

The Mistakes of Algebra made by the Prep-Year Students in Solving Inequalities

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Abstract: This paper is based on student's performances and explores the mistakes done by the Prep year students taking College Algebra course in Mathematics when finding solutions sets for inequalities. Purpose of this paper is to examine the prep year students of Jubail Industrial College, Al Jubail who have taken college algebra course. The prep year students results are very poor in these basic concepts. They are not successful in solving the problem of inequalities and graphs of the function. The most common mistake done by the students is that they multiply both sides of the inequality.

Key Words: Multiply, Algebra, Inequality, Graph, Function.

I. Introduction

The concept "inequality" and finding a solution set for an inequality is essential in algebra as well as for all the fields of mathematics. Because, the main concern of an algebra includes the concept of "function" and analyzing the properties of these functions. The analysis of some properties of the functions requires the solutions of inequalities. For instance, in

order to find the domain of the functions $f(x) = \sqrt{x^2 + 2x - 2}$ and $g(x) = \log \frac{x}{x+3}$, it is necessary to find the solution sets for the inequalities $x^2 + 2x - 2 \geq 0$ and $\frac{x}{x+3} > 0$.

Therefore, Algebra text books introduce the concept of inequalities before they cover functions and their properties (Auffmann, Baker and Notion 2005, Adams 2003, Thomas & Finney 1996). As these examples show, the performances of students in solving inequalities directly affect their result in algebra courses and so this situation encourages the researcher to work on inequalities topics (Tsamir, Tirosh, Almog 1998; Tsamir and Almog 2001; Tsamir and Bazzini 2004; Linchevski and Sfard 1991; Dreyfus & Eisenberg 1985). On the basis of these considerations, this study aims to analyze the students' performances and explore the mistakes done by them while studying algebra course at prep level.

Method

The subjects of this study are level I of prep year course (attending Math I and Math II at Industrial College at Jubail). This group, who have already been taught to solve inequalities by using algebra and geometric methods, is given a five-question exam.

The exam administered to the subjects

1.	Solve the inequality $x^2 < -x + 30$
2.	Solve the inequality $\frac{3x + 4}{x + 1} \leq 2$
3.	Find the solution set of $ 2 - 3x < 7$
4.	Sketch the graph $y = x - 2 $
5.	Find the solution set of $12 < x + 5 < 19$

During the assessment of this exam, the criteria for the correct answer was —the correct calculation done by the students and hence the correct solution set. The data gathered was presented and interpreted as following.

II. Results and Discussion

Table1 shows the students' performances in the exam given to them.

Table1: Students' Performances

The questions	1st question	2nd question	3rd question	4th question	5th question
The percentages for the correct answers	68	34	52	50	70

Discussion on question#1

68% of the students have given the correct answer to the first question. In other words, 32% of the students were not able to solve the inequality correctly. It was realized that students found the solution set by using wrong factorization.

Sample-1: The solution given by one of the students who fails to give the correct answer to the first question.

The image shows a student's handwritten work for solving the inequality $x^2 < -x + 30$. The student's steps are as follows:

$$x^2 < -x + 30$$
$$x^2 + x < 30$$
$$x(x+1) < 30$$
$$x < 30, \quad x+1 < 30$$
$$x < 29$$

Discussion on question#2

We observed that the 34% of the students answered second question correctly. According to this result we might conclude that a considerable number of students have failed to answer this question correctly. As given in the sample 2.1

Sample2.1.: The solution given by one of the students who fail to give the correct answer to the second question.

Handwritten work for Sample 2.1:

$$\frac{3x+4}{x+1} \leq 2$$

$$\frac{3x+4}{x+1} - 2 \leq 0$$

$$\frac{3x+4-2x+2}{x+1} \leq 0$$

$$\frac{x+6}{x+1} \leq 0 \quad x+6 \leq 0 \rightarrow x \leq -6$$

Here student

- is doing the mistake by not paying attention to negative sign while making the same denominator
- making another mistake while multiplying by $(x+1)$ without paying attention to the sign of $(x+1)$ which is wrong

Discussion on question#3

In the third question, nearly average 52% of the students found the correct solution set. In other words, average 48% of the students was not able to find the correct solution set. These students mostly focused on the negative and positive values of the polynomial $|2 - 3x|$. One student is confused between right and wrong and failed to solve in $|2 - 3x|$. A sample for this mistake was given in sample-3.

Sample-3: The solution given by one of the students who fail to give the correct answer to the this question.

Handwritten work for Sample 3:

③ $|2-3x| < 7$

~~$2-3x < 7$~~

~~$-7 < 2-3x < 7$~~

~~$-9 < -3x < 5$~~

$-3x < 7-2$

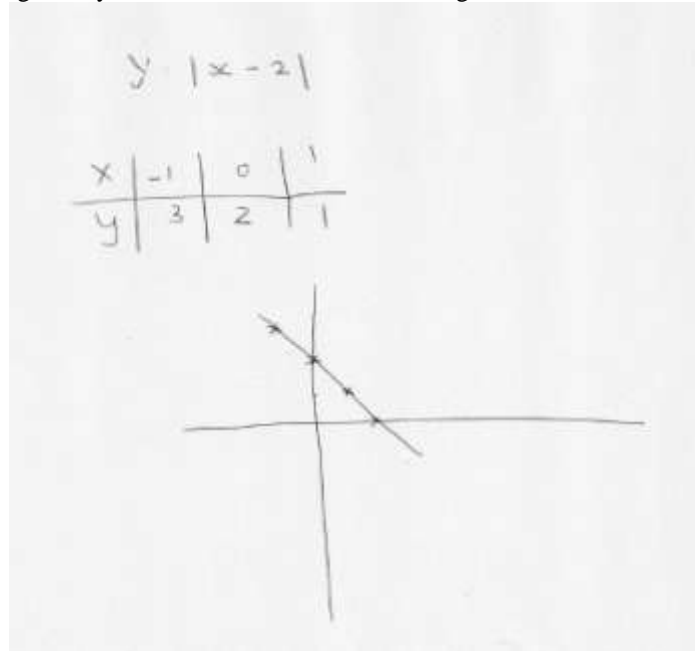
$-3x < 5 \Rightarrow \frac{-3x}{-3} < \frac{5}{-3}$

$x < -\frac{5}{3}$ ✓

Discussion on question#4

The correct solution set for the fourth question was found by 50% of the students. In other words 50% students gave the wrong answer to this question. These students obtained the solution without finding the correct table of x and y. The following sample is explaining this situation.

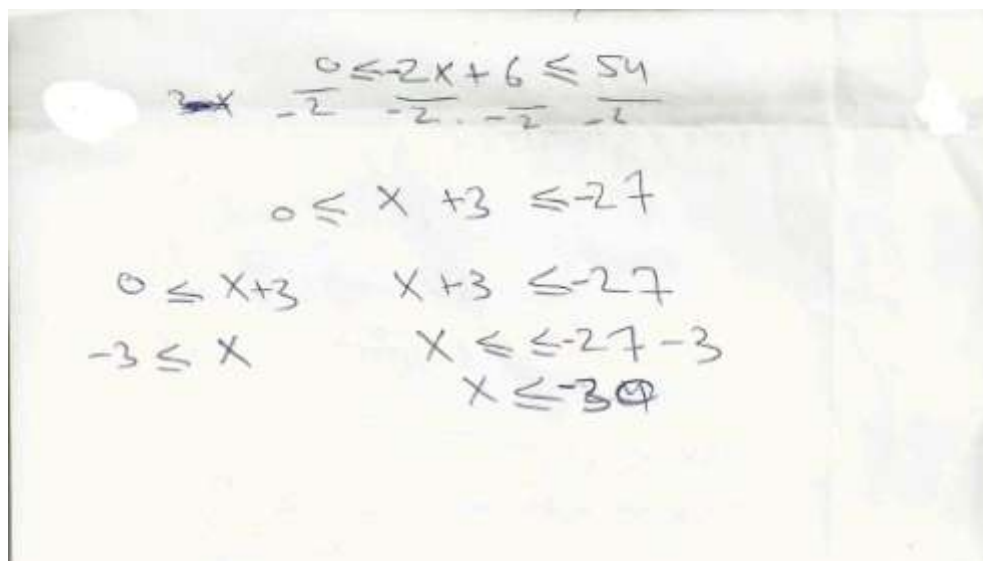
Sample-4: The solution given by one of the students who fails to give the correct answer to the 4th question.



Discussion on question#5

•Almost 70% of the students were successful in finding the correct answer to the fifth question. When compared with other questions, the students have been more successful in this question. The student did not change the sign of the inequality while dividing by the negative number so the most observed mistake in this question was given in sample

Sample5: The solution given by one of the students who failed to give the correct answer to the fifth question.



III. Conclusion / Implication / suggestion

When overall performance of students are considered over this five question test, we cannot say that the students are successful in finding the correct solution set for inequalities and thus it will have adverse effect on their success in Algebra Courses .

In addition , when the solution of all those students who failed to find the correct solution were analyzed , it was observed that the students are wring the solution to the problem as a series of algebraic calculation without considering what the solution set should be. In other words they did not check whether ‘x’ is a real number and the expression in the parenthesis is positive and factoring the polynomial expression haphazardly in the solution set of inequality .

If the students had followed this checking procedure and perform factoring properly, they could havefind the idea to get the correct solution set.

However, building on the knowledge that students draw upon as they make sense of mathematics is an essential aspect of supporting their learning. Teachers can use incorrect responses as a resource to help students to deepen and refine their thinking. Studentmistakes and errors can be used for discussion and analysis of important and challenging mathematical ideas which may be beneficial for all students.

Further studies in students’ mistakes and errors are needed for developing readable textbook and developing curriculum that meets the students’ expectations.

References

- [1]. Auffmann , Barker and Nation(2011). College Algebra & Trigonometry (7th ed.) , Brooks/Cole , 20 Devis Drive , Belmont, CA94002-3098 USA
- [2]. Adams,R.A.(2003).Calculus(5thed.),PearsonEducationCanadaInc.,Toronto,Ontario.AddisonWesley Longman.
- [3]. Dreyfus,T. & Eisenberg, T., (1985),A Graphical Approach to SolvingInequalities, School Science and Mathematics, 85, 651-662.
- [4]. Linchevski,L. &Sfard, A.,(1991),Rules without Reasons as Process without Objects-The Case of Equations and Inequalities,Proceedings of 15th International Conference for the PsychologyofMathematicsEducation, Asisi, Italy, Vol: II, 317-324.
- [5]. Sandor,J., (1997), On Means Generated by Derivatives of Functions, International Journal of Mathematical Educationin Science and Technology,Vol: 28, No:1, 146-148.
- [6]. Thomas, B. G. & Finney, R. L.,(1996),CalculusandAnalyticGeometry (9th ed.), Addison-Wesley Publishing Company.
- [7]. Tsamir, P., Tirosh, D., Almog, N.,(1998), Students’ Solutions of Inequalities, Proceedings of PME 22, Stellenbosch, South Africa, Vol:VI, 129-136.
- [8]. Tsamir, P. &Almog, N.,(2001), Students’ Strategies and Difficulties: The Case of algebraic inequalities,InternationalJournalofMathematicalEducationin Science and Technology,Vol: 32, No:4, 513-524.
- [9]. Tsamir, P. &Bazzini,L.,(2004),Consistencies and Inconsistencies in Students’ SolutionstoAlgebraic’single-value’Inequalities, International Journal of Mathematical Education in Science and Technology