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Characterization of Mathematical Connection Errors in Derivative Problem Solving

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Abstract: *This research aims to determine the types of mathematical connection errors when undergraduate students solve mathematics problems about derivative, and describe the characteristics of each type of the errors. The style of this research is descriptive exploratory, as an initial observation for disertation. Its subjects were 12 undergraduate students of Mathematics Education Study Program, Faculty of Teacher Training and Education, University of Mataram, at Second Semester in Academic Year 2014/2015. Data of research collected through the assignment sheet of mathematics problem solving about derivative of absolute value function and followed up by interview based problem solving tasks. The study resulted that the type of mathematical connection errors can be divided into errors to connect with conceptual knowledge and errors to connect with procedural knowledge. Then, each of these errors type is characterized through considering with three interconnected topic, that are absolute value function, derivative, and limits. Later, it is characterized by describing the process of thinking.*

Keywords - *characterization, derivative problems, error, mathematical connection.*

I. Introduction

Problems are often encountered when learners solve a problem in school or university is that they have learned and mastered the knowledge required to solve the problem, but can't associate or make a connection and use it to solve the problem. Therefore, the ability of mathematical connection needs attention in the learning of mathematics. To be able to solve mathematical problems, the necessary skills to connect mastered knowledge relating to the problems encountered.

Mathematical knowledge can be divided into two types, namely conceptual knowledge and procedural knowledge (Hiebert & Lefevre, 1986). The core of conceptual knowledge is to understand the relationship between ideas and concepts of mathematics. The purest form of procedural knowledge focuses on the symbolism, skills, rules and algorithms used step by step in solving a mathematical task. Learners should learn the concepts at once with the procedures so that they can see the connection.

Concepts, procedures or skills, together with facts and principles identified by Gagne (in Bell, 1978) as the direct object of mathematics. While the one of indirect objects of the mathematical is problem solving. Problems are questions that can be understood by learners but can not be answered immediately with a routine procedure that has been known to learners. So a question can be clasified as a problem when the questions give a challenge to be answered and the answer can not be done by a routine strategy. Solve the problem is the process of accepting the challenge to answer the question that is the problem (Hudojo, 1988). A problem for college students when was faced to elementary level students excluding a problem because the problem will not be understood and will not give a challenge to be answered.

Bruner's connectivity theorem stated that in mathematics every concept related to other concepts. Similarly, between the argument and the other argument, the theory with the other theory, among the topics to the topic, and between branches of the branch of mathematics sholuld be related (Ruseffendi, 1988). Therefore, in order to be more successful in learning mathematics, learners must be given the opportunity to make mathematical connection.

Mathematical connection was described by Hiebert and Carpenter (1992) as part of the network is structured like a spider web, "*The junctures, or nodes, can be thought of as pieces of represented information, and threads between them as the connections or relationships*". Mathematical connection can also be described as a component of a scheme or a linked group of schemes in mental network. Marshall (1995) argued that the characteristics of the scheme is the connection. The more connections, the greater compactness and strength of the scheme.

Studies on the connection mathematical already conducted in schools and universities (Adlakha & Kowalski, 2007; Bilotski and Subbotin, 2009; Kondratieval & Radu, 2009; Lapp et al, 2010; Presmeg 2006; Yantz, 2013). Yantz (2013), when examined undergraduate students in pre-calculus course, concluded that the subjects had not yet established a connection between algebra procedures and the nature of the underlying numbers. Results of research on the connection in linear algebra carried out by Lapp et al (2010) showed that subjects find it more difficult to make the connection between concepts like eigenvalues and eigenvectors and of other parts such as the conceptual basis and the dimension.

Research by Eli et al (2013) on middle school pre-service teachers has resulted five categories of mathematical connections, namely: (1) *categorical*, if the paticipant's explanation relied upon the use of surface features primarily as a basis for defining a group or category; (2) *characteristic/property*, if the participant's explanation for the sort involved defining the characteristics or describing the properties of concepts in terms of other concepts; (3) *curricular*, if the participant's explanation for the sort involved relating ideas or concepts in terms of the impact to curriculum, including but not limited to, the order in which one would teach concepts or topics; (4) *procedural*, if the participant's explanation for teh sort involved relating ideas based on a mathematical procedure or algorithm possible through the construction of an example, which may include a description of the mechanics involved in carrying out the procedure rather than the mathematical ideas embedded in the procedure; and (5) *derivational*, if the participant's explanation for the sort involved knowledge of one concept(s) to build upon or explain other concept(s), included but not limited to the recognition of the existence of a derivation.

Results of another study suggests three types of connections, that are referred to the most commonly in relevant literature and in their formal curriculum documents, but in practice their development of "connected knowing" could have been stronger, more frequent and more consistent. The three kinds of connections are the connection between new information and current knowledge, the connection between mathematical concepts, and the connection to everyday experience (Mousley, 2004). The types of connection is in line with the scope of connection standards in the NCTM (2000), which include recognize and use connection among mathematical ideas, understand how mathematical ideas interconnected and build on one another to produce a coherent whole, recognize and apply mathematics in contexts outside of mathematics.

Study of the source of theory and research results presented above, gave rise to the idea of research with the theme of mathematical connection. This research of mathematical connection was focused on the connection of mathematical knowledge that is the connection between new information with knowledge already mastered divided into two types of knowledge, namely conceptual knowledge and procedural knowledge. Therefore, research on the characterization of the mathematical connection errors when undergraduate students solve problems derivatives was conducted. Errors in making mathematical connection become the focus of attention because there is still a lot of errors encountered, so that it is important to be characterized as a basis for further research. The material derivative chosen because it is the core material in differential calculus, and a lot of good use in other disciplines as well as in everyday life.

This research is expected to be useful to provide an overview of the undergraduate students about the types of errors and its characterization of mathematical connection, so it can be used as a benchmark to improve problem solving skills. In addition, it is expected to be useful as inputs for lecturers about the importance of undergraduate students making mathematical connection so that it can be taken into consideration in planning and implementing of learning.

II. Method of Research

Type of research is descriptive exploratory study with qualitative approach. The subjects were twelve undergraduate students at second semester of Mathematics Education Study Program, Faculty of Teacher Training and Education, Mataram University. The main instrument in qualitative research was a researcher itself (Moleong, 2006). The support instruments are an assignment sheet and an interview guides. The assignment sheets gave a task of derivative problem solving containing the absolute value function as follows.

1. Given $f(x) = |x|$. Find $f'(0)$.
2. Given $f(x) = x|x|$. Find $f'(0)$.

The problem is given by the consideration that in order to solve not only the routine procedure using formulas derived, but also requires an understanding of the concept of derivative and also connection with other concepts, such as the absolute value itself and also the concept of limit. To explain the mathematical connection in the process of thinking when solving problems such derivatives, required structuring research problems. What is meant by the structure of the problem is the flow of problem solving is ideally compiled by researchers. If the subject completes the task of solving the problem in accordance with the structure of the problem, the result of the completion of the subjects there was nothing wrong. The structure of the problem 1 is described in Fig. 1 and the structure of problem 2 is described in Fig. 2.

Although the problem number 1 and number 2 looks almost the same, but there are substantial differences. Considered the final answer, ini the problem number 1 its derivative does not have any value or the value of its derivatives do not exist, while in the problem number 2 its derivative has a definite value. Considered from process of answering, as described in the structure of the above problems, the function of the problem number 1 could be directly coordinated with the definition of absolute value, whereas the problem number 2 should be done procedures of limit counting previously to obtain a simpler form which can be coordinated with the definition absolute value.

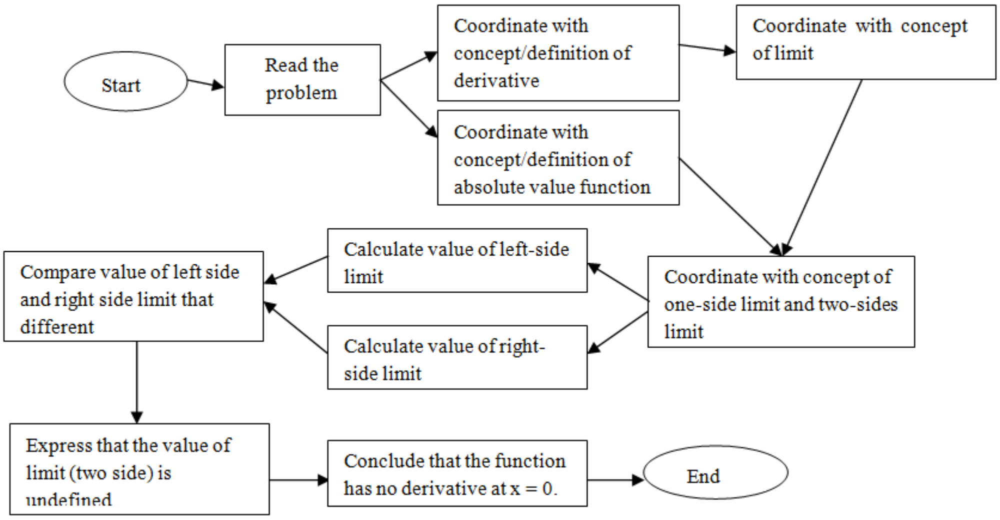


Figure 1. Structure of Derivatife Problem 1

The following figure describe structure of problem 2.

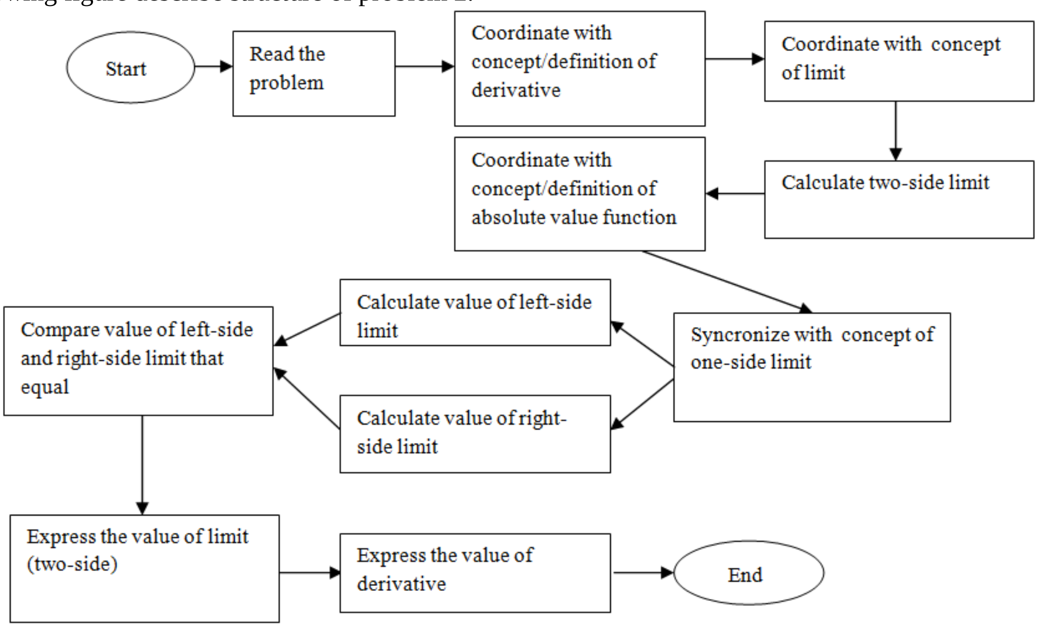


Figure 2. Structure of Derivatife Problem 2

Errors made by subjects are grouped and identified its characteristics. Considered from the type of knowledge conceptual and procedural knowledge, the mathematical connection errors divided into two types, namely errors in connect to conceptual knowledge and errors in connect to procedural knowledge. While looking at the material, there are three topics related material, namely the absolute value function, the derivative of function, and the limit of function. Therefore indicators of the type of mathematical connection errors can be expressed in Table 1 below.

Table 1. Indicators for Types of Mathematical Connection Errors

No.	Type of Mathematical Connection Errors	Indicators
1	Errors in connect to conceptual knowledge	• Coordinate with the definition of absolute value function but make mistakes to specify the definition of absolute value; or does not write the definition of absolute value. • Coordinate with the definition of a derivative but make mistakes to specify the definition of a derivative; or does not write the definition of a derivative. • Coordinate with the form of the two sides limit, but does not coordinate

		with the one side limit; or does not write teh form of limit.
2	Errors in connect to procedural knowledge	• Make mistakes when calculate derivative of absolute value function; or declare the absolute value function as the roots of quadratic forms that to cause miscalculations of derivatives. • Using derivatives rules or formulas that are not appropriate, in this case calculate derivative of the absolute value function with the power rule. • There are mistakes when calculate limit, includes error to calculate left-side or right-side limit.

III. Result and Discussion

The results of the subjects' answers to the problem solving task recapitulated based aspects of conceptual knowledge and procedural knowledge that each of the three topics related to the material, which are the absolute value function, the derivative of function and the limit of function. The connection with conceptual knowledge on the absolute function is said to be true if the concern subject associate with the correct definition of the absolute value functions. The connection with conceptual knowledge on the derivative is said to be true when the concern subject associate with the definition of the derivative that using limits. The connection of conceptual knowledge on the limit is said to be true when the concern subject associate with the left side and right side limit, because determining the value of the limit requires the calculation of one side limit. While the connection with procedural knowledge in each of the three material is said to be correct if the calculations related to each such material result the correct value. Recapitulation of answer for problem number 1 presented in Table 2, while the answer for problem number 2 was recapitulated in Table 3.

Table 2. Recapitulation of Answer for Problem 1

No. of Subjek	Connect to Conceptual Knowledge			Connect to Procedural Knowledge		
	Absolute Value	**Derivative**	**Limit**	**Absolute Value**	**Derivative**	**Limit**
1.	True	Nothing	Nothing	False	False	Nothing
2.	True	Nothing	True	False	False	False
3.	Nothing	Nothing	Nothing	Nothing	Nothing	Nothing
4.	Nothing	True	Nothing	Nothing	False	Nothing
5.	Nothing	Nothing	False	False	False	False
6.	Nothing	True	False	False	Nothing	False
7.	True	Nothing	Nothing	False	False	Nothing
8.	Nothing	Nothing	Nothing	False	Nothing	Nothing
9.	Nothing	Nothing	Nothing	False	False	Nothing
10.	False	True	Nothing	Nothing	Nothing	Nothing
11.	False	True	Nothing	Nothing	Nothing	Nothing
12.	True	True	Nothing	False	Nothing	Nothing

Analysis of the above data resulted types of connection errors mathematically, which can be divided into two types, namely errors in connect to conceptual knowledge and errors in connect to procedural knowledge. The errors in connect to both conceptual knowledge and procedural knowledge can be seen from the material related to the derivative problem of absolute value function, namely the concept of absolute value function, the concept of derivative, and the concept of limit.

Table 3. Recapitulation of Answer for Problem 2

No. of Subjek	Connect to Conceptual Knowledge			Connect to Procedural Knowledge		
	Absolute Value	**Derivative**	**Limit**	**Absolute Value**	**Derivative**	**Limit**
1.	True	True	False	True	False	Nothing
2.	Nothing	True	True	False	False	False
3.	True	Nothing	Nothing	False	False	Nothing
4.	False	Nothing	Nothing	False	False	Nothing
5.	Nothing	Nothing	False	False	False	Nothing
6.	True	True	False	False	False	False
7.	True	Nothing	Nothing	False	False	Nothing
8.	Nothing	Nothing	Nothing	False	False	Nothing
9.	Nothing	Nothing	Nothing	False	False	Nothing
10.	False	Nothing	Nothing	False	Nothing	Nothing
11.	True	True	True	False	False	Nothing
12.	True	Nothing	Nothing	False	Nothing	Nothing

The errors in connect to conceptual knowledge about absolute value function occurs when the concerned subject has been coordinating with the absolute value function but misunderstood the definition of absolute value, as shown in Fig. 3 part (a). It appears that the concerned subject can coordinate with the absolute value function, but an mistake occurred because it does not understand that when $|x|$ worth x and when $|x|$ worth -x. In the case of errors in connect to conceptual knowledge about derivatives, there are some subjects already coordinated with the concept of derivative connect properly, but the other subjects didn't coordinate with the concept of derivatives so that there is no error connection can be considered. The errors in connect to conceptual knowledge about limit occurs when the concerned subject has been coordinating with the form of the two-side limit but does not coordinate with the one-side limit, as shown in Fig. 3 part (b). It appears that the concerned subject already inscribed limit, but did not declare it became left-side or right-side limit.


(a)

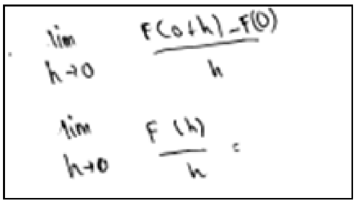
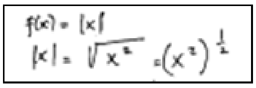
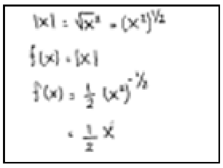

(b)

Figure 3. Errors in connect to conceptual knowledge

The errors in connect to procedural knowledge about absolute value function occurs when the concerned subject in procedure to calculate declared absolute value function as the roots of quadratic forms, as shown in Figure 4 part (a). It appears that the concerned subject used the wrong procedure in calculating an absolute value function. The errors in connect to procedural knowledge about derivatives is done by applying a not appropriate rules, as shown in Figure 4 part (b). It appears that the subject made a mistake in derivative calculation procedure of the absolute value function using the power rule. The errors in connect to procedural knowledge about limit also occurs as shown in Figure 4 part (c). The concerned subject make mistake in calculating the value at one stage of the calculation of the limit.


(a)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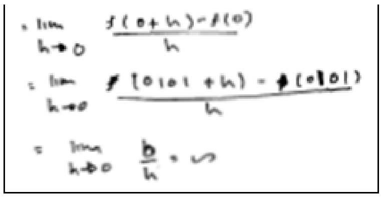
(c)

Figure 4. Errors in connect to procedural knowledge

IV. Conclusion

From the results of this study concluded that the type of mathematical connection errors when undergraduate students solve derivative problems of the absolute value function can be divided into two types, that are errors in connect to conceptual knowledge and errors in connect to procedural knowledge. Characterization of each type of errors in terms of topics related, that are absolute value function, derivatives and limits. The implications of this study expected the lecturers provide more problem solving that requires undergraduate students to explore mathematical ability as a whole in order to make the mathematical connection as a tool in problem solving.

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