study. This was used in selecting the final items of the test.

Distracter and Discrimination Index

The distracter index was the difference between the proportions of the candidates scoring an item correct in the upper group and those scoring the item correct in the lower group in the opposite direction of the discrimination index (Emaikwu, 2011). The value of a distracter index ranges from -1.00 to 1.00. A positive value indicates that the distracter was not effective (that is bad) since it was chosen by more of the candidates who performed well than those who did not perform well. A negative value of index indicates that the distracter was effective (that is good) since it was chosen by more of the candidates who did not perform well than those who perform well. A zero distracter index indicates that the distracter does not distract or confuse the students in the two groups. Any option with positive or zero distracter was modified (see Appendix E).

The experts' suggestions led to the reframing and eliminating of ambiguous and not so appropriate questions. For example the following adjustments were effected:-

- i. Concrete manipulation activities that were lined up for areas of 3D shapes were changed to 2D shapes.
- ii. Some of the questions were rephrased, for example, the diagram on question 34 on the properties of an angle at a point of intersecting straight lines was redrawn.

3.7.2 Students' Geometry Attitude Questionnaire (SGAQ)

The Students' Geometry Attitude Questionnaire (SGAQ) was also face and content validated by three experts made up of one expert in Educational Psychology, one expert in Mathematics Education and one Mathematics Teacher. These experts assessed the questionnaire to find out whether the items conformed to the subject matter they were supposed to measure or not; and whether the items are clear, readable and free from ambiguity for the level of students they were designed to answer or not. The experts also confirmed whether the items were relevant to the study objectives, language, content and if the items would answer the research questions appropriately and tests the formulated hypotheses. The experts brought out the general criticisms and suggestions for the improvement of the instrument. The experts' suggestions also led to the reframing and eliminating ambiguous and not so appropriate attitudinal statements. Out of the 45 items on the questionnaire presented for face and content validity, only 40 were later retained and used for data collection.

3.7.3 The Instructional Instrument (Lesson Plans)

The lesson plans were subjected to both face and content validity by three experts made up of one expert in Mathematics education, one Measurement and Evaluation expert and one Mathematics Teacher. The experts checked whether the lesson plans contain no ambiguous diagrams or figures and were clearly written in terms of the reading difficulty level of the students' understanding. The experts also confirmed that the lesson plans were appropriate to reveal concrete manipulative ability levels of the students

3.8 Pilot Testing

Eighty seven (87) Junior Secondary School Two (JSSII) students, made up of 48 boys and 39 girls of Demonstration Secondary School, University of Agriculture, Makurdi participated in the pilot test. This school chosen was part of the population but not one of those sampled for the main study. The aim of this pilot test was to determine the characteristics of the test items which included their item difficulty, item distraction and item discrimination indices and the reliability coefficient. The Concrete Manipulative Approach was then applied on the experimental group classes by the Researcher while the Research Assistants taught the control group classes. Treatments for the pilot test lasted ten weeks, after which the Geometry Performance Test (GPT) and the Student's Geometry Attitude Questionnaire (SGAQ) were administered. The result of the pilot test indicated that 17 items out of the 50 test items were not difficult and were reconstructed. The pilot test also indicated that 20 items out of the 50 items did not fall within the discriminatory range of 0.30 to 0.70 and were appropriately modified. As for the distracter index, only 16 of the test items did not distract and were as well reconstructed.

The Results Of The Pilot Test Were Then Used To:-

1. calculate the reliability coefficient of the instruments. Also the item distraction, discrimination and difficulty indices were determined using the scores of the pilot test.

2. adjust the test time by computing the average of the time taken by the first, fortieth and eighty seventh student to finish the test.
3. Ascertain if students would need some explanations to the items in the Student's Geometry Attitude Questionnaire.

Based on the findings of the pilot test the period of treatment for the main study was increased from ten weeks to twelve weeks. This was because it was found that the students needed time to get used to the Concrete Manipulative Approach at the initial stage.

3.9 Reliability of Instruments

The reliability coefficient of the Geometry Performance Test (GPT) and Students' Geometry Attitude Questionnaire (SGAQ) were determined as follows:

3.9.1. Reliability of Geometry Performance Test

The reliability coefficient for the Geometry Performance Test was found to be 0.71 using the test re-test reliability estimate. The scores obtained from the pilot test were used to determine the reliability coefficient of the test items. The test re-test is a method of establishing the stability of the test instrument. The researcher administered the Geometry Performance Test twice with a week interval on the subject after the pilot testing period (Emaikwu, 2011, Pallant, 2011). Two sets of scores were obtained from the same test. The resulting test scores were correlated using Pearson Product Moment Correlation Coefficient statistical tool. The reliability coefficient showed that GPT was reliable to measure what it was purported to measure. The reliability coefficient for the Geometry Performance Test is shown in Appendix f.

3.9.2. Reliability of Students' Geometry Attitude Questionnaire (SGAQ)

The reliability coefficient for the Students' Geometry Attitude Questionnaire was also found to be 0.9 using the Cronbach-alpha method of estimating reliability. The method is a more generalized method of estimating internal consistency and a reliability estimate for attitudes scales that provide responses on a continuum of "Strongly Agree, Agree, Disagree and Strongly Disagree". This reliability coefficient showed that SGAQ was also reliable to measure what it was purported to measure. The reliability coefficient for the Students' Geometry Attitude Questionnaire is as shown in Appendix F.

3.10 Administration of Treatments

The two treatments administered in this study were as follows:

- (i) Concrete Manipulative Approach (CMA)
- (ii) Conventional Lecture Method (CLM)

3.10.1 Concrete Manipulative Approach (CMA)

The Concrete Manipulative Approach as a treatment for this study was administered to the experimental group. It makes use of concrete materials in teaching/learning. The approach incorporates geometrical concepts in concrete models which can appeal to the five senses. The Concrete Manipulative Approach as an aspect dealing with the physical and concrete manipulative was preferred in this study because it is using concrete objects or observable properties and simple relationships in learning geometry. CMA puts into consideration the use of concrete manipulatives in learning which educationist believes is the best way to make sense of what is seen, heard and touched. The Concrete Manipulative Approach comprised the following steps:

- A. manipulations of its environment, understand concepts based on direct experience
- B. logical thought, manipulation of concrete objects,
- C. operational thinking predominates, irreversible and egocentric thinking, use thought and imaginative more, understand the law of conservation
- D. use of concrete examples.

Concrete Manipulative Approach is also the concrete reasoning patterns most frequently required by students for the understanding of concepts as follows:

- (1) understanding concepts defined in terms of familiar actions and examples that can be observed directly;
- (2) apply reversibility and conservation reasoning to objects;
- (3) establish one-to-one correspondences and arrange data in increasing or decreasing order;
- (4) Make simple classifications and successfully relate systems to subsystems and classes to subclasses.

Lesson plans based on the Concrete Manipulative Approach were then prepared as shown in Appendix H. At the beginning of the lessons, one week of class time was spent to revise the relevant JSSI topics on properties of plane figures and various geometric models prepared. Students were grouped and each group was asked to organize a set of colored card-board papers into a concept map. On each paper there was a plane figure,

a definition or a property. By organizing the papers, students demonstrate their prior knowledge of the relationships between plane shapes, properties and definitions. The activities for finding the areas of plane figures generally included cutting out the shapes and pasting them together in order to see similar component parts of each plane figure. The plane figures drawn on colored card board papers and different resources were used to make models of concrete manipulatives. To find out the formulas used in this study, students applied their experience studying the area of plane figures of these concrete manipulatives. At the end of these activities, the students discovered formulas for the area of the parallelogram, triangles, trapezoid, regular polygons and circular regions.

In every lesson, students were made to manipulate plane shapes and learn from models (a model of a house, an excavator, a motor circle and a grinding pepper machine among others) that are made of plane shapes by the students themselves to draw inspiration like in a natural environment. The approach was then organized into a five- step of teaching/learning; the first step being the introductory step, while other steps include instructional procedure, evaluation, conclusion and assignment. The activities of the lessons were converted into a flowchart by the researcher as illustrated in figure 3.2.

Figure 3.2 Flowchart of Concrete Manipulative Approach

3.10.2 Conventional Lecture Method (CLM)

The Conventional Lecture Method basically called for verbal presentation of ideas about the topics. This is a teaching procedure in which there is a one – way channel of communication where the teacher makes an oral presentation of the subject matter content and students react by silently listening and taking notes. In this method also the teacher gives out all the facts he wants the students to know and master, caring very little if at all whether or not the students are actively participating and contributing to the success of the lesson (Cohen & Manion, 2006). Teaching under this method is reduced to storytelling. The method reduces learners to mere note – takers and passive listeners. Learner’s perception and assimilation of the subject matter is slow. Conventional Lecture Method often inhibits active participation of students in the classroom and teacher dependence on the part of students (Ransdem, 2009). Sanders, (2005) revealed that skills are best learnt through practicals rather than mere listening.

3.11 Procedure for Data Collection

The data for the study was collected through the following:

1. The experimental group was taught using CMA teaching strategy, while the control group was taught using Conventional Lecture Method.
2. The groups were post tested using the same instruments used in the pre-test to assess Geometry performance and attitude.
3. The scripts were marked and results collated for analysis.
4. Two weeks later, the experimental and control groups were further post-posttested to assess the extent of their retention abilities

3.12 Procedure for Data Analysis

The data collected were analyzed using relevant statistics at $P \leq 0.05$ to answer research question and tested the null hypotheses using relevant statistical tools from Statistical Package for the Social Sciences (SPSS) version 16.0. The research questions were answered using (Mean, Standard deviation, Mean differences, Mean Ranks and Mean ranks Differences).

The null hypotheses were stated with the corresponding statistics at $P \leq 0.05$ for testing as follows:

Ho₁ There is no significant difference between the attitude ratings of Junior Secondary School students’ taught geometry using Concrete Manipulative Approach and Conventional Lecture Method. Statistics Used: Mann-Whitney statistical tool
Ho₂ There is no significant difference in the attitude ratings of Junior Secondary School male and female students’ taught geometry using Concrete Manipulative Approach. Statistics Used: Mann-Whitney statistical tool
Ho₃ There is no significant difference between the mean academic performance scores of Junior Secondary School students’ taught geometry using Concrete Manipulative Approach and Conventional Lecture Method.

Statistics Used: t-test statistical tool

Ho₄ There is no significant difference in mean academic performance scores of Junior Secondary School male and female students’ taught geometry using Concrete Manipulative Approach.

Statistics Used: t-test statistical tool
Ho₅ There is no significant difference between the mean retention scores of Junior Secondary School students’ taught geometry using Concrete Manipulative Approach and Conventional Lecture Method.

Statistics Used: t-test statistical tool

The Mann-Whitney statistical technique was used to analyze hypotheses one and two because the questionnaire items (attitude) are perceptions and not quantitative values in a non-parametric statistics. Moreover attitude might not be equally distributed and the group item (treatment groups) was made up of only two independent groups of experimental and control. Hence a non-parametric statistics of Mann-Whitney rank test is appropriate (Gall, et. al., 2007).

The t-test statistical technique was used to analyze hypotheses three to five because the test items (performance and retention) are not perceptions but quantitative values in a parametric statistics. The t-test was used also because the simplest way to simplify quantitative data is with the used of t-test. The tool is commonly used to determine whether the mean of a population significantly differ from a specific value (called the hypothesized mean) or from the mean of another population.

IV. Data presentation, analysis and discussions

4.1 Introduction

This chapter presented the results of the study. The responses and the scores from the test were subjected to analysis using the Statistical Package for the Social Science (SPSS), and the level of significance adopted for rejecting or retaining stated hypotheses was 0.05. The chapter is presented using the following sub-headings: Data Presentation, Data Analysis, Summary of the Major Findings and the Discussions.

4.2 Data Presentation

The study was conducted on concrete manipulative approach on attitude, retention and performance in geometry among Junior Secondary School students in Benue State. A sample size of 211 students was used for the study. Five research questions with their corresponding hypotheses were answered and tested. The data collected was used to test the stated hypotheses. The results are as presented in Tables 4.01 – 4.10 respectively.

4.3 Data Analysis

4.3.1 Post-test on Attitude change of Students’ in the Experimental and Control Groups

Data on attitude ratings change of Junior Secondary School students on Geometry using Concrete Manipulative Approach and Conventional Lecture Method.

Table 4.01: Mean Rank Scores of the Students Attitudinal change in the Experimental and Control Groups

Group	N	Mean Ranks	Sum of Ranks	Mean Ranks Diff
Experimental	103	132.08	13604.50	
				50.96
Control	108	81.12	8761.50	

Table 4.01 shows that the result of the attitude ratings of students’ in the experimental and control groups are 132.08 and 81.12 respectively. The mean rank difference in the attitude ratings was found to be 50.96. This implies that the attitude of students taught Geometry using Concrete Manipulative Approach and the Conventional Lecture Method differ from one another as can be observed from their means of 132.08 and 81.12 respectively. To find out if the difference in mean ranks was statistically significant, the corresponding hypothesis one was therefore tested using Mann-Whitney U-test. **H₀₁**: There is no significant difference between the attitude ratings of Junior Secondary School Students’ taught Geometry using Concrete Manipulative Approach and Conventional Lecture Method. To test this hypothesis, the post test rates of the subjects in the experimental and control group were compared using Mann-Whitney U-test statistical tool. Table 4.02 showed the result obtained.

Table 4.02: Mann-Whitney U-test Analysis on the Attitude Ratings of Junior Secondary School Students in the Experimental and Control Groups

Group	N	Mean Ranks	Sum of Ranks	Mann-Whitney-cal	P-value
Experimental	103	132.08	13604.5		
				2875.50	0.001*
Control	108	81.12	8761.50		

* Significant at 0.05, $P \leq 0.05$

From Table 4.02, the calculated Mann-Whitney U-test value is 2875.50 and the calculated P-value of 0.001 is less than $P \leq 0.05$ and is significant at $P \leq 0.05$ alpha level hence Mann-Whitney U-test value calculated is statistically significant and therefore the null hypothesis was rejected. This implies that there was

significant difference in the attitude ratings of Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method. The result thus showed that the Concrete Manipulative Approach is more effective in improving students' attitude in the Geometry concepts taught than the Conventional Lecture Method.

4.3.2 Post-test on Attitude of Gender in the Experimental Group

Data on attitude of Junior Secondary School male and female students taught Geometry using Concrete Manipulative Approach.

Table 4.03: Mean Rank Scores of Male and Female Students Attitude in the Experimental Group

Group	N	Mean Ranks	Sum of Ranks	Mean Ranks Diff
Male	93	106.20	9877.00	0.36
Female	118	105.84	12489.00	

Table 4.03 shows that the attitude ratings of male and female students in the experimental group are 106.20 and 105.84 respectively. The mean ranks difference for the male and female groups was found to be 0.36. To find out if the difference in mean ranks was statistically significant, the corresponding hypothesis two was therefore tested using Mann-Whitney U-test.

Ho₂: There is no significant difference between the attitude ratings of Junior Secondary School male and female students taught geometry using Concrete Manipulative Approach. To test this hypothesis, the post test scores of the male and female students in the experimental group were compared using Mann-Whitney U-test statistical tool as shown in Table 4.04.

Table 4.04: Mann-Whitney U-test Analysis on the Attitude Ratings of Junior Secondary School Male and Female Students in the Experimental Group

Group	N	Mean Ranks	Sum of Ranks	Mann-Whitney-cal	P-value
Male	93	106.20	9877.00	5468	0.965**
Female	118	105.84	12489.0		

** Not Significant, P > 0.05

Table 4.04 shows the calculated Mann-Whitney U-test value is 5468 and the calculated P-value of 0.965 is greater than 0.05 alpha level and is not significant at P > 0.05 alpha level hence Mann-Whitney U-test value calculated is statistically not significant and therefore the null hypothesis was retained. This implies that there was no significant difference in the attitude of male and female students taught geometry using Concrete Manipulative Approach. This implies that significant difference did not exist in the gender factor for the group that received lessons with the Concrete Manipulative Approach. The result thus shows that the Concrete Manipulative Approach did not have significant effect on the gender factor in terms of their attitude in the Geometry concepts taught using Concrete Manipulative Approach.

4.3.3 Post-test Scores for Experimental and Control Groups

Data on Post-test scores of Junior Secondary School student taught Geometry using Concrete Manipulative Approach and Conventional Lecture Method

Table 4.05: Result of Mean and Standard Deviation of Post-test for Experimental and Control Groups

Group	N	\bar{X}	Standard Deviation	Mean Diff
Experimental	103	84.01	3.94	36.44
Control	108	47.57	3.04	

Table 4.05 shows that the result of the means and Standard Deviations of experimental group is 84.01 and 3.945 and that of control group is 47.57 and 3.045. The mean difference in the mean performance scores was found to be 36.44. To find out if the difference in mean was statistically significant, the corresponding hypothesis three was therefore tested.

Ho₃: There is no significant difference between the mean academic performance scores of Junior Secondary School Students taught Geometry using Concrete Manipulative Approach and Conventional Lecture

Method. To test this hypothesis, the post test scores of the subjects in the experimental and control groups were compared using t-test statistics. Table 4.06 shows the result obtained.

Table 4.06: t-test Analysis on the Mean Academic Performance of Junior Secondary School Students in the Experimental and Control Groups

Group	N	\bar{X}	Standard Deviation	Standard Error	Df	t-cal	t-crit	P-value
Experimental	103	84.01	3.945	0.38878	209	75.31	1.96	.001*
Control	108	47.57	3.045	0.29305				

* Significant, $P \leq 0.05$

From Table 4.06, the calculated t-test value is 75.31 and the calculated P-value of 0.001 is less than $P \leq 0.05$ and is significant at $P \leq 0.05$ alpha levels hence t-test value calculated is statistically significant and therefore the null hypothesis was rejected. This implies that there was significant difference in the mean academic performance of Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method. The result thus showed that the Concrete Manipulative Approach improved more on students' academic performance in the Geometry concepts taught than the Conventional Lecture Method.

4.3.4 Post-test Scores for Gender in the Experimental Group.

Data on the means and standard deviation scores of Junior Secondary School male and female students taught Geometry using Concrete Manipulative Approach.

Table 4.07: Result of Mean and Standard Deviation of Male and Female Students Post-test for Experimental Group

Group	N	\bar{X}	Standard Deviation	Mean Diff
Male	93	65.72	18.54	0.64
Female	118	65.08	18.70	

Table 4.07 shows that the result of the means and standard deviation scores of male students' in the experimental group are 65.72 and 18.54 and that of female students are 65.08 and 18.70. The mean difference in the mean performance scores was found to be 0.64. To find out if the difference in mean was statistically significant, the corresponding hypothesis four was therefore tested. **H₀₄:** There is no significant difference between the mean academic performance scores of Junior Secondary School Male and Female students taught geometry using Concrete Manipulative Approach. To test this hypothesis, the post test scores of the male and female students in the experimental group were compared using t-test statistics. Table 4.08 shows the result obtained.

Table 4.08: t-test Analysis on the mean academic performance of Male and Female Students in the Experimental Group

Group	N	\bar{X}	Standard Deviation	Standard Error	df	t-cal	t-crit	P-value
Male	93	65.72	18.548	1.923	209	-0.246	1.96	0.806**
Female	118	65.08	18.704	1.721				

** Not Significant, $P > 0.05$

From Table 4.08, the calculated t-test value is -0.246 with a critical value of 1.64. The calculated P-value of 0.806 is greater than 0.05 alpha level and is not significant at $P > 0.05$ alpha level hence, t-test value calculated is statistically not significant and therefore the null hypothesis was retained. This implies that there was no significant difference in the mean academic performance of male and female students taught geometry using Concrete Manipulative Approach. This implies significant difference did not exist in the gender factor for the group that received lesson with the Concrete Manipulative Approach. The result thus showed that the Concrete Manipulative Approach did not have significant difference on the male and female students' performance in the Geometry concepts taught.

4.3.5 Post Post-test on Retention of Experimental and Control Groups.

Data on retention ability of Junior Secondary School students taught Geometry using Concrete Manipulative Approach and Conventional Lecture Method

Table 4.09: Result of Mean and Standard Deviation of Post Post-test for Experimental and Control Groups

Group	N	\bar{X}	Standard Deviation	Mean Diff
Experimental	103	86.69	5.22	
				38.27
Control	108	48.42	5.55	

Table 4.09 shows the result of the means and Standard Deviation scores of Junior Secondary School students' retention in the experimental group is 86.69 and 5.22 and that of control group is 48.42 and 5.55. The mean difference in the mean retention scores was found to be 38.27. To find out if the difference in means was statistically significant, the corresponding hypothesis five was therefore tested. **H₀₅:** There is no significant difference between the mean retention scores of Junior Secondary School Students taught Geometry using Concrete Manipulative Approach and Conventional Lecture Method. To test this hypothesis, the Post post-test scores of the subjects in the experimental and control groups were compared using t-test statistics as shown in Table 4.10.

Table 4.10: t-test Analysis of Post Post-test Scores in the Experimental and Control Groups

Group	N	\bar{X}	Standard Deviation	Standard Error	df	t-cal	crit t-	P-value
Experimental	103	86.69	5.222	0.514				
					209	51.49	1.96	0.001*
Control	108	48.42	5.557	0.534				

*Significant, $P \leq 0.05$

From Table 4.10, the t-calculated is 51.49 at 209 degree of freedom and the calculated P-value of 0.001 is less than $P \leq 0.05$ and is significant at $P \leq 0.05$ alpha level hence t-calculated is statistically significant and therefore the null hypothesis was rejected. This implies that there was significant difference in the mean retention scores of Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method. The result thus showed that the Concrete Manipulative Approach raised the retention scores of students in the Geometry concepts taught higher than the Conventional Lecture Method.

4.4 Summary of Major Findings

The summary of the major findings from the results are as follows:

- (1) Students' attitude was rated high towards the learning of the geometry concepts taught due to the use of Concrete Manipulative Approach.
- (2) Female students were rated equally with their male counterparts in their attitude towards geometry when taught using Concrete Manipulative Approach.
- (3) The students' academic performance was high in the geometry concepts taught due to the use of Concrete Manipulative Approach.
- (4) The female students performed equally with their male counterpart when taught geometry using Concrete Manipulative Approach.
- (5) The retention ability of students exposed to CMA teaching was found to be better than that of the students exposed to Conventional Lecture Method.

IV. Discussions

The study investigated the effect of concrete manipulative approach on attitude, retention and performance in geometry among Junior Secondary School Students in Benue State. Several findings were made and were briefly discussed and related to the available literature. The results of data analyzed on Table 4.01 showed a mean attitude rating of 132.08 for the experimental group which was ranked higher than the control group with a mean value of 81.12. The result of hypothesis one indicated that there was a significant difference in the mean attitude ratings of students taught Geometry using Concrete Manipulative Approach and Conventional Lecture Method. The presence of concrete models in the experimental class alone increased the students' interest and curiosity. Moreover the touching, feeling, seen and putting together these models raised the concentration of the students in the class. An increase in attitude rate was recorded when Concrete Manipulative Approach was used in instructional delivery because using this approach in teaching, the teacher assumes the roles of a facilitator, mediator and assessor of learning. This increase in attitude was attributed to the Concrete Manipulative Approach used for teaching the experimental group. The increase in attitude was also

attributed to the educational benefits the subjects derived from the use of the concrete manipulatives: the esthetics, beauty, curiosity, interest and all the affective characters the approach had on the attitude of the students while learning the geometry concepts taught. The free atmosphere and relaxed relationship between learners and objects and between a learner and the teacher who acted only as a guide and facilitator was also a benefit. This therefore supports earlier report by Ajewole (2006), Herwit (2007), Yara (2009) and Yagci (2010) who asserted that children learn best by doing, not just by sitting and listening.

This present finding also confirms the studies of Zan & Martino, (2007) and Walker, et al. (2011) who found that the use of practical skills and certain amount of relevant concrete manipulatives raised the level of concentration, developed positive attitude and enhanced learning. In addition to this, they further observed that students who learned with concrete manipulatives were more interested, patient and curious in the classroom. It is therefore expedient that for effective teaching of geometry to occur, the teacher should get the learners' interest as much as possible through the use of concrete manipulative that will enable them to develop the needed attitude relevant to life.

From Table 4.04, since the Mann-Whitney value of gender was 5468 and the calculated P-value of 0.965 was greater than $P \leq 0.05$, the null hypothesis that stated that there is no significant difference between the mean attitude of the male and female students taught Geometry using the Concrete Manipulative Approach was retained. This implies that the same feelings the male students had on the geometry concepts taught was the same the females students had in the class. The mean attitude ratings of male and female students were statistically equal and that male students were rated same as the female students in the Geometry concepts taught when Concrete Manipulative Approach was used as a pedagogical approach. This research report that the males were rated equally to their female counterparts could therefore be attributed to the avoidance of the persistent use of the often heavily criticized Conventional Lecture Method in the teaching/learning of Geometry.

This finding agrees with Bayram (2004) who was of the view that attitudes are formed through direct experience. He proposed that since the person trying to form or change attitude usually lacked direct knowledge of the internal stimuli available to him or her, it was necessary to rely on external cues in order to understand the concept. Thus the use of concrete manipulative as external cues and observable behaviours led both the male and female students to the increase in attitude. This finding agrees also with that of Halat (2006) who found out that there was no significant difference between male and female adults students attitude towards the study of geometry. Halat, (2006) noted that gender's attitude was not a factor in geometric learning. The findings also agrees with that of the Second International Mathematics Study (SIMS, 1982-1983) that gender difference do not appear in Mathematics learning except in the poorly taught areas such as geometry and measurement. Given their reasons that since girls often do not play games that enhance their visual and spatial knowledge or have prior out-of-class experience, they may be disadvantaged in a geometry class. However, according to Hyde & Mertz, (2009) using concrete manipulatives in a geometry class would provide opportunities for the formation of good attitude towards geometry.

From Table 4.6, since the t-calculated was 75.31 at 209 degree of freedom and the P-value was 0.001 which is less than $P \leq 0.05$, the hypothesis that stated that there is no significant difference in the mean performance scores of students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method was rejected. A high performance score of 84.01 was recorded in the experimental group. This implies that there was a great improvement in the academic performance of Junior Secondary School Students exposed to the Concrete Manipulative Approach than the Conventional Lecture Methods. Various researchers as noted in the literature reviewed in this study refer to conventional lecture method as a telling method and are of the opinion that telling is not teaching or learning even though the method affords the class, opportunities of obtaining useful and essential facts, information and knowledge at the minimum expense of time. Ogwuozor, (2006) asserted that the most prominently used approach in the teaching of mathematics in secondary schools is the often criticized conventional lecture method as against the Concrete Manipulative Approach as advocated by some educationists. Therefore for higher performance, the teacher should involve the learners in manipulatives to enable them acquire the needed manipulative skills relevant in solving geometry problems.

This position agrees with that of White, (2012) who reported that whatever content is taught, whichever environment the school is situated and whatever kind of pupils are given to teach, the important and vital role of the type of teaching approach and concrete manipulatives used cannot be over-emphasized. The high performance in the experimental group than the control group was because of the environment and the pedagogy given to the students to work with. The experimental group had concrete manipulatives in a Concrete Manipulative Approach and they performed higher than the students who received instruction using the Conventional Lecture Method. While at pretest both groups had no experience with concrete manipulatives or approached geometry in a Concrete Manipulative way and so performed equally.

This finding is also similar to that of Eze, (2011), who found out that, the experimental group performed higher than the control group because they visualized and employed enough spatial skills especially the use of the steps of the Concrete Manipulative Approach to learn geometry. Students' ability to accurately

measure, cut plain shapes and observe their geometric properties, patterns and draw simple relationships from the shapes was responsible for this high academic performance. Knowledge retention which serves as a prerequisite to good performance according to Clair, (2004) was also another reason that brought in the high performance in this study. The students were able to recall vividly the concepts taught using Concrete Manipulative Approach (Clair, 2004).

The finding of the study is in support of Bayram, (2004) who found out that there was a statistically significant mean difference between students that received instruction with concrete models and those who received instruction with traditional method. Lending credence to this result, Meng and Idris (2012) reports that a child introduced to any knowledge using concrete manipulative receives a permanent and irreversible knowledge and end up having long term memory of what he or she has learnt; a reversibility property of the Concrete Manipulative Approach. In their submission, Yara (2009) and Yagci (2010) again observed that students who learned using concrete manipulative were more attentive, engaged in learning and participated more actively in the classroom than those who do not.

The high performance in this study is also a reflection of Nnaobi, (2007) and Federal Ministry of Education, (2006) goals. They believe that attainment of set objectives of instruction in a learning process and goals of Mathematics Education in particular are to be achieved because this is a major concern of education policy makers in Nigeria. This therefore refutes earlier report by Odubunmi, (2006) who reported that students considered the learning of geometry to be difficult and therefore creates hatred for it instead. The Concrete Manipulative Approach used in learning the geometry concepts improved the performance in geometry more at posttest than at pretest.

Table 4.07 showed the mean academic performance scores of male and female students taught geometry using Concrete Manipulative Approach was 65.08 and 65.72 respectively. The result of testing hypothesis four indicated that there was no significant difference in the mean performance scores of students taught geometry using the Concrete Manipulative Approach. This implies that the mean performance scores of male and female students are not statistically different when Concrete Manipulative Approach was used as a pedagogical approach. This implies that the treatment given to the experimental group did not affect gender as a factor as far as performance in the geometry concepts taught in this study are concerned.

This finding disagrees with the findings of Guven & Kosa, (2008); Halat, (2006) and Yang & Chen, (2010) who reported that male students performed better than their female counterparts in geometry. That geometry learning is a reflection of different steps taken by male and female students, which was found to be related to more of their masculine abilities than any other factor. According to Connie, et al (2006) and Zheng, (2007) complex variables such as biological, psychological, socio-cultural and environmental variables are taken care of in a Concrete Manipulative Approach which also applied in this study. The approach controlled the gender variable in the learning of the geometry concepts taught.

On Table 4.10, since the t-calculated was 51.49 at 209 degree of freedom and the calculated P-value of 0.001 was less than $P \leq 0.05$ hence t-calculated is statistically significant and therefore the null hypothesis was rejected. This implies that there was significant difference in the mean retention scores of Junior Secondary School students taught geometry using Concrete Manipulative Approach and Conventional Lecture Method. The result thus shows that the Concrete Manipulative Approach raised the retention scores of students in the Geometry concepts taught higher than the Conventional Lecture Method. The high retention scores in the experimental group can be attributed to the learning approach used. Using concrete manipulative as a step in the learning process enabled the students visualize and organize their thoughts well. Such act is in line with Connie, et.al. (2006) who suggested various techniques that when adhered strictly to, helped the students retain in the long-run.

This research finding also agree with that of Steyn, (2004) who argued that knowledge retention is related to the way the concept is taught to the learners and that the teacher's method of teaching is responsible for guiding students in the process of learning. A teaching approach that uses concrete manipulatives, that require practical skills that bring out geometric properties, patterns and assimilating the knowledge and produce a long term memory is useful (Steyn, 2004). Moreover, the finding agrees with that of Cohen, & Manion, (2006) and Ransdem, (2009) who were of the view that students who are taught without the use of concrete manipulatives, tend to only listen and take notes while the teacher continues talking is not necessary. Learners are reduced to mere note-takers and there is no active participation by students in the classroom. This means more of what was heard verbally through conventional lecture method was easily forgotten, thus remembering was very little after wards.

V. Summary, Conclusion And Recommendations

5.1 Introduction

In this chapter the summary of the study, methodology for data collection and analysis are presented. Also presented are the conclusion, recommendations, limitations of the study, the contribution of the study to knowledge and suggestions for further studies.

5.2 Summary

This study was carried out to investigate the effect of concrete manipulative approach on attitude, retention and performance in Geometry among Junior Secondary School students in Benue State, Nigeria. The Concrete Manipulative Approach was used to enhance positive geometric attitude in the students as well as improve the students' retention and academic performance in geometry while the Conventional Lecture Method was used as a control. The study involved learning, using the Concrete Manipulative Approach as the treatment in an attempt to positively change the subjects' attitude, retention and performance to a better one in the selected geometry concepts. The Conventional Lecture Method used as a treatment, served as an external validation. Two hundred and eleven (211) students were used for the data collection. The geometry concepts used for the study were the properties of polygons, perimeter of a square and a rectangle. The area of a square, a rectangle, a parallelogram, a trapezium, a triangle and a circle were also considered. Others were the circumference of a circle, angle properties of parallel lines, properties of an angle at a point of intersecting straight lines and at a point and construction of triangles. Five research questions were asked and answered while five null hypotheses were formulated and tested at $P \leq 0.05$ alpha levels. A forty-item Student's Geometry Attitude Questionnaire was also validated and used to find out the attitudinal change of the subjects towards the Geometry concepts taught in the experimental and control groups. Geometry Performance Test made up of 50 multiple choice type questions was also validated and used to measure the performance of the students at pre-test and at the post-test. The same questions were later reshuffled and used as post-posttest for recall test.

The data collected from the Students' Geometry Attitude Questionnaire, Geometry Performance Test and Geometry Retention Test were analyzed using mean ranks, means, standard deviations, t-test and Mann-Whitney statistical tools to answer the research questions and test the stated hypotheses. The results and discussions were reported in chapter four.

5.3 Conclusion

When CMA teaching is compared with Conventional Lecture Method, students' attitude, retention and conceptual ability in geometry were highly enhanced as follows:

1. Students' were actively engaged in class activities and discussions. Their active participation significantly improves their understanding of geometry concepts.
2. as a result of class activities, the students have the ability to retain geometry concepts better and for a longer time.
3. A friendly and democratic learning environment was created as a result enhanced better class discussions with peers and the teacher.
4. females exposed to the Concrete Manipulative Approach, performed equally with the males in this study.
5. Concrete Manipulative Approach improved the retention level of students in the Geometry concepts better than the Conventional Lecture Method.

5.4 Contribution to Knowledge

The gaps filled by this study are:

- (1) Concrete Manipulative Approach as a student's centered approach can have a tremendous effect when used to teach Geometry especially at the Junior Secondary School level. This approach raises students' manipulative talent and improves academic performance.
- (2) Through CMA, the students especially the junior classes will find Mathematics especially geometry very interesting and focused since they will also be involved in the teaching/learning process. The students will also be curious, confident as well as search, criticize and think creatively.
- (3) CMA will help the students to think with feelings and act with thinking as they feel, see, touch and manipulate the objects themselves. That is, the students will understand geometry from both their affective, cognitive and psychomotor skills.

5.5 Recommendations

Based on the findings from this study the following recommendations were made:

- (1) Concrete Manipulative Approach should be used by students of Geometry in learning as it helps to form Students' positive attitude in learning Geometry.
- (2) Concrete Manipulative Approach should be used by Teachers of Mathematics, as the use of it will aid students to retain much of what is learnt.

- (3) Concrete Manipulative Approach should be used to teach Junior Secondary School Students in order to clear the beliefs that the study of geometry is masculine.
- (4) Teacher training institute like the Colleges of Education and Universities should incorporate the Concrete Manipulative Approach, into their methodology curricular at all levels.

5.6 Limitations of the Study

There are many constraints beyond the researcher during the research. Factors that can affect change in attitude, retention and performance of students forms the limitations to this study. Some of these constraints include:

- (1) The findings of this study only described what was observed during a twelve weeks period.
- (2) Using the research assistants to teach the control group and the researchers' involvement in teaching only the experimental group.

5.7 Suggestions for further Studies

The following suggestions for further studies are made:

1. Similar studies using CMA as teaching strategy can be conducted in science and mathematics to determine their efficacy or the subjects' performance.
2. The study can be replicated in Nigerian tertiary institutions to determine the efficacy and effectiveness of CMA as a teaching strategy on students' performance in all subjects.
3. There is need to assess the efficacy of the CMA strategy relative to other teaching strategies such as demonstration method, group method, inquiry method, concepts mapping and so on.

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**Appendix A
Population of the Study**

S/N	Name of School	Total No. of Students	Pop. Of Male students	Pop. Of Female Students	Education zone	Total students by zone
1	Govt. Sec. Sch. Ihugh	218	144	74	A	
2	Devine Love Cath. Girls College K/Ala	57		57	A	
3	Govt. College K/Ala	203	126	77	A	
4	G S S Tsar	197	127	70	A	
5	Mbawuar Sec. Sch. Ihugh	211	143	78	A	
6	St. Peters Sec. Sch. V/Ikya	198	131	77	A	
7	Mbagba High Sch. Ihugh	238	159	79	A	
8	Fed. Govt. College V/Ikya	112	112		A	
9	G S S Kornya	207	138	69	A	
10	Tilley Gyado College Ihugh	190	126	64	A	
11	N. K. S. T. Sec Sch Zaki Biam	212	146	66	A	
Total			1352	724		2076
12	Govt. Sec. Sch. Gboko	270	183	87	B	
13	St. John Sec.	274	204	70	B	

	Sch. Gboko					
14	Anglican Sec. Sch. Gboko	268	194	74	B	
15	Mbakor Com. Sec. Sch. Wannune	238	154	84	B	
16	Fed. Govt. Girls College Gboko	61		61	B	
17	Bristow Sec. Sch. Gboko	298	214	84	B	
18	Govt. College Makurdi	210	210		B	
19	Vaatia College Makurdi	239	158	81	B	
20	Tilley Gyado College Makurdi	310	234	76	B	
Total			1551	617		2168
21	Govt. Sec. Sch. Otukpo.	209	181	28	C	
22	Otukpa Comprehensive Sec Sch. Otukpa	197	167	30	C	
23	Govt. Sec. Sch. Oju.	206	179	27	C	
24	St. Gerald Sec. Sch. Otukpo.	198	169	29	C	
25	St. Paul Sec Sch. Otukpo	212	181	31	C	
26	Wesley High Sch. Otukpo	175	150	25	C	
27	St. Anns Girl Sec Sch. Otukpo	169	141	28	C	
28	St. Francis Sec Sch. Otukpo	231	200	31	C	
29	Orkama High Sch. Otukpa	222	201	21	C	
30	Fed. Govt. College Otukpo	167	132	35	C	
Total			1701	285		1986
Grand Total			4604	1626		6230

Source: Teaching Service Board, Makurdi (2014).

Appendix b Geometry performance test

Time: $1\frac{1}{2}$ hours

INSTRUCTIONS:

- (a) Write in capital letters your **surname** followed by your **other names**.
- (b) In the box marked gender, indicate with an **M** if you are male, or **F** if you are female
- (c) Answer **All** the questions
- (d) Each question is followed by **four** options lettered A to D. Find out the correct option for **each** question and write on your answer booklet the letter which bears the same letter as the options you have chosen. Give only one answer to **each** question.

1. Calculate the area of a sector of a circle which subtends an angle of 45° at the Centre of the circle, radius 12cm.

(a) $92.86 \pi \text{ cm}^2$ (b) $42.677 \pi \text{ cm}^2$ (c) $187 \pi \text{ cm}^2$ (d) $72.03 \pi \text{ cm}^2$

2. Calculate the area of the trapezium ABCD in Fig. 1.

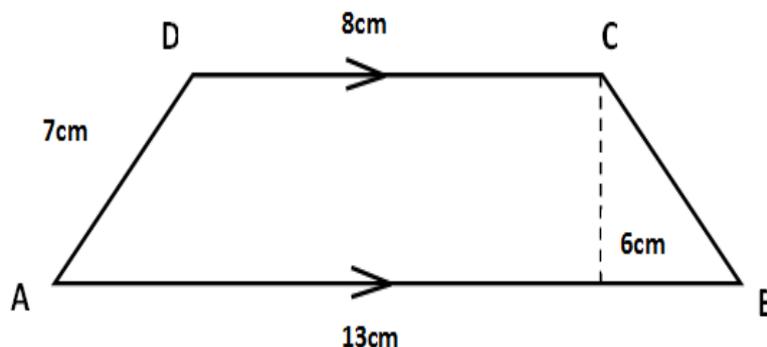


Fig. 1

- (a) 48cm^2 (b) 40cm^2 (c) 63cm^2 (d) 34cm^2
3. Calculate the perimeter of a segment whose radius is 7cm, and the sector angle is 60° and $\pi = \frac{22}{7}$
- (a) 12.2cm (b) 14.33cm (c) 31cm (d) 10.38cm
4. Calculate the area of a segment of a circle radius 4cm, where the sector angle is 45° and $\pi = \frac{22}{7}$.
- (a) 1.68cm^2 (b) 1.63cm^2 (c) 0.68cm^2 (d) 0.63cm^2
5. Calculate the area of the triangle in Fig. 2.

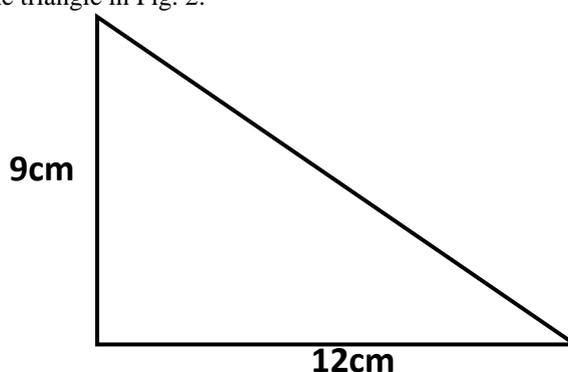


Fig. 2.

- (a) 102cm^2 (b) 54cm^2 (c) 61cm^2 (d) 35cm^2
6. Angle 60 is an angle bisected from
(a) 30 (b) 120 (c) 135 (d) 90
7. The area of circle PQR with centre O is 72cm^2 . What is the area of sector POB if angle $POQ = 40^\circ$?
- (a) 8cm^2 (b) 11cm^2 (c) 33cm^2 (d) 7cm^2
8. Calculate the perimeter of a sector of a circle of radius 28cm with 80° as the sector angle.
- (a) 96.11cm^2 (b) 91.11cm^2 (c) 65.11cm^2 (d) 95.11cm^2
9. Calculate the area of the shaded segment of the circle with radius 20cm and angle 63° .
- (a) 41.8cm^2 (b) 91.8cm^2 (c) 66.1cm^2 (d) 95.11cm^2
10. Angle 45 is an angle bisected from
(a) 30 (b) 45 (c) 22.5 (d) 90

11. A goat is tied to a peg in the ground. The rope is 3m long, what area of grass can the goat eat?
 (a) 4.84cm^2 (b) 27.9cm^2 (c) 33.84cm^2 (d) 3.86cm^2

12. Using the diagram calculate the perimeter of the minor segment ACB to 4s.f.

Fig. 3

- (a) 20.47cm (b) 25.24cm (c) 22.25cm (d) 20.34cm

13. Calculate the shaded area in Fig. 4.

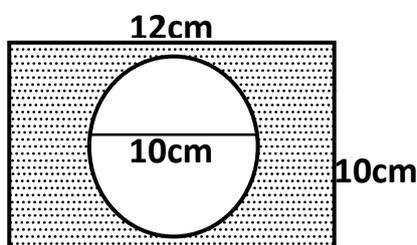


Fig. 4.

- (a) 41.5cm^3 (b) 11.5cm^3 (c) 33.5cm^3 (d) 13.5cm^3

14. Find the area of the shaded portion in Fig. 5.

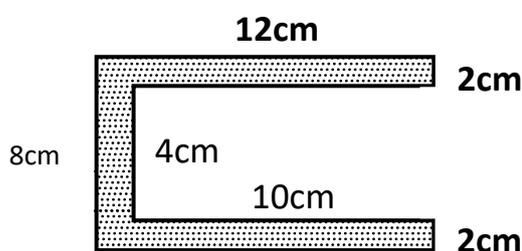


Fig. 5.

- (a) 65cm^2 (b) 50cm^2 (c) 66cm^2 (d) 56cm^2

15. The sports field shown in Fig. 6 has a $90\text{m} \times 70\text{m}$ football field with a semi circular area at each end. A track runs round the perimeter of the field. Use $\pi = \frac{22}{7}$ to calculate the area of the sports field.

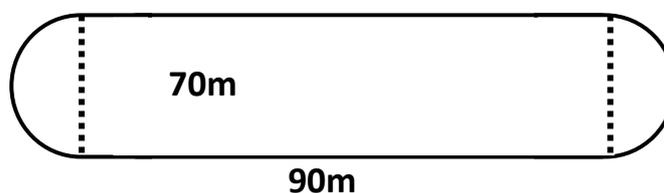


Fig. 6.

- (a) 10140m^2 (b) 10150m^2 (c) 10250m^2 (d) 10155m^2

16. Find the area of a circle of radius $3\frac{1}{2}$ (taking $\pi = \frac{22}{7}$).

- (a) $105\frac{1}{2}\text{cm}^2$ (b) $16\frac{1}{2}\text{cm}^2$ (c) $30\frac{1}{2}\text{cm}^2$ (d) $38\frac{1}{2}\text{cm}^2$

17. Find the area of a circle of diameter 14cm (taking $\pi = \frac{22}{7}$).

- (a) 54cm^2 (b) 155cm^2 (c) 154cm^2 (d) 254cm^2

18. _____ is a step in construction triangle UVW of length UV = 12cm,
VW = 11cm and UW = 10cm.

- (a) With radius 11cm draw a line segment VW.
- (b) With v as centre of convenient radius, draw an arc to meet VW at A.
- (c) Draw an angle UVW
- (d) All of the above.

19. Calculate the area of a sector of a circle which subtends an angle of 60° at the centre of the circle, radius 15cm.

- (a) $41.57 \pi \text{ cm}^2$ (b) $9.1 \pi \text{ cm}^2$ (c) $35.17 \pi \text{ cm}^2$ (d) $37.57 \pi \text{ cm}^2$

20. Calculate the perimeter of a segment whose radius is 12cm, and the sector angle is 60° and $\pi = \frac{22}{7}$

- (a) 12.2cm (b) 24.57cm (c) 31cm (d) 10.38cm

21. Calculate the area of the shaded portion to 3s.f (taking $\pi = \frac{22}{7}$).

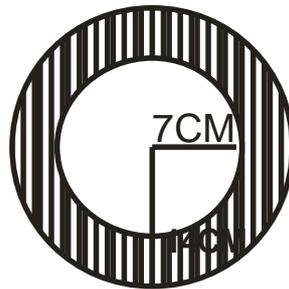


Fig. 7

- (a) 18.2cm^2 (b) 462cm^2 (c) 461cm^2 (d) 16.2cm^2

22. How many steps are involved when constructing a triangle were only two sides are given?

- (a) 6 (b) none of the above (c) 5 (d) 4

23. How many steps are needed in constructing a triangle XYZ of length XY = 8, YZ = 7cm and XZ = 6cm?

- (a) 5 (b) 6 (c) 4 (d) none of the above

24. The shaded portion in the diagram in fig 8 is the

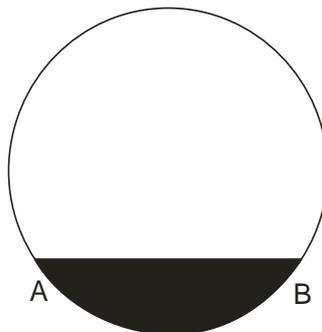


Fig. 8

- (a) Minor segment (b) major segment (c) minor sector (d) major sector

25. Find the area of the shaded portion in Fig. 9.

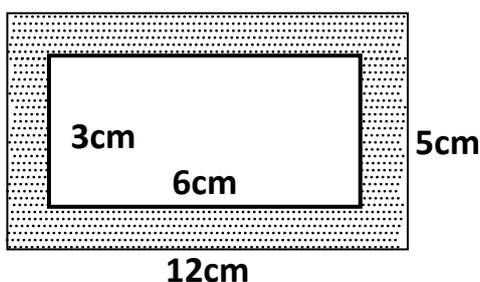


Fig. 9

(a) 43 cm^2 (b) 42 cm^2 (c) 41 cm^2 (d) 21 cm^2

26. The sum of angles on a straight line is

(a) 90° (b) 36° (c) 180° (d) 260°

27. Calculate the perimeter of a sector of a circle of radius 14cm where the sector angle is 60° .

(a) 43.02 cm^2 (b) 37.14 cm^2 (c) 42.7 cm^2 (d) 105.14 cm^2

28. Calculate the area of the diagram in Fig. 10

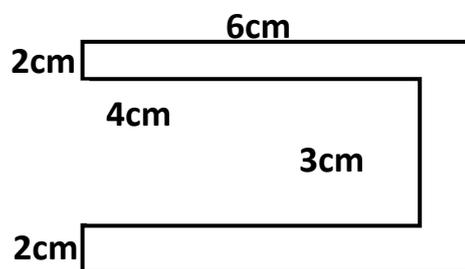


Fig. 10

(a) 30 cm^3 (b) 1056 cm^3 (c) 920 cm^3 (d) 1011 cm^3

29. Which of the following angles when bisected will give you 22.5

(a) 30 (b) 45 (c) 22.5 (d) 90

30. Calculate the area of the shape in Fig 11 (taking $\pi = 3.14$).

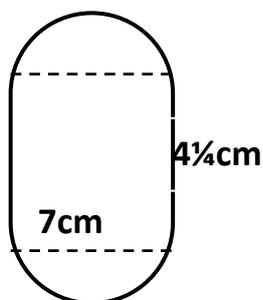


Fig. 11

(a) 180 cm^2 (b) 17 cm^2 (c) 69.9 cm^2 (d) 14 cm^2

31. Find the size of the marked angle in Fig. 10.

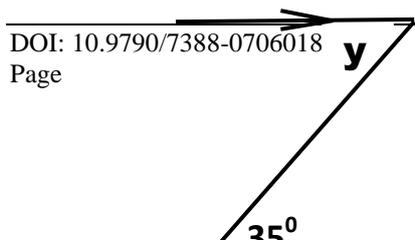


Fig. 12

- (a) 70° (b) 45° (c) 60° (d) 35°

32. Find the value of x in the diagram below.

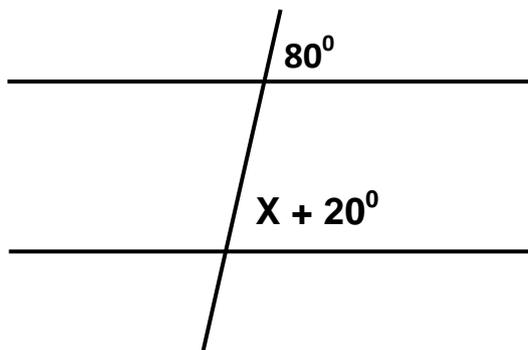


Fig. 13

- (a) 50° (b) 60° (c) 45° (d) 120°

33. Find the value of C in the figure below

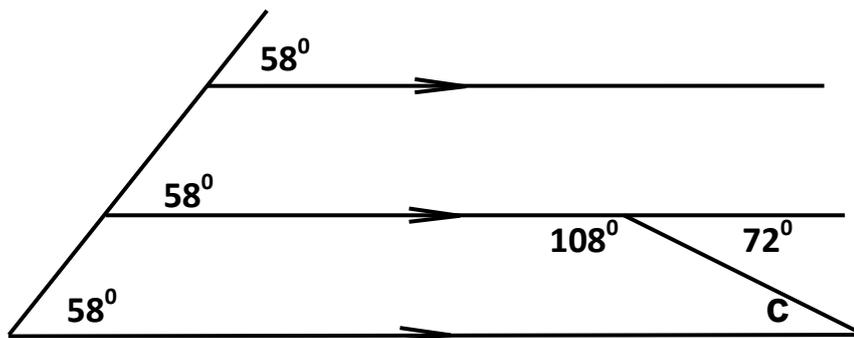


Fig. 14

- (a) 72° (b) 58° (c) 108° (d) 135°

34. Find the value of the angle w in the diagram below:

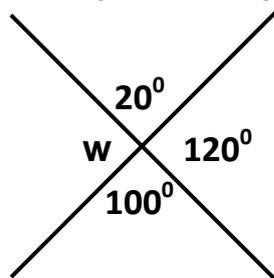


Fig. 15

- (a) 335° (b) 20° (c) 120° (d) 100°

35. Calculate the area of the trapezium in the diagram below:

Fig. 16

- (a) 50cm^2 (b) 42cm^2 (c) 28cm^2 (d) 32cm^2

36. Find the area of the semi-circular disc whose diameter is 28cm.

- (a) 436cm^2 (b) 2.639cm^2 (c) 0.497cm^2 (d) 4.378cm^2

37. The area of metal sheet required to make a circular washer whose internal radius is 2.2cm and external radius is 7.8cm is

zz

Fig. 17

- (a) 176cm^2 (b) 202cm^2 (c) 102cm^2 (d) 340cm^2

Use Fig. 18 to answer questions 38 and 39.

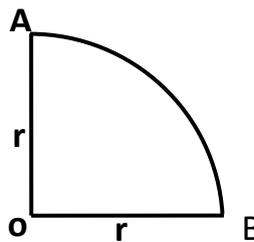


Fig. 18

38. In the quadrant of the circle AOB, if the length of the arc = 33cm, calculate the radius of the arc.

- (a) 20cm (b) 21cm (c) 12cm (d) 58cm

39. Calculate the area of the quadrant of the circle.

- (a) $342\frac{1}{2}\text{cm}^2$ (b) 346cm^2 (c) $346\frac{1}{2}\text{cm}^2$ (d) 343cm^2

40. Draw ΔABC in which $AB = 4\text{cm}$ $AC = 5\text{cm}$ and $BC = 6\text{cm}$. Through M, the midpoint of BC, draw lines parallel to AC and AB respectively to meet AB and AC at H and K respectively. Measure line HK.

- (a) 3cm (b) 2cm (c) 4cm (d) 5cm

41. Calculate the area of ΔABC in the diagram below:

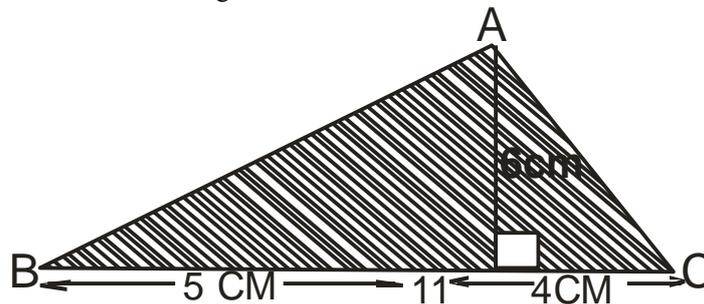


Fig. 19

- (a) 50cm^2 (b) 30cm^2 (c) 27cm^2 (d) 27cm^3

42. A hexagon has how many sizes?

- (a) 8 (b) 4 (c) 5 (d) 6

43. An intercept is the part of the transversal that cut off

- (a) Before the lines
 (b) Between the lines
 (c) After the lines
 (d) Into the three lines

44. A line drawn parallel to one side of a triangle divides the other sides in

- (a) Three parts
 (b) The same ratio

- (c) The same size
 (d) The same figure
45. The angle which a chord subtends at the centre of the circle is twice the angle it subtends at
 (a) The chord
 (b) The sector
 (c) The segment
 (d) The circumference
46. The diagram shows a cyclic quadrilateral PQRS with its diagonals intersecting at K. Which of the following triangles is similar to triangle QKR?

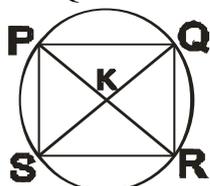


Fig. 20

- (a) ΔPQK (b) ΔSKR (c) ΔPSR (d) ΔPKS
47. Calculate the ΔABC below:

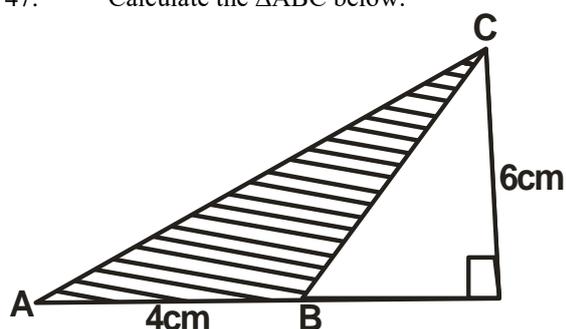


Fig. 21

- (a) 14 cm^2 (b) 12 cm^2 (c) 13 cm^2 (d) 15 cm^2
48. A rectangular polygon of a n sides has each exterior angle equal to 45° , find the value of n
 (a) 8 (b) 15 (c) 6 (d) 12
49. Find the size of the angle marked x in the diagram.

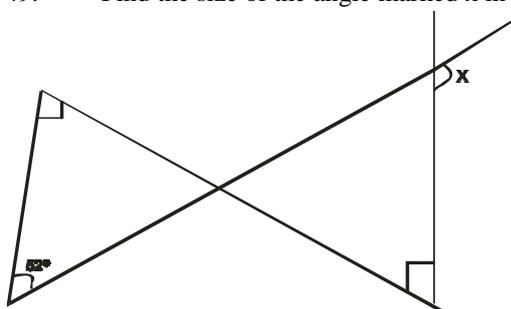


Fig. 22

- (a) 108° (b) 128° (c) 132° (d) 142°
50. What are the properties common to both a square and a rectangle?
 (a) All the properties of a square.
 (b) All the properties of a parallelogram.
 (c) All the properties of a rectangle.
 (d) All of the above.

Marking Scheme for GPT

1. C	26 C
2. C	27 C

- | | |
|-------|------|
| 3. B | 28 A |
| 4. D | 29 B |
| 5. B | 30 C |
| 6. B | 31 D |
| 7. A | 32 B |
| 8. D | 33 A |
| 9. A | 34 C |
| 10. D | 35 C |
| 11. B | 36 A |
| 12. A | 37 A |
| 13. A | 38 B |
| 14. D | 39 C |
| 15. B | 40 A |
| 16. D | 41 C |
| 17. C | 42 D |
| 18. A | 43 B |
| 19. D | 44 B |
| 20. B | 45 D |
| 21. B | 46 D |
| 22. C | 47 B |
| 23. A | 48 A |
| 24. C | 49 B |
| 25. B | 50 D |

Appendix c
Geometry retention test

Time: $1\frac{1}{2}$ hours

Instructions:

- Write in capital letters your **surname** followed by your **other names**.
- In the box marked gender, indicate with an **M** if you are male, or **F** if you are female
- Answer **All** the questions
- Each question is followed by **four** options lettered A to D. Find out the correct option for **each** question and write on your answer booklet the letter which bears the same letter as the options you have chosen. Give only one answer to **each** question.

1. Find the area of the shaded portion in Fig. 1.

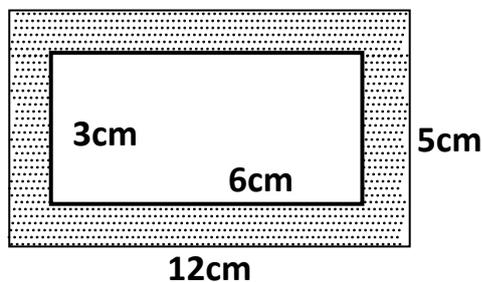


Fig.1

- (a) 43 cm^2 (b) 42 cm^2 (c) 41 cm^2 (d) 21 cm^2
2. The sum of angles on a straight line is
(a) 90° (b) 36° (c) 180° (d) 260°
3. Calculate the perimeter of a sector of a circle of radius 14cm where the sector angle is 60° .

(a) 43.02cm^2 (b) 37.14cm^2 (c) 42.7cm^2 (d) 105.14cm^2

4. Calculate the area of the figure in Fig. 2.

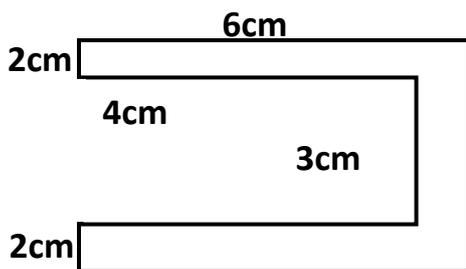


Fig. 2

(a) 30cm^3 (b) 1056cm^3 (c) 920cm^3 (d) 1011cm^3

5. Which of the following angles when bisected will give you 22.5

(a) 30 (b) 45 (c) 22.5 (d) 90

6. Calculate the area of the shape in Fig 3. (taking $\pi = 3.14$).

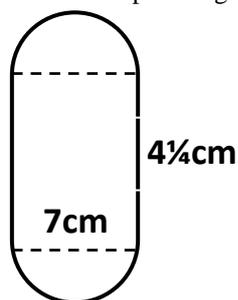


Fig. 3

(a) 180cm^2 (b) 17cm^2 (c) 69.9cm^2 (d) 14cm^2

7. Find the size of the marked angle in Fig. 4.

8.

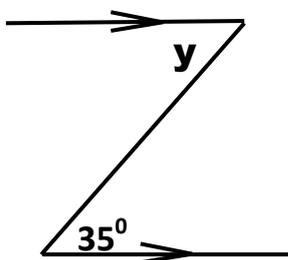
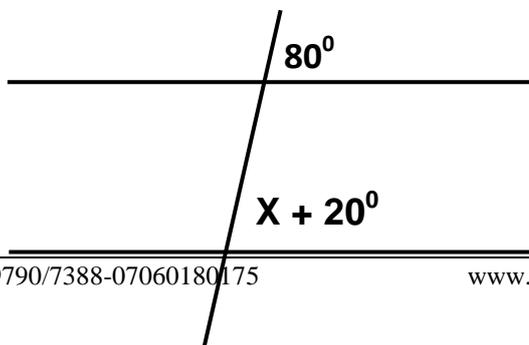


Fig. 4

(a) 70° (b) 45° (c) 60° (d) 35°

8. Find the value of x in the diagram below.



- Fig. 5
 (a) 50° (b) 60° (c) 45° (d) 120°
 9. Find the value of C in the figure below

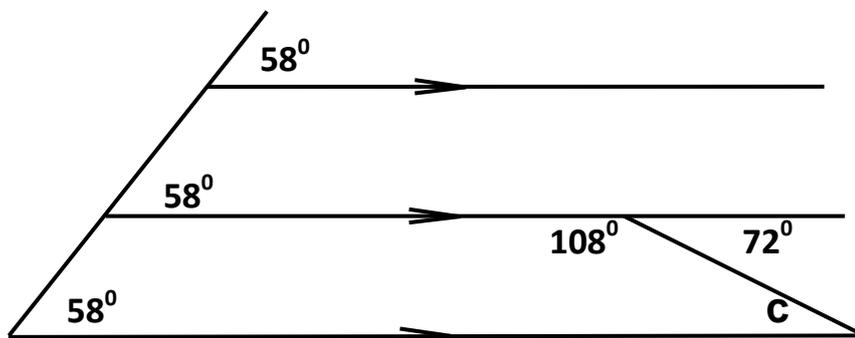
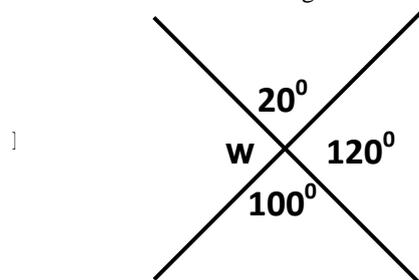
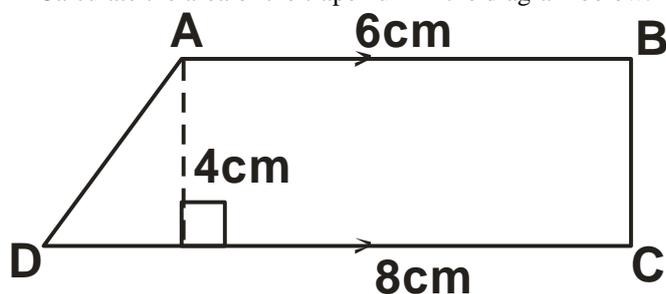


Fig. 6

- (a) 72° (b) 58° (c) 108° (d) 135°
 10. Find the value of the angle w in the diagram below:



- (a) 335° (b) 20° (c) 120° (d) 100°
 11 Calculate the area of the trapezium in the diagram below:



- Fig. 8
 (a) 50cm^2 (b) 42cm^2 (c) 28cm^2 (d) 32cm^2

- 12 Find the area of the semi-circular disc whose diameter is 28cm.
 (a) 436cm^2 (b) 2.639cm^2 (c) 0.497cm^2 (d) 4.378cm^2
 13 The area of metal sheet required to make a circular washer whose internal radius is 2.2cm and external radius is 7.8cm is

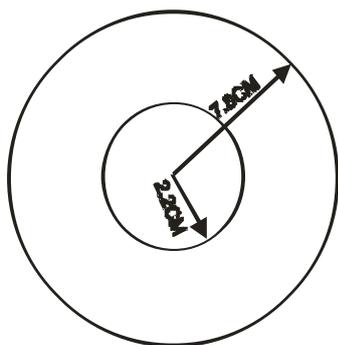


Fig. 9

- (a) 308cm^2 (b) 202cm^2 (c) 102cm^2 (d) 340cm^2

Use Fig. 10 to answer questions 14 and 15.

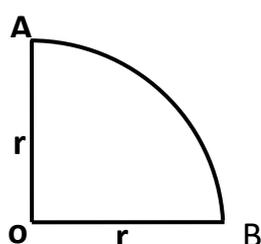


Fig. 10

- 14 In the quadrant of the circle AOB, if the length of the arc = 33cm, calculate the radius of the arc.
 (a) 20cm (b) 21cm (c) 12cm (d) 58cm
- 15 Calculate the area of the quadrant of the circle.
 (a) $342\frac{1}{2}\text{cm}^2$ (b) 346cm^2 (c) $346\frac{1}{2}\text{cm}^2$ (d) 343cm^2
- 16 Draw ΔABC in which $AB = 4\text{cm}$ $AC = 5\text{cm}$ and $BC = 6\text{cm}$. Through M, the midpoint of BC, draw lines parallel to AC and AB respectively to meet AB and AC at H and K respectively. Measure line HK.
 (a) 3cm (b) 2cm (c) 4cm (d) 5cm
- 17 Calculate the area of ΔABC in the diagram below:

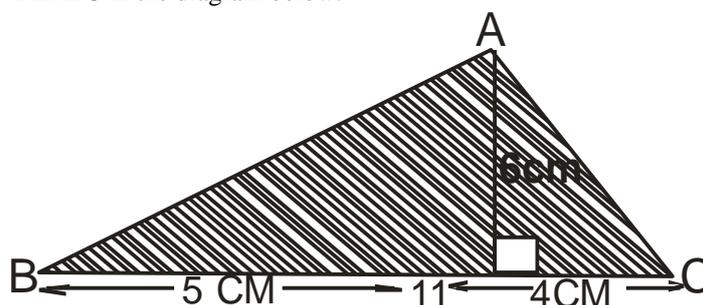


Fig. 11

- (a) 50cm^2 (b) 30cm^2 (c) 27cm^2 (d) 27cm^3
- 18 A hexagon has how many sides?
 (a) 8 (b) 4 (c) 5 (d) 6
- 19 An intercept is the part of the transversal that cut off
 (a) Before the lines
 (b) Between the lines
 (c) After the lines
 (d) Into the three lines
- 20 A line drawn parallel to one side of a triangle divides the other side in
 (a) Three parts

- (b) The same ratio
- (c) The same size
- (d) The same figure

21 The angle which a chord subtends at the centre of the circle is twice the angle it subtends at

- (a) The chord
- (b) The sector
- (c) The segment
- (d) The circumference

22 The diagram shows a cyclic quadrilateral PQRS with its diagonals intersecting at K. Which of the following triangles is similar to triangle QKR?

Fig. 12

- (a) $\Delta P Q K$
- (b) $\Delta S K R$
- (c) $\Delta P S R$
- (d) $\Delta P K S$

23 Calculate the ΔABC below:

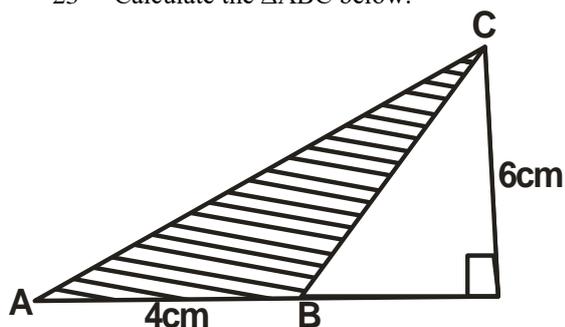


Fig. 13

- (a) 14 cm^2
- (b) 12 cm^2
- (c) 13 cm^2
- (d) 15 cm^2

24 A rectangular polygon of a n sides has each exterior angle equal to 45° , find the value of n

- (a) 8
- (b) 15
- (c) 6
- (d) 12

25 Find the size of the angle marked x in the diagram.

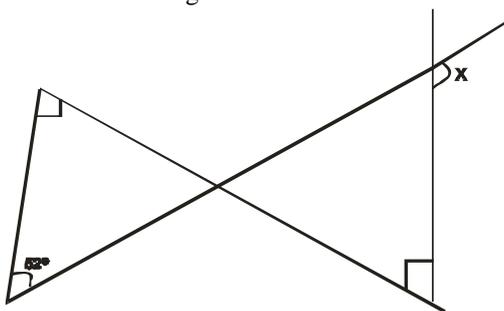


Fig. 14

- (a) 108°
- (b) 128°
- (c) 132°
- (d) 142°

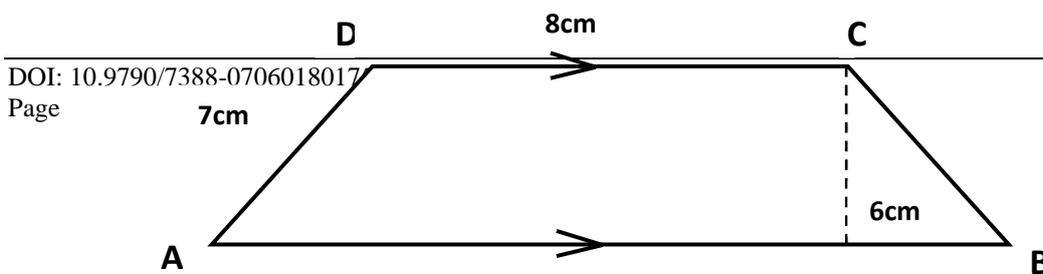
26 What are the properties common to both a square and a rectangle?

- (a) All the properties of a square.
- (b) All the properties of a parallelogram.
- (c) All the properties of a rectangle.
- (d) All of the above.

27 Calculate the area of a sector of a circle which subtends an angle of 45° at the Centre of the circle, radius 12cm.

- (a) $92.86 \pi \text{ cm}^2$
- (b) $42.677 \pi \text{ cm}^2$
- (c) $187 \pi \text{ cm}^2$
- (d) $72.03 \pi \text{ cm}^2$

28 Calculate the area of the trapezium ABCD in Fig. 1.



- (a) 48cm^2 (b) 40cm^2 (c) 63cm^2 (d) 34cm^2

Fig. 15

- 29 Calculate the perimeter of a segment whose radius is 7cm, and the sector angle is 60° and $\pi = \frac{22}{7}$
 (a) 12.2cm (b) 14.33cm (c) 31cm (d) 10.38cm
- 30 Calculate the area of a segment of a circle radius 4cm, where the sector angle is 45° and $\pi = \frac{22}{7}$.
 (a) 1.68cm^2 (b) 1.63cm^2 (c) 0.68cm^2 (d) 0.63cm^2
- 31 Calculate the area of the triangle in Fig. 2.

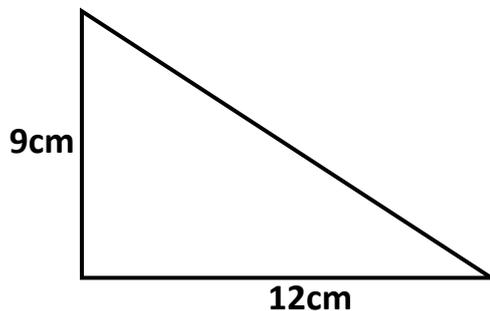


Fig. 16

- (a) 102cm^2 (b) 54cm^2 (c) 61cm^2 (d) 35cm^2

- 32 Angle 60 is an angle bisected from
 (a) 30 (b) 120 (c) 135 (d) 90
- 33 The area of circle PQR with centre O is 72cm^2 . What is the area of sector POB if angle $POQ = 40^\circ$?
 (a) 8cm^2 (b) 11cm^2 (c) 33cm^2 (d) 7cm^2
- 34 Calculate the perimeter of a sector of a circle of radius 28cm with 80° as the sector angle.
 (a) 96.11cm^2 (b) 91.11cm^2 (c) 65.11cm^2 (d) 95.11cm^2
- 35 Calculate the area of the shaded segment of the circle with radius 20cm and angle 63° .
 (a) 41.8cm^2 (b) 91.8cm^2 (c) 66.1cm^2 (d) 95.11cm^2
- 36 Angle 45 degrees is an angle bisected from
 (a) 30 (b) 45 (c) 22.5 (d) 90
- 37 A goat is tied to a peg in the ground. The rope is 3m long, what area of grass can the goat eat?
 (a) 4.84cm^2 (b) 27.9cm^2 (c) 33.84cm^2 (d) 3.86cm^2
- 38 Using the diagram calculate the perimeter of the minor segment ACB to 4s.f.

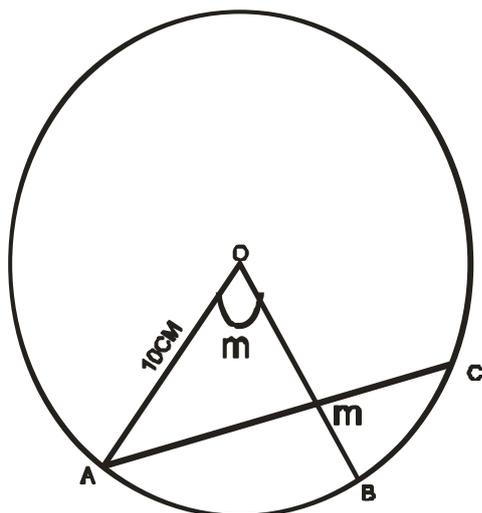


Fig. 17

- (a) 20.47cm (b) 25.24cm (c) 22.25cm (d) 20.34cm

39 Calculate the shaded area in Fig. 4.

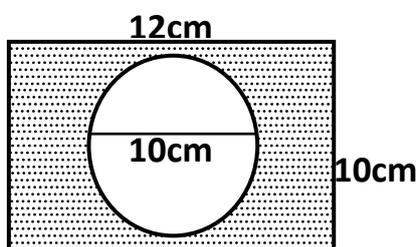


Fig. 18

- (a) 41.5cm^3 (b) 11.5cm^3 (c) 33.5cm^3 (d) 13.5cm^3

40 Find the area of the shaded portion in Fig. 19

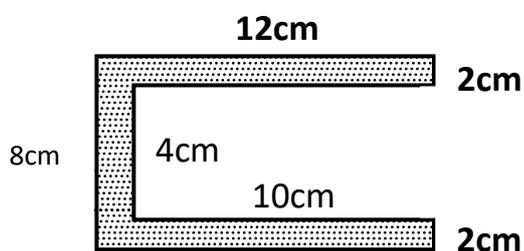


Fig. 19

- (a) 65cm^2 (b) 50cm^2 (c) 66cm^2 (d) 56cm^2

41 The sports field shown in Fig. 6 has a 90m x 70m football field with a semi circular area at each end. A track runs round the perimeter of the field. Use $\pi = \frac{22}{7}$ to calculate the area of the sports field.

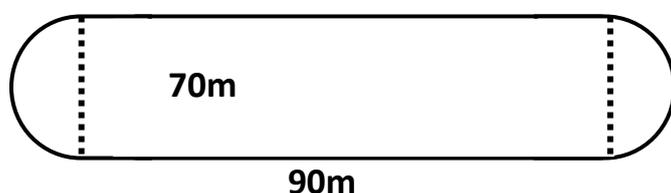


Fig. 20

- (a) 10140m^2 (b) 10150m^2 (c) 10250m^2 (d) 10155m^2
- 42 Find the area of a circle of radius $3\frac{1}{2}$ (taking $\pi = \frac{22}{7}$).
- (a) $105\frac{1}{2}\text{cm}^2$ (b) $16\frac{1}{2}\text{cm}^2$ (c) $30\frac{1}{2}\text{cm}^2$ (d) $38\frac{1}{2}\text{cm}^2$
- 43 Find the area of a circle of diameter 14cm (taking $\pi = \frac{22}{7}$).
- (a) 54cm^2 (b) 155cm^2 (c) 154cm^2 (d) 254cm^2
- 44 _____ is a step in construction triangle UVW of length UV = 12cm, VW = 11cm and UW= 10cm.
- (a) With radius 11 cm draw a line segment VW.
 (b) With v as centre of convenient radius, draw an arc to meet VW at A.
 (c) Draw an angle UVW
 (d) All of the above.
- 45 Calculate the area of a sector of a circle which subtends an angle of 60° at the centre of the circle, radius 15cm.
- (a) $41.57 \pi \text{ c m}^2$ (b) $9.1 \pi \text{ cm}^2$ (c) $35.17 \pi \text{ cm}^2$ (d) $37.57 \pi \text{ cm}^2$
- 46 Calculate the perimeter of a segment whose radius is 12cm, and the sector angle is 60° and $\pi = \frac{22}{7}$
- (a) 12.2cm (b) 24.57cm (c) 31cm (d) 10.38cm
- 47 Calculate the area of the shaded portion to 3s.f (taking $\pi = \frac{22}{7}$).

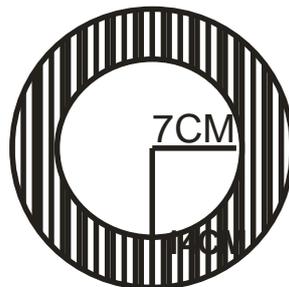


Fig. 21

- (a) 18.2cm^2 (b) 462cm^2 (c) 461cm^2 (d) 16.2cm^2
- 48 How many steps are involved when constructing a triangle were only two sides are given?
- (a) 6 (b) none of the above (c) 5 (d) 4
- 49 How many steps are needed in constructing a triangle XYZ of length XY = 8, YZ = 7cm and XZ = 6cm?
- (a) 5 (b) 6 (c) 4 (d) none of the above
- 50 The shaded portion in the diagram is the

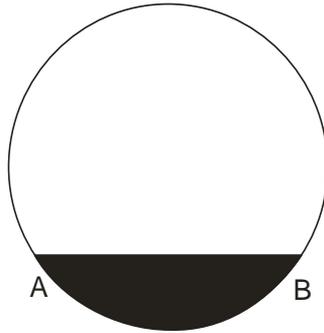


Fig. 22

(a) Minor segment (b) major segment (c) minor sector (d) major sector

Marking Scheme for GRT

- | | |
|-------|-------|
| 1. C | 26. C |
| 2. C | 27. C |
| 3. B | 28. A |
| 4. D | 29. B |
| 5. B | 30. C |
| 6. B | 31. D |
| 7. A | 32. B |
| 8. D | 33. A |
| 9. A | 34. A |
| 10. D | 35. C |
| 11. B | 36. A |
| 12. A | 37. A |
| 13. A | 38. B |
| 14. D | 39. C |
| 15. B | 40. A |
| 16. D | 41. C |
| 17. C | 42. D |
| 18. A | 43. B |
| 19. D | 44. B |
| 20. B | 45. D |
| 21. B | 46. D |
| 22. C | 47. B |
| 23. A | 48. A |
| 24. C | 49. B |
| 25. B | 50. D |

Appendix d

Students' geometry attitude questionnaire (sgaq)

Name of student-----Class-----Age-----Sex-----

Name of School-----

This is a perception of your feelings and self-confidence towards geometry which is an aspect of mathematics. It is not to measure your cognitive or psychomotor abilities but your affective abilities. You are requested to tick the appropriate option out of the four alternatives provided for each statement. The options are Strongly Agreed (SA), Agreed (A), Disagree (D) and Strongly Disagree (SD).

Your response will be treated confidentially. Please be truthful in your response to each statement. Do not tick two options in one statement.

S/N	STATEMENTS	SA	A	D	SD
1.	I am sure that I can learn geometry				
2.	My teacher has been interested in my progress in geometry				
3.	I am doing geometry because I know it is necessary for my future career				
4.	I don't think I could do advanced geometry				
5.	Geometry will not be important to me in my life's work				
6.	Getting a teacher to take me seriously in geometry is a problem				
7.	Geometry is hard for me				
8.	I will need geometry for my future work				
9.	I am sure of myself when I do geometry				
10.	I do not expect to use much geometry when I get out of school				
11.	I would talk to my geometry teacher about a career which uses geometry				
12.	Geometry is a worthwhile, necessary area in Mathematics				
13.	I am not the type to do well in geometry				
14.	My teacher have encouraged me to study more geometry				
15.	Taking geometry is a waste of time				
16.	Geometry has been my worst area in Mathematics				
17.	I think I could handle more difficult geometry				
18.	My teachers think advanced geometry will be a waste of time for me				
19.	I will use geometry in many ways as an adult				
20.	I see geometry as something I won't use very often when I get out of school				
21.	I feel that geometry teachers ignore me when I try to talk about geometry				
22.	Most subjects I can handle very well, but I cannot do a good job with geometry				
23.	I can get good grades in geometry				
24.	I will need a good understanding of geometry for my future work				
25.	My teachers want me to talk all the geometry I can				
26.	I know I can do well in geometry				
27.	Doing well in geometry is not important for my future				
28.	My teachers would not take me seriously if I told them I was interested in a career in geometry				
29.	I am sure I could do advanced work in geometry				
30.	Geometry is not import for my life				
31.	I am good in geometry				
32.	I study geometry because I know how useful it is				
33.	Geometry teachers have made me feel I have the ability to go on in geometry				
34.	My teacher think I am the kind of person who would do well in geometry				
35.	I understand geometry better because it deals with concrete manipulatives				
36.	I like geometry because it deals with observable properties				
37.	Females can do just as well as males in geometry				
38.	Females are as good as males in geometry				
39.	Studying geometry is just as good for female as for male				
40.	Females who enjoy studying geometry are a little bit strange in our society				

Appendix e
Item analysis of geometry performance test

Item NO	(i) $DI = \frac{u+L}{N}$	(ii) $DII = \frac{U-L}{\frac{1}{2}N}$	(iii) $DIII = \frac{U-L}{\frac{1}{2}N}$
1.	0.38	0.15	-0.18
2.	0.66	-0.36	-0.03

3.	0.41	0.33	0.24*
4.	0.56	-0.09	0.02*
5.	0.23*	-0.15	-0.27
6.	0.39	-0.18	0.58**
7.	0.55	-0.24	0.18*
8.	0.79*	0.18	0.06*
9.	0.45	-0.60	-0.45
10.	0.30	-0.06	0.21*
11.	0.47	-0.33	-0.21
12.	0.98*	0.09	-0.09
13.	1.36*	1.36	-0.21
14.	0.41	0.90	0.01*
15.	0.24*	-0.45	0.09*
16.	0.23*	-0.12	-0.09
17.	0.75*	-0.15	-0.12
18.	0.65	1.21	-1.06
19.	0.62	-0.27	0.12*
20.	0.18*	-0.70	0.12*
21.	0.03*	0	-0.06
22.	0.37	-0.45	-0.24
23.	0.65	-0.09	0.09*
24.	0.70	-1.03	0*
25.	0.27*	-0.06	-0.30
26.	0.30	0	-0.42
27.	0.24*	0.12	0.21*
28.	0.88*	-0.55	0.21*
29.	0.58	-0.55	0.15*
30.	0.76*	-0.15	-0.27
31.	0.45	-0.24	-0.24
32.	0.56	-0.09	0*
33.	0.58	0.54	-0.61
34.	0.56	0.39	-0.30
35.	0.45	0.18	-0.03
36.	0.71*	-0.33	0.09*
37.	0.69	-0.18	-0.03
38.	0.89*	0.03	-0.79
39.	0.60	-0.61	-0.52
40.	0.80*	-0.09	-0.58
41.	0.50	0.45	-0.58
42.	0.43	0.51	-0.67
43.	0.61	0.12	-0.24
44.	0.55	-0.12	-0.06
45.	0.28*	-0.03	-0.30
46.	0.68	0.03	-0.15
47.	0.62	0.09	-0.09
48.	0.59	0.52	-0.58
49.	0.56	-0.39	-0.24
50.	0.65	-0.03	-0.03

DI - Item Difficulty Index

DII - Item Discrimination Index

DIII - Item Distracter Index

*- Item which where reconstructed

Appendix f Reliability of the research instruments

A. Test Instrument

The test re-test method (measure of stability) of reliability estimate was used in this study to arrive at the Reliability Coefficient.

GET

FILE='C:\Users\AKAAZUA\Documents\AFAMDATA2.sav'.

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RELIABILITY

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/SCALE('ALL VARIABLES') ALL
 /MODEL=ALPHA
 /STATISTICS=CORR.

Reliability

Notes		
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Comments		
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	Weight	<none>
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	N of Rows in Working Data File	87
	Matrix Input	
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on all cases with valid data for all variables in the procedure.
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Resources	Processor Time	00:00:00.000
	Elapsed Time	00:00:00.000

[DataSet1] C:\Users\AKAAZUA\Documents\AFAMDATA2.sav

Scale: ALL VARIABLES

Case Processing Summary			
		N	%
Cases	Valid	87	100.0
	Excluded ^a	0	.0
	Total	87	100.0

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics		
Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.718	.723	2

Inter-Item Correlation Matrix		
	TEST1	TEST2
TEST1	1.000	.566
TEST2	.566	1.000

B. Students Geometry Attitude Questionnaire

The Cronbach alpha method (measure of internal consistency) of reliability estimate was used in this study to compute the reliability coefficient for the questionnaire using Statistical Package for the Social Sciences (SPSS) as shown in the computer print out.

Reliability

/variables=v1 v2 v3 v4 v5 v6 v7 v8 v9 v10 v11 v12 v13 v14 v15 v16 v17 v18 v19 v20 v21 v22 v23 v24 v25 v26 v27 v28 v29 v30 v31 v32 v33 v34 v35 v36 v37 v38 v39 v40
 /scale('all variables') all
 /model=alpha
 /statistics=scale.

Reliability

Notes		
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Comments		
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	Split File	<none>
	N of Rows in Working Data File	87
	Matrix Input	
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on all cases with valid data for all variables in the procedure.
Syntax	RELIABILITY /VARIABLES=V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11 V12 V13 V14 V15 V16 V17 V18 V19 V20 V21 V22 V23 V24 V25 V26 V27 V28 V29 V30 V31 V32 V33 V34 V35 V36 V37 V38 V39 V40 /SCALE('ALL VARIABLES') ALL /MODEL=ALPHA /STATISTICS=SCALE.	
Resources	Processor Time	00:00:00.016
	Elapsed Time	00:00:00.014

[DataSet1] C:\Users\AKAAZUA\Documents\Tertseareliabilitydata.sav

Scale: All Variables

Case Processing Summary			
		N	%
Cases	Valid	87	100
	Excluded ^a	0	0
	Total	87	100.0

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics	
Cronbach's Alpha	N of Items
.935	40

Scale Statistics			
Mean	Variance	Std. Deviation	N of Items
95.4000	423.743	20.58502	40

Appendix g

Letters Letter of request for validation of a research instrument

Validation Of A Research Instrument

I am developing a research instrument to generate data for my PhD Mathematics Education Thesis. The instrument for data collection is a structured questionnaire for student's which is titled: Students' Geometry Attitude Questionnaire (SGAQ) which is divided into three sections A, B and C with a total of thirty 20 structured items. Section A is designed to provide background information about respondents and section B is a four point rating scale of Strongly Agree (SA) Agree (A) Disagree (D) and Strongly Disagree (SD) that seeks information required to answering the research questions and testing of the formulated hypotheses. Section C requires respondents to provide brief suggestions on ways to improve the handling of geometry at Junior Secondary School two. The drafts of the instruments are attached.

As an experienced Expert, could you please examine and assess all the items with regards to the following:

- (i) Whether the items conform with the subject matter they are supposed to measure or not
- (ii) Whether the items are clear, readable and free from ambiguity for the level of students' they are designed to answer or not
- (iii) Whether the items are relevant to the study objectives, content and if the items will answer the research questions appropriately and tests the formulated hypothesis.
- (iv) Whether the items satisfy conditions of constructing questionnaire items or not and
- (v) Any general criticisms and suggestions that could be made for the improvement of the instrument.
- (vi) Whether or not the items tackle basic ideas on attitude expected at secondary school geometry;
- (vii) Whether or not the ideas tackled by the instrument are relevant to and pre-requisite to solving attitude

Validation Of A Research Instrument

I am developing a research instrument to generate data for my PhD Mathematics Education thesis. The instrument for data collection is a test for student's which is titled: Geometry Performance Test (GPT). A total of fifty five multiple choice test-items with four options, one correct option and three distracters, were developed by the researcher. The test items constructed by the researcher covered all the units to be taught by the researcher on Geometry. The test items were also constructed to test the subjects' retention and performance in geometry. The development of the items was also based also on the instructional objectives contained in JSSII Mathematics Curriculum (FRN, 2007).

The points of reference to be used in assessing the test are to check whether:

1. The test items conform to the table of specification.
2. The test items are clearly stated and appropriate in terms of the reading difficulty level of the students' understanding.
3. The test items are appropriate to reveal cognitive, affective and psychomotor conflicts in the students.
4. To confirm the item difficulty, item discrimination and item distraction indices of the test items.
5. The test items include ambiguous statements that can confuse or be misinterpreted by the examinees.
6. The test items are arranged in order of difficulty from simple to complex.
7. The test does not have identifiable pattern of answers at the time of testing.

**Letter Of Request For Permission To Meet Students
Request For Permission To Meet With Your Students.**

The bearer of this letter is Mr. Jacob T. Akaazua, a PhD Student in Science Education Department. I will be most grateful if you could permit him to meet some of your students at a convenient time. This is for him to be able to administer his research instruments to JSSII students in your school.

Mr. Akaazua will personally discuss further details of his work with you.

Thanking you for your cooperation.

Yours faithfully,

Head of Department

**APPENDIX H
Lesson Plans for the Experimental Group**

Lesson One

Subject: Mathematics
Topic: Properties of Polygons
Class: JSS II
Date: 12th. September, 2014
No. of Students: 61 and 42 students respectively
Age: Average of 13 years old

Teaching/Learning Materials:

- 5 x 5 geoboards (1 per student)
- Geobands (elastic bands) (1 per student)
- Chart paper
- Index cards (1 per student)
- Large Vertical Mirror
- Coloured cardboard papers and scissors

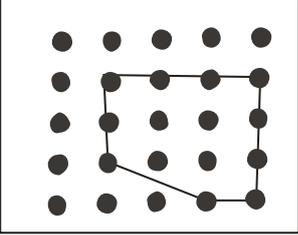
Specific Objectives

At the end of the lesson, the students should be able to cut and compare various polygons (i.e. triangles, quadrilaterals, pentagons, hexagons, heptagons, octagons) using coloured cardboard papers and sort them by their geometric properties (i.e. number of sides, number of interior angles number of right angles).

Previous Knowledge

Students have already known how to recognize and identify two-dimensional shapes by their appearance as a whole.

Content Development (CD)	Teachers Performance Activity	Students Performance Activity
CD Step I	Teacher lead the students to cut different coloured card board papers in triangular,	Students cut different coloured card board papers in triangular, quadrilateral,

Introduction	quadrilateral, pentagonal, hexagonal, heptagonal and octagonal shapes and sort them by their geometric properties (i.e. number of sides, number of interior angles and number of right angles). Teacher again leads the student in manipulating a 5 x 5 geoboard using geobands.	pentagonal, hexagonal, heptagonal and octagonal shapes and sort them by their geometric properties (i.e. number of sides, number of interior angles and number of right angles). Students manipulate a 5 x 5 geoboard using geobands
<p align="center">CD Step II Instructional Procedure</p>	<ul style="list-style-type: none"> • Teacher instructs students to form plane shapes that has from three to eight straight sides on their geoboard using their geobands. • Teacher instructs students to identify their shapes. • Teachers ask students the following questions; <ul style="list-style-type: none"> - Which family does your shape belong to? - What do all your family members have in common? - What are some differences among the family members? • Teacher show the following geoboard to all the students and ask them to describe it and record their findings on chart paper. <div style="text-align: center;">  </div>	<ul style="list-style-type: none"> • Students form plane shapes that have from three to eight straight sides on their geoboard using their geobands. • Students found their family to be the triangles, the quadrilaterals, the pentagon's, the hexagons or the octagons • Students answer that the similarities and differences in terms of the number of sides, side length, number of interior angles, number of right angles and lines of symmetries. <p>Students record on their chart paper;</p> <ul style="list-style-type: none"> • The shape is a pentagon. • The shape has five vertices. • Three angles are right angles. • There are four pins in the middle of the shape. • Two-sides are the same length. • Students describe their geoboard shape based on their recorded findings.
<p align="center">CD Step III Evaluation</p>	Teacher leads the students to stand upright before a large vertical mirror in which they can see all of their selves and record their symmetry properties.	The students stand upright before a large vertical mirror in which they can see all of their selves. Every part of their front appears in twos from head to toe. The two are balanced with each other about a line. The line runs between their two eye-brows, between their two eyes, down the middle of their nose, their lips, chest and between their legs. Because of this balance about a middle line, human being is said to have <u>mirror symmetry</u> or to be <u>symmetric</u> .
<p align="center">CD Step IV Conclusion</p>	The Teacher leads the students to draw appropriate conclusions.	<p>The students draw the conclusion that:</p> <ol style="list-style-type: none"> 1. A shape has mirror symmetry or just symmetry when it can be divided into 2 halves by a line. The halves must be identical but in reversed order. The dividing line is called the mirror line or line of symmetry. 2. A circle has an infinite number of lines of symmetry. 3. A rectangle has two lines of symmetry 4. An isosceles triangle has one line of symmetry. 5. All corners of a rectangle are identical and are each equal to a right angle. 6. Each pair of opposite sides of a rectangle are parallel and equal. 7. The diagonals of a rectangle are equal and they bisect each other at their point of intersection. 8. Two side of an isosceles triangle are equal. 9. The base angles of an isosceles

		triangle are identical. 10. The line of symmetry of an isosceles triangle bisects the base line at right angles.
CD Step V Assignments	The teacher assigns the following question to students to do at home. 1. What are the properties common to both a square and a rectangle?	Answer to Assignment 1. All the properties of a rectangle.

Lesson Two

Subject: Mathematics
Topic: Perimeter of a square and a rectangle.
Class: JSS II
Date: 19th. September, 2014
No. of Students: 61 and 42 students respectively
Age: Average of 13 years old

Teaching/Learning Materials

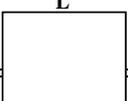
- A ruler
- A string
- A pencil
- Cover of your exercise book
- An envelope
- A model of a house
- One of the windows of the classroom
- Graph sheet of paper (1 sheet per student)

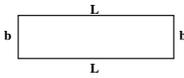
Specific Objectives

At the end of the lesson, the students should be able to derive the formulae to calculate the perimeter of a square and a rectangle.

Previous Knowledge

Students have already known how to wound round the curves of an irregular shape using a string to measure its perimeter.

Content Development (CD)	Teachers Performance Activity	Student Performance Activity
CD Step I Introduction	Teacher leads the students to cut different coloured card board papers in triangular and rectangular shapes and define the perimeter of a square and a rectangle	Students cut different coloured card board papers in triangular and rectangular shapes and then defined the perimeter of a square as a figure whose length and breadth are equal and the perimeter of rectangle to be a four sided figure in which the opposite sides are equal.
CD Step II Instructional Procedure	The Teacher guides the students to derive the formula of the perimeter of a squares and rectangles.	The students trace five different rectangles on a square (graph) sheet of paper using the ruler and the pencil and then add the number of squares on the side to find the distance round the shape and record it. The value of the perimeter of a rectangle of length 4cm and breadth 4cm is obtained as 16cm. The perimeter contains units' square cm: 4 rows of 4 units each. We say that the perimeter is $4\text{cm} + 4\text{cm} + 4\text{cm} + 4\text{cm} = 16\text{cm}$. The students discovered that the formula for finding the perimeter of a rectangle is known when the length and breadth are known. This is also used to find the perimeter of a square: Perimeter = Length + Breadth + Length x Breadth A Square: 

		<p>Perimeter of a square = (Length + Breadth + Length x Breadth)</p> $P = (L + B + L + B)$ $P = 2(L + B)$ <p>Or $L = \frac{P - B}{2}$</p> <p>i.e Length = $\frac{P - B}{2}$</p> <p>A Rectangle</p>  <p>Perimeter of a rectangle = (Length + Breadth + Length x Breadth)</p> $P = (L + B + L + B)$ $P = 2(L + B)$ <p>Or $L = \frac{P - B}{2}$</p> <p>i.e Length = $\frac{P - B}{2}$</p> $\text{Breadth} = \frac{P - L}{2}$
CD Step III Evaluation	<p>Teacher leads the students in applying the discovered formulas for the perimeter of a square and a rectangle:</p> <ol style="list-style-type: none"> Calculate the perimeter of a square advertising board of length 5m Calculate the perimeter of a rectangle 6cm by 3.5cm Measure the length and breadth of <ol style="list-style-type: none"> This page Cover of your exercise book An envelope One of the windows of the classroom 	<p>The students evaluate the following:</p> <p>Perimeter = $L + B + L + B$</p> $P = 5m + 5m + 5m + 5m$ $P = 2(5m + 5m)$ $P = 20m$ <p>Perimeter = $6cm + 3.5cm + 6cm + 3.5cm$</p> $P = 2(6cm + 3.5cm)$ $A = 19cm.$
CD Step IV Conclusion	The Teacher leads the students to draw appropriate conclusions.	The students draw the conclusion that the perimeter of a shape is the length of its boundaries, that is, the distance around its edges.
CD Step V Assignments	<ol style="list-style-type: none"> The perimeter of a square plot is $144m^2$. Calculate the length of a side of the plot. 	<p>Answers to Assignments</p> <p>Length = 36m</p>

Lesson Three

Subject: Mathematics

Topic: Area of a square and a rectangle

Class: JSSII

Date: 26th. September, 2014

No. of Students: 61 and 42 students respectively

Age: Average of 13 years old

Instructional Materials:

- A ruler
- A pencil
- A square (graph) sheet of paper
- Models of a house made of plane shapes

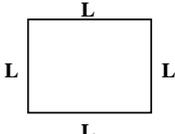
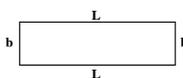
Specific Objectives

At the end of the lesson, the students should be able to:

- Derive the appropriate formula for the area of a given square and a rectangle
- Apply appropriate formula to calculate the area of a square and a rectangle.

Previous knowledge:

The students can calculate the perimeter of plane shapes.

Content Development (CD)	Teachers Performance Activity	Student Performance Activity
CD Step I Introduction	Teacher leads the students to cut different card board papers in squares and rectangles and define the area of a square and a rectangle.	Students cut different card board papers in squares and rectangles and define the area of a square and a rectangle as a measure of its surface, that is, the amount of surface enclosed within its boundary.
CD Step II Instructional Procedure	The Teacher guides the students to derive the formula of a squares and rectangles.	<p>The students trace five different rectangles on a square (graph) sheet of paper using the ruler and the pencil and then count the number of squares enclosed by each to find their area and record it.</p> <p>The value of the area of a rectangle of length 4cm and breadth 4cm is obtained as 16cm^2.</p> <p>The area contains units' square cm: 4 rows of 4 units each. Students discovered that the area is $4\text{cm} \times 4\text{cm} = 16\text{cm}^2$. This gives a simple formula for finding the area of a rectangle when the length and breadth are known.</p> <p>This is also used to find the area of a square: Area = Length x Breadth A Square:</p> <div style="text-align: center;">  </div> <p>Area of a square = (length of side)²</p> $A = L \times L$ $A = L^2$ <p>Or $L = \sqrt{A}$ i.e Length = \sqrt{A}</p> <p>A Rectangle:</p> <div style="text-align: center;">  </div> <p>Area of a rectangle = Length x Breadth</p> $\text{Length} = \frac{\text{Area}}{\text{breadt } h}$ $\text{Breadth} = \frac{\text{Area}}{\text{leangt } hh}$
CD Step III Evaluation	Teacher leads the students in applying the discovered formulas for the area of a square and a rectangle: 4. Calculate the area of a square advertizing board of length 5m 5. Calculate the area of of a rectangle 6cm by 3.5cm 6. The area of a rectangle is 24cm^2 and one side is 6cm in length. Find its breadth	The students evaluates the following: 1. Area = $L \times L$ A = $5\text{m} \times 5\text{m}$ A = $5 \times 5\text{m} \times \text{m}$ A = 25m^2 2. Area = $6\text{cm} \times 3.5\text{cm}$ A = $6 \times 3.5\text{cm} \times \text{cm}$ A = 21cm^2 . 3. Breadth = $\frac{\text{Area}}{\text{leangt } hh}$

		$= \frac{24cm^2}{6cm}$ $= 4cm$
CD Step IV Conclusion	The Teacher leads the students to draw appropriate conclusions.	The students draw the conclusion that the perimeter of a shape is the length of its boundaries, that is, the distance around its edges while the area is the amount of surface enclosed within its boundary.
CD Step V Assignments	2. The area of a square plot is $144m^2$. Calculate the length of a side of the plot.	Answers to Assignments Length = 12m

Lesson Four

Subject: Mathematics
Topic: Area of a Parallelogram, Trapezium and a Triangle
Class: JSSII
Date: 3rd. October, 2014
No. of Students: 61 and 42 students respectively
Age: Average of 13 years old

Instructional Materials

- A ruler
- A pencil
- Scissors
- Pattern blocks of Triangles and Squares
- Models of a house

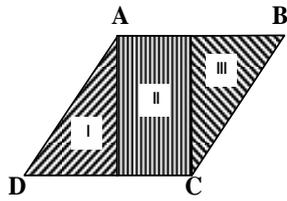
Specific Objectives

At the end of the lesson, the students should be able to:

- Derive the appropriate formula of a given parallelogram, trapezium and triangles.
- Apply appropriate formula to calculate the area of a parallelogram, trapezium and triangles.

Previous knowledge:

The students can calculate the perimeter of plane shapes.

Content Development (CD)	Teachers Achievement Activity	Student Achievement Activity
CD Step I Introduction	Teacher leads the students to cut different card board papers in parallelograms, trapeziums and triangular shapes and define the area of parallelogram, trapezium and triangles.	Students cut different card board papers in parallelograms, trapeziums and triangular shapes and then defined the parallelogram as a closed four-sided plane shape that has its opposite sides parallel and equal. The diagonals of a parallelogram bisect each other. The trapezium as a quadrilateral with two opposite parallel sides. The perpendicular distance between the parallel sides is the height. While the students define a triangle to be a 3-sided closed plane shape. The students therefore defined area of a parallelogram, a trapezium and a triangle as a measure of its surface. That is, the amount of surface enclosed within its boundary.
CD Step II Instructional Procedure	The Teacher guides the students to derive the formula of a parallelogram, trapezium and triangles.	Area of the Parallelogram. With the pencil and ruler the students draw parallelogram ABCD on the pattern blocks as shown in Figure A 

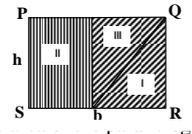
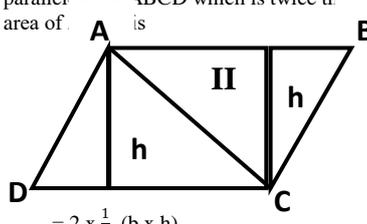
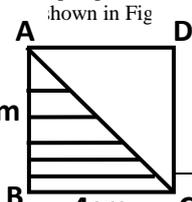
		<p>Fig. A. The students divide the parallelogram into three parts as shown in Figure A. The students using the pair of scissors cut out the triangle labeled I and re-arrange the three parts by fixing I at the side of III to form the rectangle as shown in Figure B and label the rectangle as PQRS.</p>  <p>remains the same as the area of rectangle PQRS, h is the height of both the rectangle and the parallelogram, b is the base of the rectangle PQRS which is also the base of parallelogram ABCD. $DC=AB=PQ=SR=b$. Hence for any parallelogram with base b and height h, the students discovered that the formula for the area of a parallelogram is $b \times h$. The students deduce also that this formula can also be derived if the area of parallelogram ABCD is considered as the sum of two equal triangles ABC and ADC as shown in Figure C.</p> <p>Fig. C. The area of $\triangle ADC$ with base b and height h is $\frac{1}{2} (b \times h)$. Hence the area of the parallelogram ABCD which is twice the area of $\triangle ADC$ is</p>  $= 2 \times \frac{1}{2} (b \times h)$ $= b \times h.$ <p>The Area of a Trapezium Similarly, the area of the trapezium is the sum of the areas of the two triangles ADC and ABC. The area of $\triangle ADC$ is $\frac{1}{2} (b \times h)$. Also the area of $\triangle ABC$ is $\frac{1}{2} (b \times h)$. Hence, the area of the trapezium is $= \frac{1}{2} ah + \frac{1}{2} bh$ $= \frac{1}{2} h (a + b).$ This can be expressed as Area = $\frac{1}{2}$ (sum of parallel sides) x perpendicular distance between the parallel sides.</p> <p>The Area of a Triangle In the case of the triangle, the students using the scissors bisect the rectangle ABCD into equal parts along its AC diagonal as shown in Fig</p> 
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		Fig. D. The area of a triangle can be expressed as $= \frac{1}{2} \times \text{base} \times \text{height}$
CD Step III Evaluation	Teacher leads the students in applying the discovered formulas for the area of a parallelogram, trapezium and a triangle 1. Calculate the area of a parallelogram of height 4cm and base 3cm 2. Calculate the area of a trapezium whose parallel sides a and b are 6cm and 8cm and the height h is 4cm. 3. Calculate the area of a triangle whose base is 4cm and height is 6cm.	The students evaluates the following: 1. Area = b x h A = 3cm x 4cm A = 12cm ² 2. Area = $\frac{1}{2} h (a + b)$. A = $\frac{1}{2} (8 + 6)4\text{cm}^2$ A = 28cm ² . 3. Area = $\frac{1}{2} \times b \times h$ = $\frac{1}{2} (4\text{cm} \times 6\text{cm})$ = 12cm ²
CD Step IV Conclusion	The Teacher leads the students to draw appropriate conclusions.	The students draw the conclusion that the area of parallelogram, trapezium and a triangle is the amount of surface enclosed within its boundary.
CD Step V Assignments	1. Calculate the base of a parallelogram whose area is 60cm ² and height is 4cm. 2. Calculate the area of a trapezium whose parallel sides are 6cm and 14cm and the height h is 5cm. 3. Calculate the area of a triangle whose base is 2cm and height is 6cm.	Answers to Assignments 1. 15cm. 2. 50cm ² 3. 6cm ²

Lesson Five

Subject: Mathematics
Topic: The circumference of a circle.
Class: JSS II
Date: 10th. October, 2014
Duration: 40 minutes
No. of Students: 61 and 42 students respectively

Age: Average of 13 years old

Teaching /Learning Materials:

- String
- Peak milk tin (varying size)
- Ruler
- Scissors

Specific Objectives:

At the end of the lesson, students should be able to derive the formula for the circumference (perimeter) of the circle of any size.

Previous Knowledge:

The students have already known how to measure.

Content Development (CD)	Teachers Performance Activity	Student Performance Activity
CD Step I Introduction	Teacher leads students to identify circles using various peak milk tins, models of solid shapes and the school buildings and define a circle, a radius and a diameter.	Students identify circles from the various peak milk tins, the solid shapes and school buildings and discover that: 1. In rectangles, the lines have definite length or breadth but in circles no length or breadth. 2. A circle is a space enclosed by a curve having all points equidistant from a fixed point called the centre. A circle is continuous unlike a rectangle.

		<p>3. The radius is the straight line distance between the centre and the point on the circumference.</p> <p>4. The diameter is the straight line from one point on the circumference to the other through the centre. It is twice the radius.</p>
<p>CD Step II Instructional Procedure</p>	<p>Teacher acting as a facilitator leads the students to derive the formula for calculating the perimeter of a circle more commonly called the circumference of a circle.</p>	<p>Students with the use of a string, measure and compare the circumference of each of the circles in the various peak milk tins and record how many times the length of the diameter of a circle will go into the distance round the circle edge of peak milk tin. That is $\frac{\text{Circumference}}{\text{Diameter}}$ or $\frac{C}{D}$.</p> <p>Again the students draw a circle of radius 7cm in their exercise books measure the circumference of the circle and find the value of $\frac{C}{D}$. Students observed that this gives approximately 3 times plus about $\frac{1}{7}$ of the length of the diameter. The constant value $\frac{C}{D}$ denoted by π is generally used as $\frac{22}{7}$ or 3.14</p> <p>Thus: $\frac{C}{D} = \pi$ $C = \pi D$ or $C = 2 \pi R$ or</p> <p>Circumference = $\frac{1}{37}$ of diameter So circumference is always equal to $\pi \times \text{diameter}$ $C = \pi d$ and written as $C = \pi \times 2 \times \text{radius}$ Or $C = \pi \times 2 r$ $C = 2 \pi r$</p>
<p>CD Step III Evaluation</p>	<p>Teacher leads the students in applying the discovered formulas for the circumference of a circle:</p> <p>1. Calculate the circumference of a circle whose radius is 7cm. Take $\pi = 3.14$</p>	<p>The students evaluates the circumference of the circle as following:</p> <p>1. $C = 2 \pi r$ $C = 2 \times 3.14 \times 7\text{cm}$. $C = 43.96\text{cm}$</p>
<p>CD Step IV Conclusion</p>	<p>The Teacher leads the students to draw appropriate conclusions.</p>	<p>The students draw the conclusion that the circumference or perimeter of a circle is the length of its boundary, that is, the distance around its edges.</p>
<p>CD Step V Assignments</p>	<p>What is the circumference of an object whose diameter is $\frac{1}{6}$ cm?</p>	<p>Answers to Assignments is: Circumference = 0.5238cm</p>

Lesson Six

Subject: Mathematics

Topic: Area of a circle

Class: JSS II

Date: 17th. October, 2014

No. of Students: 61 and 42 students respectively

Age: Average of 13 years old

Teaching /Learning Materials:

- Cardboard paper
- Scissors
- Compass
- Ruler
- Pencil
- various peak milk tins

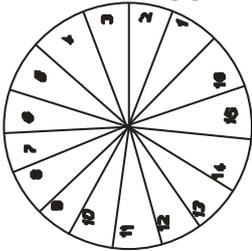
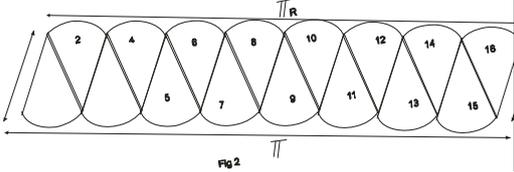
- models of solid shapes

Specific Objectives:

At the end of the lesson, students should be able to calculate the area of any circle

Previous Knowledge:

The students have already known the formula for finding the area of a parallelogram, the properties of a circle and the formula for the circumference of a circle.

Content Development (CD)	Teachers Performance Activity	Student Performance Activity
CD Step I Introduction	Teacher leads the students to identify circles using the models of solid shapes and define the area of a circle.	Students identify the circles from the models of solid shapes and then defined the area of a circle as a measure of its surface, that is, the amount of surface enclosed within its boundary.
CD Step II Instructional Procedure	The Teacher guides the students to derive the formula for finding the area of a circle.	<p>Students draw a circle of 7cm radius using the compass and a pencil on a cardboard paper.</p>  <p>From the centre of the circle, the students cut the entire circle into sixteen equal sectors. These sectors are rearranged side-by-side as in Fig. 2.</p>  <p>The resulting figure is approximately a parallelogram. Now the students measure the base and the altitude of Fig. 2. Students can then find the area of the parallelogram as 22×7. The altitude is approximately equal to the radius r \Rightarrow The base is $\frac{1}{2}$ the circumference the students may then write Area of circle $= r \times \frac{1}{2}C$ $= \frac{1}{2}rc$ But circumference $= 2\pi r$ \therefore Area of circle $= r \times \frac{1}{2}(2\pi r)$ $= \pi r^2$</p>
CD Step III Evaluation	Teacher leads the students in applying the discovered formulas for the area of a circle: 1. Find the area of a circle of radius 7cm. 2. Find the radius of a circle whose area is $38\frac{1}{2}\text{cm}^2$.	The students' evaluate the area of the circle as follows: 1. Area $= \pi r^2$ $A = 3.14 \times 7 \times 7$ $A = 154\text{cm}^2$ 2. Area $= 38\frac{1}{2}\text{cm}^2$ $\pi r^2 = 38\frac{1}{2}\text{cm}^2$ $r^2 = 3.14 \times 3.14 = 9.86$ $r = \sqrt{9.86\text{cm}}$ $r = 3.5\text{cm}$
CD	The Teacher leads the students to	The students draw the conclusion that the area of a circle

Step IV Conclusion	draw appropriate conclusions.	is the amount of surface enclosed within its boundary.
CD Step V Assignments	The area of a semi circular disc whose diameter is 28cm.	Answers to Assignments is 308cm^2 .

Lesson seven

Subject: Mathematics
Topic: Angle Properties of Parallel Lines.
Class: JSS II
Date: 24th. October, 2014
No. of Students: 61 and 42 students respectively
Age: Average of 13 years old

Teaching /Learning Materials:

- Ruler
- Pencil
- Paper
- protractors

Specific Objectives:

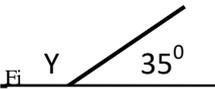
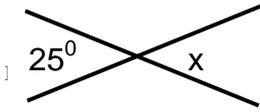
At the end of the lesson, students should be able to identify and apply

- i. alternate and corresponding angles and
- ii. adjacent angles on a straight line.

Previous Knowledge:

Students can draw given angles and measure using protractor.

Content Development (CD)	Teachers Performance Activity	Student Performance Activity
CD Step I Introduction	Teacher leads the students to identify alternate, corresponding and adjacent angles on a straight line using the models of solid shapes and define the angles.	Students identify the alternate, corresponding and adjacent angles from the models of solid shapes then defined and differentiates between the angles as: <ol style="list-style-type: none"> 1. A pair of angles c and d is called alternate angles if they are equal. 2. A pair of angles a and e are called corresponding angles if they are equal 3. A pair of angles a and d is called interior opposite angles. 4. A pair of angles b and c are also called interior opposite angles . 5. A pair of angles a + b are called adjacent angles if their sum equals 180°
CD Step II Instructional Procedure	The Teacher guides the students to elicit the idea of alternate, corresponding and adjacent angles on a straight line.	The students draw on paper several different pairs of alternate, corresponding and adjacent angles on a straight line as shown in figure J and measure them with a protractor. <div style="text-align: center;"> </div> <p>Fig. J. Students observe that the angles marked a and b ; c and d are alternate angles. Similarly, the students draw on paper several different pairs of corresponding</p>

		<p>angles as shown in figure J. Students observed that angles with the following pairs of marked angles are called corresponding angles a and e, b and f, c and g, d and h The pair of angles marked a and d, b and a are called interior opposite angles. Again, the students draw on paper a pair of adjacent angles as shown in figure J (ii).</p>  <p>Fig J (ii) Students observed that angles with the following pairs of marked angles are called adjacent angles $a + b = 180^\circ$</p>
<p>CD Step III Evaluation</p>	<p>Teacher leads the students in applying the discovered ideas to find the value of the unknown angles in each of the diagram in Fig L (i) and L (ii).</p>  	<p>The students evaluates that a. $X=25^\circ$ (vertically opposite angles are equal) b. $Y+35^\circ = 180^\circ$ (sum of adjacent angles) Therefore, $Y = 180^\circ - 35^\circ$ $Y = 145^\circ$</p>
<p>CD Step IV Conclusion</p>	<p>The Teacher leads the students to draw appropriate conclusions.</p>	<p>The students draw the conclusion that the following are true if two parallel lines are cut by a transversal:</p> <ol style="list-style-type: none"> i. The alternate angles are equal. ii. The corresponding angles are equal. iii. The interior angles on the same side of the transversal are supplementary. <p>The students also observed that there were pairs of alternate and corresponding angles from lines that are not parallel and the pairs of interior angles too were not supplementary.</p>

CD Step V Assignments	1. Find the value of the lettered angles in the diagram.	Answers to Assignments A = 58° --- Corrs s B = 58° --- Corrs s C = 72° --- Alt. s D = $180^\circ - 72^\circ = 108^\circ$ --- Adj. s on a str.

Lesson Eight

Subject: Mathematics
Topic: Properties of an angle at the point of intersecting straight lines
Class: JSS II
Date: 31st. October, 2014
No. of Students: 61 and 42 students respectively
Age: Average of 13 years old

Teaching /Learning Materials:

- Ruler
- Pencil
- Paper
- Protractors
- models of solid shapes

Specific Objectives:

At the end of the lesson, students should be able to identify, define and apply vertically opposite angles, adjacent, complementary and supplementary angles to solve problems.

Previous Knowledge:

Students can draw given angles and measure using protractor.

Content Development (CD)	Teachers Performance Activity	Student Performance Activity
CD Step I Introduction	Teacher leads the students to identify vertically opposite and adjacent angles and differentiates between them. Also complementary and supplementary angles are identified using the models of solid shapes and the school buildings and define the angles.	Students identify the vertically opposite and adjacent angles from the models of solid shapes and the school buildings then defined and differentiates between the angles that: A pair of angles x and y are called vertically opposite angles if they are equal
CD Step II Instructional Procedure	The Teacher guides the students to elicit the idea of vertically opposite angle, complementary and supplementary angles.	The students draw on paper several different pairs of vertically opposite angles as shown in figure G and measure them with a protractor.

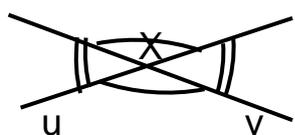
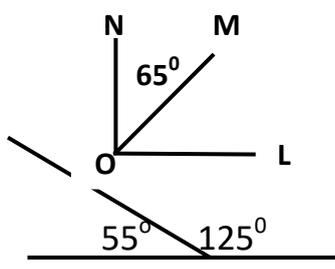
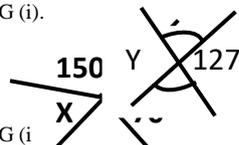
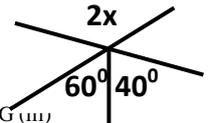
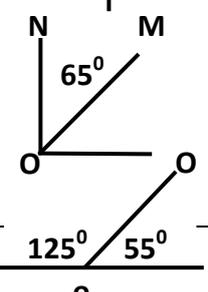
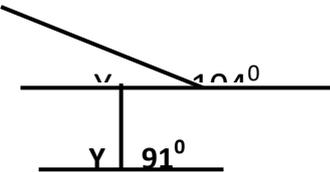
		 <p>Fig. G. Students observe that the angle marked x = y and u = v are opposite to each other. Similarly, the students draw on paper several different pairs of complementary and supplementary angles as shown in figure H and measure them with a protractor.</p>  <p>Fig. H. Students observed that angle 65° complementary to 25° 125° is supplementary to 55°.</p>
<p>CD Step III Evaluation</p>	<p>Teacher leads the students in applying the discovered ideas to find the value of the lettered angles in the diagram in fig G (i), G (ii) and G (iii) and Fig H (i), H (ii) and H (iii).</p>  <p>Fig. G (i)</p>  <p>Fig. G (ii)</p>  <p>Fig.</p>	<p>The students evaluates that</p> <ul style="list-style-type: none"> c. $X=40^\circ$ (vertically opposite angles are equal) d. $Y=127^\circ$ (vertically opposite angles are equal) e. $70^\circ + y = 150^\circ$ (vertically opposite angles are equal), $y=80^\circ$ f. $2x = 60^\circ + 40^\circ$ (vertically opposite sides are equal) $2x=100^\circ$, $x=50^\circ$

	Fig. H (ii). Fig. H (iii)	
CD Step IV Conclusion	The Teacher leads the students to draw appropriate conclusions.	The students draw the conclusion that: 1. Vertically opposite angles are equal. 2. Two angles are complementary if their sum is 90° 3. Two angles are supplementary if their sum is 180°
CD Step V Assignments	Find the value of y in each of the following angles. a.  b. 	Answers to Assignments a. $Y=76^\circ$ b. 97° c. 129°

Lesson Nine

Subject: Mathematics
Topic: Properties of Angles at Point.
Class: JSS II
Date: 7th. November, 2014
No. of Students: 61 and 42 students respectively
Age: Average of 13 years old

Teaching /Learning Materials:

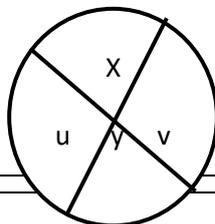
- Ruler
- Pencil
- Paper
- protractors

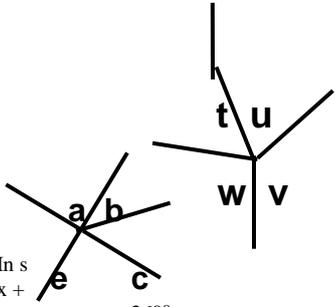
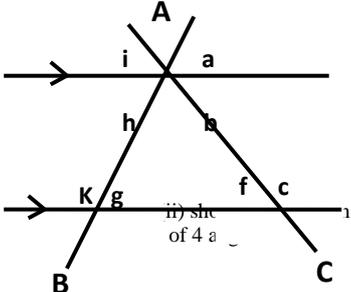
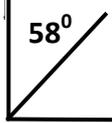
Specific Objectives:

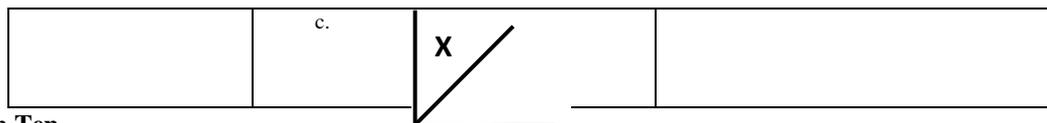
At the end of the lesson, students should be able to solve problems using the fact that angles at a point add up to 360° .

Previous Knowledge:

Students can draw given angles and measure using protractor.

Content Development (CD)	Teachers Performance Activity	Student Performance Activity
CD Step I Introduction	Teacher leads the students to identify angles at a point using the models of solid shapes and the school buildings.	Students identify angles at a point from the models of solid shapes and the school buildings then defined the angles that the sum of angles at a point is 360° .
CD Step II Instructional Procedure	The Teacher guides the students to elicit the idea of angles at a point	The students draw on paper several different sides of angles in a circle using a pair of compasses, a ruler and a protractor as shown in figure M and measure them with a protractor. 

		<p>Fig. M.</p> <p>Students observe that all the angles marked u, v, x and y are called angles at a point.</p> <p>Since u and x are adjacent angles on a straight line then $u + x = 180^\circ$</p> <p>Therefore, $u + v + x + y = 360^\circ$.</p> <p>This is also possible for the number of angles at point to be 3 or 4 or 5, and so on as in Fig. M(i)</p>  <p>In s $x +$ $t + u + v + w = 360^\circ$ $a + b + c + d + e = 360^\circ$.</p>
<p>CD Step III Evaluation</p>	<p>Teacher leads the students in applying the discovered ideas of angles at a point thus:</p> <ol style="list-style-type: none"> State which pairs of the marked angles in Fig. N (i) are adjacent angles on a straight line.  <p>Find the value of x and find the sizes of each of the angles.</p>	<p>Fig. N (i).</p> <p>The students observed that the pairs of adjacent angles on a straight line in Fig N (i) are:</p> <p>c and d, c and f, d and e, e and f.</p> <p>The students observed also that by adding; $3x + 3x + 2x + x = 360^\circ$ (sum of angles at a point) $9x = 360^\circ$ By dividing both sides by 9; $X = 40^\circ$ Hence, $3x = 3 \times 40^\circ = 120^\circ$ $2x = 2 \times 40^\circ = 80^\circ$ $X = 40^\circ$ Therefore the sizes of the angles so represented are $120^\circ, 120^\circ, 80^\circ$ and 40° respectively.</p>
<p>CD Step IV Conclusion</p>	<p>The Teacher leads the students to draw appropriate conclusions.</p>	<p>The students draw the conclusion that: The sum of angles at a point is 360°.</p>
<p>CD Step V Assignments</p>	<p>Find the value of x in the diagram below.</p> <p>a.</p>  <p>b.</p> 	<p>Answers to Assignments</p> <p>a. 57° b. 22° c. 72°</p>



Lesson Ten

Subject: Mathematics
Topic: Construction of Triangles
Class: JSS II
Date: 14th. November, 2014
No. of Students: 61 and 42 students respectively
Age: Average of 13 years old

Teaching materials:

- Ruler
- Pair of compass
- Pencil
- Eraser/Cleaner
- Protractor
- Sharpener

Specific objectives:

At the end of the lesson, the students should be able to construct any triangle.

Previous knowledge:

The students have already known how to mark and sketch the geometrical figure to be constructed with their free hand.

Content Development (CD)	Teachers Performance Activity	Student Performance Activity
CD Step I Introduction	Teacher leads the students to identify the various geometric construction instruments used in constructing triangles. The teacher demonstrates how to use the geometric construction instruments and lead the students to describe their uses.	Students identify the geometric construction instruments then describe their uses as: (a) A ruler should be used to draw and measure line segment. (b) A pair of divider is used for measuring the length of line segment. (c) A pair of compasses is used to draw a circle. (d) A protractor is used to measure an angle. (e) Set Square is used to draw triangles, line segments and parallel lines. (f) A pencil is used in drawing straight lines and curves. (g) An eraser is used to clean up areas not needed.
CD Step II Instructional Procedure	The Teacher guides the students to construct triangles when: i. 3 sides are given ii. 2 sides and the included angle are given iii. 2 angles and one side are given.	Given Length of the 3-sides The students draw a horizontal line of length 8 cm using a ruler and a pair of compasses. With one end of the line as centre, students use a pair of compasses of radius 7cm to draw an arc of a circle. Similarly, with the other end of the line as centre, again students

		<p>use a pair of compasses of radius 7cm to draw another arc of a circle to meet the former arc. Then join the point of intersection of the two arcs to each of the two ends of the horizontal line. The students then obtain a triangle (given its three sides).</p> <p><u>Given the Length of 2-sides and the included angle</u></p> <p>The students draw a horizontal line of length 6cm using a ruler and pair of compasses. At one end of the line, draw angle 55° using a protractor. On the new line, cut off a segment equal to 5cm long using a pair of compasses. The students then join the third side and obtain a triangle (given two sides and the included angle).</p> <p><u>Given Two Angles and One Side</u></p> <p>The students draw a horizontal line of length 7cm using a ruler and a pair of compasses. At one end of the line, draw angle 40° using a protractor. At the other end of the line, draw angle 50° again using a protractor. The students then obtain a triangle (given a side and its angles).</p>
<p>CD Step III Evaluation</p>	<p>Teacher leads the students in constructing the following triangles:</p> <ol style="list-style-type: none"> 1. Construct a triangle having sides 3cm, 4cm and 5cm long. 2. Draw a triangle ABC where $AB = 2.6\text{cm}$, $AC = 2.1\text{cm}$ and $\angle BAC = 115^\circ$. Measure $\angle BC$, $\angle ACB$ and $\angle BAC$. 3. Construct a triangle which has angles 40° and 55° at the end of a side 3.9 cm long. Measure the other sides. 	<p>The students evaluates as: The angles are 90°, 53° and 37°.</p>
<p>CD Step IV</p>	<p>The Teacher leads the students to conclude appropriately.</p>	<p>The students draw the conclusion that in construction, the following</p>

Conclusion		points are important. i. Use a hard pencil with a sharp point to enable lines and points to be as fine as possible. ii. In order to draw accurately, great care is needed in drawing a line through a point. iii. All construction lines, arcs leading to the final result must be left visible. They should not be cleaned.
CD Step V Assignments	1. Construct triangle XYZ with $XY = 4\text{cm}$, $YZ = 7\text{cm}$ and $XZ = 3\text{cm}$. Measure the two base angles in the triangle. 2. Construct a triangle ABC where $AB = 8\text{cm}$, $AC = 5\text{cm}$ and $\angle BAC = 110^\circ$. Measure BC, $\angle ABC$ and $\angle ACB$. 3. Construct a triangle which has angle 55° and 65° at the end of a side 7cm long. Measure the other sides.	Answers to Assignments 1. $X = 33^\circ$, $Y = 65^\circ$ and $Z = 52^\circ$ 2. $BC = 10.8\text{cm}$, $\angle ABC = 27^\circ$, $\angle ACB = 43^\circ$ 3. 6.65cm and 7.4cm .

**APPENDIX I
Lesson Plans for the Control Group**

Lesson One

Subject: Mathematics
Topic: Properties of Polygons
Class: JSS II
Date: 12th. September, 2014
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old
Reference:

1. New General Mathematics for JSSII (Longman)
2. MAN Mathematics for JSSII (UPPLC)

Specific Objectives

At the end of the lesson, the students should be able to compare various polygons (i.e. triangles, quadrilaterals, pentagons, hexagons, heptagons, octagons) using coloured cardboard papers and sort them by their geometric properties (i.e. number of sides, number of interior angles number of right angles).

Previous Knowledge

Students have already known how to recognize and identify two-dimensional shapes by their appearance as a whole.

Content Development (CD)	Teachers Performance Activity
CD Step I Introduction	Teacher writes the topic on the white board. Teacher leads the students to identify polygons by their geometric properties (i.e. number of sides, number of

CD Step II Instructional Procedure	interior angles and number of right angles. <ul style="list-style-type: none"> Teacher instructs students to draw plane shapes that has from three to eight straight sides on their note books Teacher ask students the following questions: <ul style="list-style-type: none"> Which family does your shape belong to? What do all your family members have in common? What are some differences among the family members
CD Step III Evaluation	<ul style="list-style-type: none"> Teacher draw a 5 x 5 geoboard on the white board and ask students to describe it,
CD Step IV Conclusion	The Teacher leads the students to draw appropriate conclusions.
CD Step V Assignment	MAN Mathematics work book 2 page 54 exercise E

Lesson Two

Subject: Mathematics
Topic: Perimeter of a square and a rectangle.
Class: JSS II
Date: 19th. September, 2014
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old

Reference:

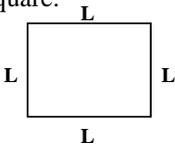
1. **New General Mathematics for JSSII (Longman)**
2. **Man Mathematics for JSSII (UPPLC)**

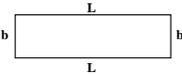
Specific Objectives

At the end of the lesson, the students should be able to derive the formulae to calculate the perimeter of a square and a rectangle.

Previous Knowledge

Students have already known how to wound round the curves of an irregular shape using a string to measure its perimeter.

Content Development (CD)	Teachers Performance Activity
CD Step I Introduction	Teacher write the topic on the white board
CD Step II Instructional Procedure	<p>The Teacher guides the students to derive the formula of the perimeter of a squares and rectangles as follows: Perimeter = Length + Breadth + Length x Breadth A Square:</p>  <p>Perimeter of a square = (Length + Breadth + Length x Breadth)</p> $P = (L + B + L + B)$ $P = 2(L + B)$ <p>Or $L = \frac{P - B}{2}$</p>

	<p>i.e Length = $\frac{P-B}{2}$</p> <p>A Rectangle</p>  <p>Perimeter of a rectangle = (Length + Breadth + Length x Breadth)</p> $P = (L + B + L + B)$ $P = 2(L+B)$ <p>Or $L = \frac{P-B}{2}$</p> <p>i.e Length = $\frac{P-B}{2}$</p> $\text{Breadth} = \frac{P-L}{2}$
CD Step III Evaluation	<p>Teacher ask the students to work out the perimeter of a square advertizing board of length 5m as follows :</p> <p>Perimeter = L + B + L + B</p> $P = 5m + 5m + 5m + 5m$ $P = 2(5m+5m)$ $P = 20m$ <p>and</p> <p>Calculate the perimeter of a rectangle 6cm by 3.5cm as follows:</p> <p>Perimeter = 6cm + 3.5cm+ 6cm + 3.5cm</p> $P = 2(6cm+3.5cm)$ $A = 19cm.$
CD Step IV Conclusion	<p>The Teacher draws a conclusion that the perimeter of a shape is the length of its boundaries. It is the distance around its edges</p>
CD Step V Assignment	<p>1. The perimeter of a square plot is 144m². Calculate the length of a side of the plot.</p>

Lesson Three

Subject: Mathematics
Topic: Area of a square and a rectangle
Class: JSSII
Date: 26th. September, 2014
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old

Reference:

1. **New General Mathematics for JSSII (Longman)**
2. **Man Mathematics for JSSII (UPPLC)**

Specific Objectives

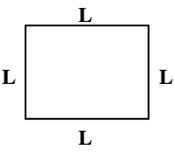
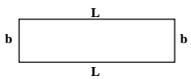
At the end of the lesson, the students should be able to:

- Derive the appropriate formula for the area of a given square and a rectangle
- Apply appropriate formula to calculate the area of a square and a rectangle.

Previous knowledge:

The students can calculate the perimeter of plane shapes.

Content Development (CD)	Teachers Performance Activity
CD Step I	Teacher ask the students to define the area of a square and a rectangle.

<p>Introduction</p> <p>CD Step II Instructional Procedure</p>	<p>The Teacher guides the students to derive the formula of a squares and rectangles. The area of a square: Area = Length x Breadth A Square:</p>  <p>Area of a square = (length of side)² $A = L \times L$ $A = L^2$ Or $L = \sqrt{A}$ i.e Length= \sqrt{A}</p> <p>A Rectangle:</p>  <p>Area of a rectangle = Length x Breadth Length = $\frac{Area}{breadth}$ Breadth = $\frac{Area}{length}$</p>
<p>CD Step III Evaluation</p>	<p>Teacher leads the students in applying the discovered formulas for the area of a square and a rectangle: Area = L x L $A = 5m \times 5m$ $A = 5 \times 5m \times m$ $A = 25m^2$</p> <p>4. $Area = 6cm \times 3.5cm$ $A = 6 \times 3.5cm \times cm$ $A = 21cm^2$.</p> <p>5. Breadth = $\frac{Area}{length}$ $= \frac{24cm^2}{6cm}$ $= 4cm$</p>
<p>CD Step IV Conclusion</p>	<p>The Teacher draws the conclusion that the area is the amount of surface enclosed within its boundary.</p>
<p>CD Step V Assignment</p>	<p>MAN Mathematics work book2 Page 52 exercise D</p>

Lesson Four

Subject: Mathematics
Topic: Area of a Parallelogram, Trapezium and a Triangle
Class: JSSII
Date: 3rd. October, 2014
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old
Reference:
1. **High Standard Mathematics for JSSII (Spectrum)**
2. **Man Mathematics for JSSII (UPPLC)**

Specific Objectives

At the end of the lesson, the students should be able to:

- Derive the appropriate formula of a given parallelogram, trapezium and triangles.
- Apply appropriate formula to calculate the area of a parallelogram, trapezium and triangles.

Previous knowledge:

The students can calculate the perimeter of plane shapes.

Content Development (CD)	Teachers Performance Activity
CD Step I Introduction	Teacher writes the topic on the white board and asks the students to define the area of parallelogram, trapezium and triangles.
CD Step II Instructional Procedure	<p>The Teacher guides the students to derive the formula of a parallelogram, trapezium and triangles. The teacher write on the white board that in any parallelogram with base b and height h, the formula for the area of a parallelogram is $b \times h$</p> <p>The area of $\triangle ADC$ with base b and height h is $\frac{1}{2} (b \times h)$. Hence the area of the parallelogram ABCD which is twice the area of $\triangle ADC$ is</p> $= 2 \times \frac{1}{2} (b \times h)$ $= b \times h.$ <p>Similarly, the area of the trapezium is the sum of the areas of the two triangles ADC and ABC. The area of $\triangle ADC$ is $\frac{1}{2} (b \times h)$. Also the area of $\triangle ABC$ is $\frac{1}{2} (b \times h)$.</p> <p>Hence, the area of the trapezium is</p> $= \frac{1}{2} ah + \frac{1}{2} bh$ $= \frac{1}{2} h (a + b).$ <p>This can be expressed as</p> <p>Area = $\frac{1}{2}$ (sum of parallel sides) x perpendicular distance between the paralalled sides.</p> <p>In the case of the triangle, the area of a triangle can be expressed as</p> $= \frac{1}{2} \times \text{base} \times \text{height}$
CD Step III Evaluation	<p>Teacher leads the students in applying the discovered formulas for the area of a parallelogram, trapezium and a triangle</p> <ol style="list-style-type: none"> 1. Area = $b \times h$ $A = 3\text{cm} \times 4\text{cm}$ $A = 3 \times 4\text{cm} \times \text{cm}$ $A = 12\text{cm}^2$ 2. Area = $\frac{1}{2} h (a + b)$. $A = \frac{1}{2} (8 + 6)4\text{cm}^2$ $A = 28\text{cm}^2$.

	$3. \text{ Area} = \frac{1}{2} \times b \times h$ $= \frac{1}{2} (4\text{cm} \times 6\text{cm})$ $= 12\text{cm}^2$
CD Step IV Conclusion	The Teacher draws the conclusion that the area of parallelogram, trapezium and a triangle is the amount of surface enclosed within its boundary.
CD Step V Assignment	<ol style="list-style-type: none"> (1) Calculate the base of a parallelogram whose area is 60cm² and height is 4cm. (2) Calculate the area of a trapezium whose parallel sides are 6cm and 14cm and the height h is 5cm. (3) Calculate the area of a triangle whose base is 2cm and height is 6cm.

Lesson Five

Subject: Mathematics
Topic: The circumference of a circle.
Class: JSS II
Date: 10th. October, 2014
Duration: 40 minutes
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old

Reference:

- New General Mathematics for JSSII (Longman)
- Man Mathematics for JSSII (UPPLC)

Specific Objectives:

At the end of the lesson, students should be able to derive the formula for the circumference (perimeter) of the circle of any size.

Previous Knowledge:

The students have already known how to measure.

Content Development (CD)	Teachers Performance Activity
CD Step I Introduction	Teacher writes the topic on the white board and ask students to draw and label a circle (radius, diameter, sector, segment and chord)
CD Step II Instructional Procedure	Teacher writes on the white board circumference is always equal to $\pi \times \text{diameter}$ $C = \pi d$ and written as $C = \pi \times 2 \times \text{radius}$ Or $C = \pi \times 2 r$ $C = 2 \pi r$
CD Step III Evaluation	Teacher evaluates the circumference of a circle whose radius is 7cm. Take $\pi = 3.14$ $C = 2 \pi r$ $C = 2 \times 3.14 \times 7\text{cm.}$ $C = 43.96\text{cm}$
CD Step IV Conclusion	The Teacher draws the conclusion that the circumference or perimeter of a circle is the length of its boundary, that is, the distance around its edges.
CD Step V Assignment	Find the circumference of a circle whose diameter is 14cm. Take $\pi = 3.14$

Lesson Six

Subject: Mathematics
Topic: Area of a circle
Class: JSS II
Date: 17th. October, 2014
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old

- Reference:**
1. New General Mathematics for JSSII (Longman)
 2. Man Mathematics for JSSII (UPPLC)

Specific Objectives:

At the end of the lesson, students should be able to calculate the area of any circle

Previous Knowledge:

The students have already known the properties of a circle and the formula for the circumference of a circle.

Content Development (CD)	Teachers Performance Activity
CD Step I Introduction	Teacher writes the topic on the white board and ask students to draw and label a circle (radius, diameter, sector, segment and chord)
CD Step II Instructional Procedure	The teacher leads the students to derive the formula for finding the area of a circle. $\text{Area of circle} = r \times \frac{1}{2} C$ $= \frac{1}{2} rc$ But circumference = $2\pi r$ \therefore Area of circle = $r \times \frac{1}{2} (2\pi r)$ $= \pi r^2$
CD Step III Evaluation	Teacher leads the students to evaluates $\text{Area} = \pi r^2$ $A = 3.14 \times 7 \times 7$ $A = 154\text{cm}^2$ $\text{Area} = 38\frac{1}{2} \text{cm}^2$ $\pi r^2 = 38\frac{1}{2} \text{cm}^2$ $r^2 = 3.14 \times 3.14 = 9.86$ $r = \sqrt{9.86}\text{cm}$ $r = 3.5\text{cm}$
CD Step IV Conclusion	The Teacher draws the conclusion that the area of a circle is the amount of surface enclosed within its boundary.
CD Step V Assignment	What is the area of a semi circular disc whose diameter is 28cm?

Lesson seven

Subject: Mathematics
Topic: Angle Properties of Parallel Lines.
Class: JSS II
Date: 24th. October, 2014
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old

- Reference:**
1. New General Mathematics for JSSII (Longman)

2. Man Mathematics for JSSII (UPPLC)

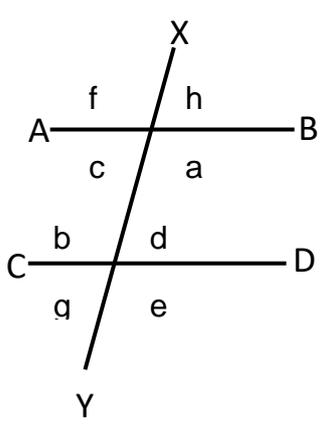
Specific Objectives:

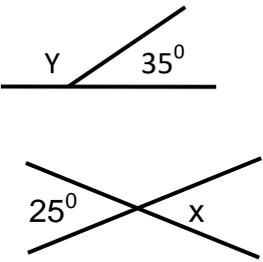
At the end of the lesson, students should be able to identify and apply

- i. alternate and corresponding angles and
- ii. adjacent angles on a straight line.

Previous Knowledge:

Students can draw given angles.

Content Development (CD)	Teachers Performance Activity
<p>CD Step I Introduction</p>	<p>Teacher writes the topic on the white board and defines an angle to be a meeting point of two straight lines.</p>
<p>CD Step II Instructional Procedure</p>	<p>The Teacher draw on the white board several different pairs of alternate, corresponding and adjacent angles on a straight line and measure them with a protractor.</p> <div style="text-align: center;">  </div> <p>The angles marked a and b ; c and d are alternate angles. angles with the following pairs of marked angles are called corresponding angles a and e, b and f, c and g, d and h The pair of angles marked a and d, b and a are called interior opposite angles. The teacher draws on the white board a pair of adjacent angles</p> <div style="text-align: center;">  </div> <p>Students observed that angles with the following pairs of marked angles are called adjacent angles $a + b = 180^\circ$</p>
<p>CD</p>	<p>Teacher leads the students in applying the</p>

<p>Step III Evaluation</p>	<p>discovered ideas to find the value of the unknown angles in each of the diagrams</p>  <p>The teacher evaluates that</p> <ol style="list-style-type: none"> $X=25^\circ$ (vertically opposite angles are equal) $Y+35^\circ = 180^\circ$ (sum of adjacent angles) <p>Therefore, $Y = 180^\circ - 35^\circ$ $Y = 145^\circ$</p>
<p>CD Step IV Conclusion</p>	<p>The Teacher draws conclusion that the following are true if two parallel lines are cut by a transversal:</p> <ol style="list-style-type: none"> The alternate angles are equal. The corresponding angles are equal. The interior angles on the same side of the transversal are supplementary. <p>The teacher also concludes that there were pairs of alternate and corresponding angles from lines that are not parallel and the pairs of interior angles too were not supplementary.</p>
<p>CD Step V Assignment</p>	<p>Work book page 64 exercise C No 1, 3 and 4</p>

Lesson Eight

Subject: Mathematics
Topic: Properties of an angle at the point of intersecting straight lines
Class: JSS II
Date: 31st. October, 2014
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old
Reference:
 1. Man Mathematics for JSSII (UPPLC)
 2. High Standard Mathematics for JSSII (Spectrum)

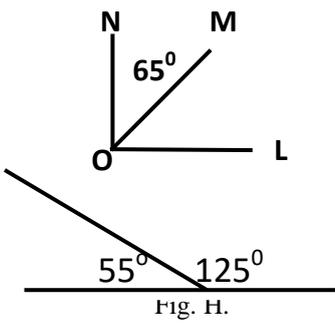
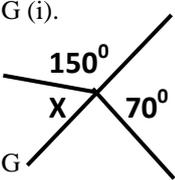
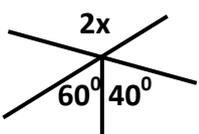
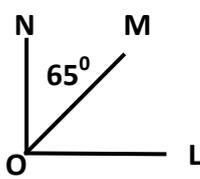
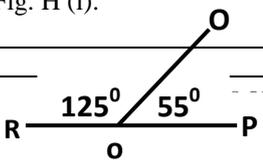
Specific Objectives:

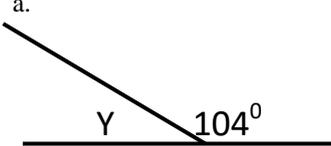
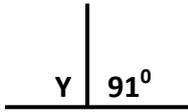
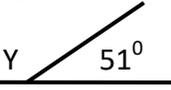
At the end of the lesson, students should be able to identify, define and apply vertically opposite angles, adjacent, complementary and supplementary angles to solve problems.

Previous Knowledge:

Students can draw given angles and measure using protractor.

<p>Content Development (CD)</p>	<p>Teachers Performance Activity</p>
<p>CD Step I Introduction</p>	<p>Teacher leads the students to identify vertically opposite and adjacent angles and differentiates between them. Also complementary and</p>

	<p>supplementary angles are identified using the models of solid shapes and the school buildings and define the angles.</p>
<p>CD Step II Instructional Procedure</p>	<p>The Teacher guides the students to elicit the idea of vertically opposite angle, complementary and supplementary angles. draw on paper several different pairs of vertically opposite angles as shown in figure G and measure them with a protractor.</p> <p>Teacher show that the angle marked $x = y$ and $u = v$ are opposite to each other. Similarly, the teacher draw on the white board several different pairs of complementary and supplementary angles as shown in figure H and measure them with a protractor.</p>  <p>Teacher show that angle 65° complementary to 25° 125° is supplementary to 55°.</p>
<p>CD Step III Evaluation</p>	<p>Teacher leads the students in applying the discovered ideas to find the value of the lettered angles in the diagram in fig G (i), G (ii) and G (iii) and Fig H (i), H (ii) and H (iii).</p>  <p>Fig. G (i).</p>  <p>Fig. G</p>  <p>Fig. G (iii)</p>  <p>Fig. H (i).</p> 

CD Step IV Conclusion	The teacher draws conclusion that: 1. Vertically opposite angles are equal. 2. Two angles are complementary if their sum is 90° 3. Two angles are supplementary if their sum is 180°
CD Step V Assignment	Find the value of y in each of the following angles. a.  b.  c. 

Lesson Nine

Subject: Mathematics
Topic: Properties of Angles at Point.
Class: JSS II
Date: 7th. November, 2014
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old

- Reference:**
1. New General Mathematics for JSSII (Longman)
 2. Man Mathematics for JSSII (UPPLC)

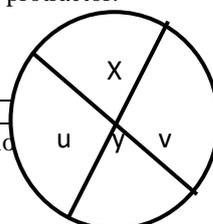
Specific Objectives:

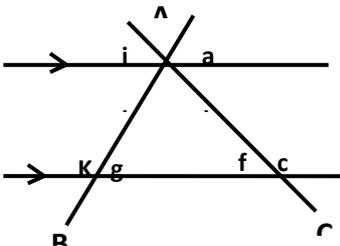
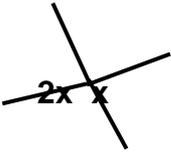
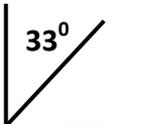
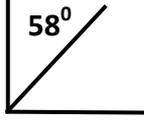
At the end of the lesson, students should be able to solve problems using the fact that angles at a point add up to 360° .

Previous Knowledge:

Students can draw given angles and measure.

Content Development (CD)	Teachers Performance Activity
CD Step I Introduction	Teacher writes the topic on the white board
CD Step II Instructional Procedure	The Teacher draw on paper several different sides of angles in a circle using a pair of compasses, a ruler and a protractor as shown in figure M and measure them with a protractor.



	<p>Teacher show that all the angles marked u, v, x and y are called angles at a point. Since u and x are adjacent angles on a straight line then $u + x = 180^\circ$ Therefore, $u + v + x + y = 360^\circ$.</p>
<p>CD Step III Evaluation</p>	<p>Teacher leads the students in applying the discovered ideas of angles at a point thus:</p> <ol style="list-style-type: none"> 1. State which pairs of the marked angles in Fig. N (i) are adjacent angles on a straight line.  <p>Fig. N (ii) shows that sizes in degrees of 4 angles.</p>  <p>Find the value of x and find the sizes of each of the angles.</p>
<p>CD Step IV Conclusion</p>	<p>The Teacher leads the students to draw appropriate conclusions.</p>
<p>CD Step V Assignment</p>	<p>Find the value of x in the diagram below.</p> <p>d.</p>  <p>e.</p> 

Lesson Ten

Subject: Mathematics
Topic: Construction of Triangles
Class: JSS II
Date: 14th. November, 2014
No. of Students: 51 and 57 students respectively
Age: Average of 13 years old

Reference:

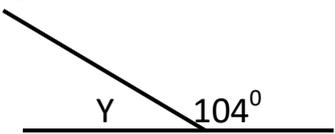
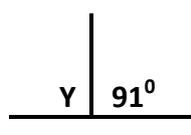
- **New General Mathematics for JSSII (Longman)**
- **Man Mathematics for JSSII (UPPLC)**

Specific objectives:

At the end of the lesson, the students should be able to construct any triangle.

Previous knowledge:

The students have already known how to mark and sketch the geometrical figure to be constructed with their free hand.

Content Development (CD)	Teachers Performance Activity
CD Step I Introduction	Teacher asks the students to name the various geometric construction instruments used in constructing triangles. The teacher demonstrates how to use the geometric construction instruments and lead the students to describe their uses.
CD Step II Instructional Procedure	The teacher guides the students to construct triangles when: <ol style="list-style-type: none"> 1. 3 sides are given 2. 2 sides and the included angle are given 3. 2 angles and one side are given.
CD Step III Evaluation	Teacher leads the students in constructing the following triangles: <ol style="list-style-type: none"> 1. Construct a triangle having sides 3cm, 4cm and 5cm long. 2. Draw a triangle ABC where AB = 2.6cm, AC = 2.1cm and $\angle BAC = 115^\circ$. Measure BC, $\angle ACB$ and $\angle BAC$. 3. Construct a triangle which has angles 40° and 55° at the end of a side 3.9 cm long. Measure the other sides.
CD Step IV Conclusion	The students draw the conclusion that: <ol style="list-style-type: none"> 1. Vertically opposite angles are equal. 2. Two angles are complementary if their sum is 90° Two angles are supplementary if their sum is 180°
CD Step V Assignment	Find the value of y in each of the following angles. <p>a.</p>  <p>b.</p> 

Appendix J

Comparison of Pretest Mean Scores of Experimental and Control Groups.

A t-test analysis of the pretest Performance scores of the experimental and control groups was carried out. This was to determine the equivalence of the two groups at the start of the treatment. The result obtained is in the table below.

Pretest Mean Scores of Experimental and Control Groups.

Group	N	\bar{X}	Standard Deviation	Standard Error	df	t-cal	t-crit	P-value
Experimental	103	45.37	1.456	0.143				
					209	0.928	1.96	0.355**
Control	108	45.14	2.081	0.200				

** Not Significant, $P > 0.05$

Thus there is no significant difference in the pretest performance mean score of the experimental and control groups. This therefore shows that the two groups were equivalent at the start of research treatments.

Appendix K

Students' Performance in Mathematics (BECE) 2003-2014.

Year of Exam	Total Sat	No. of Pass (A1-C6)	% Pass	% Fail
2003	3,371	2,264	32.81	67.19
2004	3,102	1,968	36.55	63.45
2005	3,907	2,559	34.50	65.50
2006	3,506	2,211	36.91	63.09
2007	3,524	2,326	33.97	66.03
2008	4,406	2,722	38.20	61.80
2009	4,028	2,420	39.92	60.08
2010	4,907	2,612	46.75	53.25
2011	3,009	1,286	57.24	42.76
2012	3,557	2,632	25.99	74.01
2013	4,250	3,190	24.94	75.06
2014	4,853	2,961	38.98	61.02

Source: Benue State Examination Board (2003-2014)

Appendix L

Benue State Examination Board, Chief Examiner's Report (Areas of Weakness) 2007 - 2014

Year	Examiner's Comment	Area of Weakness
May/June, 2007	Majority of the candidates could not apply the basic concepts	Measurement of the two dimensional shapes and Geometrical construction
May/June 2008	Candidates exhibited weakness in their interpretation and application	Geometrical statement
May/June, 2009	Apart from not adhering to instructions and accuracies required, candidates weaknesses were observed in some areas	Probability, Geometry, Sets and Algebraic graph.
May/June 2010	Majority of candidates avoided questions on topics in geometry and the few that attempted them perform poorly	Reading Geometric shapes and spatial abilities
May/June 2011	Candidates Performance need to improve in some areas	Geometrical construction, and reading from the graphs.
May/June 2012	Candidates responses indicates poor understanding and application of some basic mathematical principles and concepts	Circle geometry, geometrical constructions, measurement in two dimensional graphs, writing answers to the required degree of accuracies
May/June 2013	Their Performance in areas of the syllabus such as geometry continued to remain poor	Geometrical construction and visualization
May/June 2014	There appear to be no improvement in the areas of the syllabus where candidates' Performance have been observed to be poor	Word problems, geometrical construction, bearing-representing the given information on a correct diagram and measurement of plane shapes

Source: Benue State Examination Board, Makurdi (Mathematics Section)

Appendix M

Modified Stages of Piaget's Learning Theory of Cognitive Development

Stage	Age of Period	Description
Sensory-motor Stage	Infancy (0-2 years)	Intelligence is present, motor activity but no symbols, knowledge is developing, yet limited knowledge is based on experiences/interactions, mobility allows child to learn new things, some language skills are developed at the end of this stage.
Pre-Operational Stage	Toddler and Early Childhood (2-10 years)	Symbols or language skills are present, memory and imagination are developed, non-reversible and non-logical thinking,
Concrete Operational Stage	Elementary and Early Adolescence (10-15 years)	Operations on its environment and Logical thought, understand concepts based on direct experience, manipulation of symbols related to concrete objects, operational thinking predominates, irreversible and egocentric thinking, understand law of conservation, use thought and imagination more, use of concrete examples.
Formal Operational Stage	Adolescence and Adulthood	Form of an argument without being distracted by the content, Logical use of symbols related to abstract concepts, logical and

(15 years and above) formal thinking is uncommon.

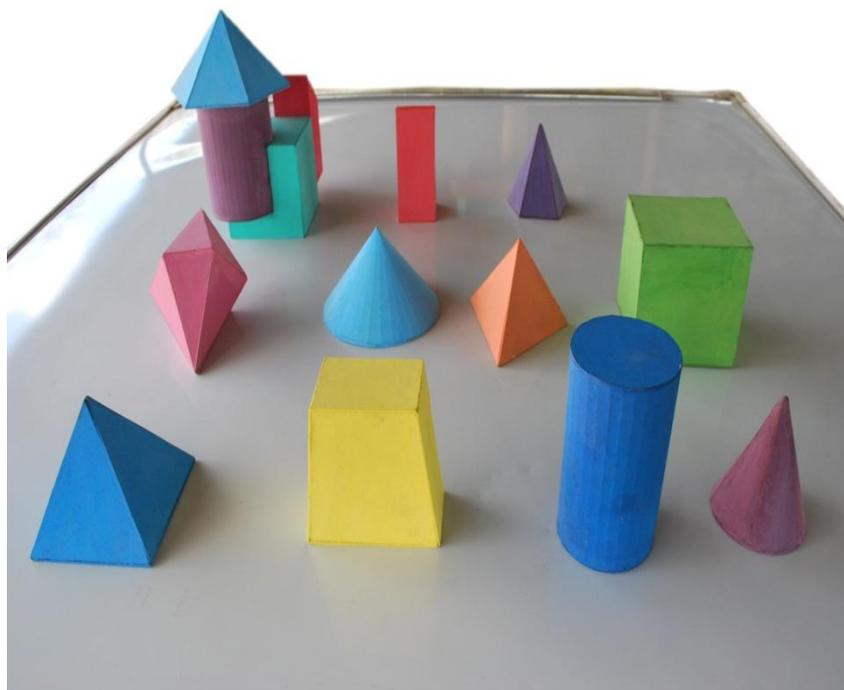
Source: Piaget (1966) Theory of Cognitive Development

Appendix n

Models of manipulatives by students



Plane shapes of various sizes



Models of Plane shapes



A Geoboard with a Pentagon Band

Appendix o
Spss results print out

NPART TESTS
/M-W= ATTITUDE BY GROUP(1 2)
/STATISTICS=DESCRIPTIVES
/MISSING ANALYSIS.

NPART TESTS
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	Cases Used	Statistics for each test are based on all cases with valid data for the variable(s) used in that test.
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a. Based on availability of workspace memory.		

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	N	Mean	Std. Deviation	Minimum	Maximum
ATTITUDE	211	66.4265	18.70875	44.00	114.00
GENDER	211	1.4408	.49766	1.00	2.00

Mann-Whitney Test

Ranks				
	GENDER	N	Mean Rank	Sum of Ranks
ATTITUDE	MALE	93	106.20	9877.00
	FEMALE	118	105.84	12489.00
	Total	211		

Test Statistics ^a	
	ATTITUDE
Mann-Whitney U	5468.000
Wilcoxon W	1.249E4
Z	-.044
Asymp. Sig. (2-tailed)	.965

a. Grouping Variable: GENDER

NPar Tests

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	Cases Used	Statistics for each test are based on all cases with valid data for the variable(s) used in that test.
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a. Based on availability of workspace memory.

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Descriptive Statistics					
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Attitude	211	66.4265	18.70875	44.00	114.00
Group	211	1.5118	.50105	1.00	2.00

Mann-Whitney Test

Ranks				
	Group	N	Mean Rank	Sum Of Ranks
Attitude	Exoerimental	103	132.08	13604.50
	Control	108	81.12	8761.50
	Total	211		

Test Statistics ^a	
	Attitude
Mann-Whitney U	2875.500
Wilcoxon W	8761.500
Z	-6.193
Asymp. Sig. (2-Tailed)	.000

A. Grouping Variable: Group

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/criteria=ci(.9500).

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T-Test

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	Cases Used	Statistics For Each Analysis Are Based On The Cases With No Missing Or Out-Of-Range Data For Any Variable In The Analysis.
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	Elapsed Time	00:00:00.016

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Group Statistics					
	Gender	N	Mean	Std. Deviation	Std. Error Mean
Performance	Male	93	65.7204	18.54862	1.92340
	Female	118	65.0847	18.70444	1.72188

Independent Samples Test											
		Levene's Test for Equality of Variances		t-test for Equality of Means							
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference		
										Lower	Upper
PERFORMANCE	Equal variances assumed	.378	.539	-.246	209	.806	-.63568	2.58411	-5.72995	4.45859	
	Equal variances not assumed			-.246	198.3	.806	-.63568	2.58154	-5.72647	4.45510	

T-Test

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N Of Rows In Working Data File		211
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	Cases Used	Statistics For Each Analysis Are Based On The Cases With No Missing Or Out-Of-Range Data For Any Variable In The Analysis.
Syntax		T-TEST GROUPS=GROUP(1 2) /MISSING=ANALYSIS /VARIABLES=PERFORMANCE /CRITERIA=CI(.9500).
Resources	Processor Time	00:00:00.000
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Group Statistics					
	Group	N	Mean	Std. Deviation	Std. Error Mean
Performance	Exoerimental	103	84.0194	3.94566	.38878
	Control	108	47.5741	3.04546	.29305

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
PERFORMANCE	Equal variances assumed	37.473	.000	75.312	209	.000	36.44534	.48393	35.49134	37.39934
	Equal variances not assumed			74.859	191.809	.000	36.44534	.48685	35.48507	37.40562

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
PERFORMANCE	Equal variances assumed	5.811	.017	.928	209	.355	.23049	.24840	-.25919	.72018
	Equal variances not assumed			.936	191.950	.351	.23049	.24638	-.25546	.71645

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	Cases Used	Statistics for each test are based on all cases with valid data for the variable(s) used in that test.
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	Number of Cases Allowed	112347
a. Based on availability of workspace memory.		

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GROUP	211	1.5118	.50105	1.00	2.00

Mann-Whitney Test

Ranks				
	GROUP	N	Mean Rank	Sum of Ranks
ATTITUDE	EXOERIMENTAL	103	123.22	12691.50
	CONTROL	108	89.58	9674.50
	Total	211		

Test Statistics ^a	
	ATTITUDE
Mann-Whitney U	3788.500
Wilcoxon W	9674.500
Z	-4.082
Asymp. Sig. (2-tailed)	.000
a. Grouping Variable: GROUP	

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	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	211
Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
Syntax	T-TEST GROUPS=GROUP(1 2) /MISSING=ANALYSIS /VARIABLES=TEST /CRITERIA=CI(.9500).	
Resources	Processor Time	00:00:00.000
	Elapsed Time	00:00:00.250

[DataSet1] C:\Users\AKAAZUA\Documents\TERTSEAPRETESTDATA1.sav

Group Statistics					
	GROUP	N	Mean	Std. Deviation	Std. Error Mean
TEST	EXP	103	45.3786	1.45600	.14346
	CONTL	108	45.1481	2.08158	.20030

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
TE ST	Equal variances assumed	5.811	.017	.928	209	.355	.23049	.24840	-.25919	.72018
	Equal variances not assumed			.936	191.950	.351	.23049	.24638	-.25546	.71645

Oneway pretest by school
/missing analysis
/posthoc=scheffe alpha(0.05).

Oneway Notes		
Output Created		22-Aug-2014 17:16:06
Comments		
Input	Data	C:\Users\AKAAZUA\Documents\TERTSEAPRETESTDATA.sav
	Active Dataset	DataSet1
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	440
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on cases with no missing data for any variable in the analysis.
Syntax		ONEWAY PRETEST BY SCHOOL /MISSING ANALYSIS /POSTHOC=SCHEFFE ALPHA(0.05).
Resources	Processor Time	00:00:00.000
	Elapsed Time	00:00:00.000

[DataSet1] C:\Users\AKAAZUA\Documents\TERTSEAPRETESTDATA.sav

ANOVA					
PRETEST	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	10.222	3	3.407	1.484	.218
Within Groups	1001.333	436	2.297		
Total	1011.555	439			

Post Hoc Tests

Multiple Comparisons						
Test Scheffe	(J) School	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Fed. Govt. Girls College, Gboko	Govt. College Makurdi	.06386	.22042	.994	-.5547	.6825
	Fed. Govt. College Vandeikya	.18648	.24115	.897	-.4903	.8633
	Devine Love Cath. Girls College, K/Ala	.50446	.27918	.354	-.2790	1.2880
Govt. College Makurdi	Fed. Govt. Girls College, Gboko	-.06386	.22042	.994	-.6825	.5547
	Fed. Govt. College Vandeikya	.12262	.17732	.924	-.3750	.6203
	Devine Love Cath. Girls College, K/Ala	.44060	.22634	.287	-.1946	1.0758
Fed. Govt. College Vandeikya	Fed. Govt. Girls College, Gboko	-.18648	.24115	.897	-.8633	.4903
	Govt. College Makurdi	-.12262	.17732	.924	-.6203	.3750
	Devine Love Cath. Girls College, K/Ala	.31798	.24657	.645	-.3740	1.0100
Devine Love Cath. Girls College, K/Ala	Fed. Govt. Girls College, Gboko	-.50446	.27918	.354	-1.2880	.2790
	Govt. College Makurdi	-.44060	.22634	.287	-1.0758	.1946
	Fed. Govt. College Vandeikya	-.31798	.24657	.645	-1.0100	.3740

Homogeneous Subsets

Pretest		
Scheffe		
School	N	Subset For Alpha = 0.05
		1
Devine Love Cath. Girls College, K/Ala	57	44.8070
Fed. Govt. College Vandeikya	112	45.1250
Govt. College Makurdi	210	45.2476
Fed. Govt. Girls College, Gboko	61	45.3115
Sig.		.201
Means For Groups In Homogeneous Subsets Are Displayed.		

T-test groups=group(1 2)
 /missing=analysis
 /variables=postposttest
 /criteria=ci(.9500).

T-Test

Notes		
Output Created		30-Apr-2017 11:20:56
Comments		
Input	Data	C:\Users\AKAAZUA\Documents\TertseaDisertationDdata7A.sav
	Active Dataset	DataSet1
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	211
Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
Syntax		T-TEST GROUPS=GROUP(1 2)

		/MISSING=ANALYSIS /VARIABLES=POSTPOSTTEST /CRITERIA=CI(.9500).
Resources	Processor Time	00:00:00.000
	Elapsed Time	00:00:00.000

[DataSet1] C:\Users\AKAAZUA\Documents\TertseaDisertationDtata7A.sav

Group Statistics					
	Group	N	Mean	Std. Deviation	Std. Error Mean
Postposttest	Exoerimental	103	86.6990	5.22219	.51456
	Control	108	48.4259	5.55719	.53474

ndependent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
POST POSTTE ST	Equal variances assumed	11.391	.001	51.498	209	.000	38.27310	.74320	36.80797	39.73823
	Equal variances not assumed			51.574	208.956	.000	38.27310	.74210	36.81013	39.73607

Jacob Tertsea Akaazua Effect of Concrete Manipulative Approach on Attitude, Retention And Performance in Geometry Among Junior Secondary School Students in Benue State, Nigeria.” IOSR Journal of Research & Method in Education (IOSR-JRME) , vol. 7, no. 6, 2017, pp. 80-175.