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Effects Of Language And Conceptual Errors In Logarithms Of Numbers On Students' Performance In Mathematics In Public Secondary Schools In Mwala Sub-County, Machakos County, Kenya.

Edward M. Kiilu, David M. Mulwa, Richard Kimiti
1. Master Of Education Candidate, Machakos University
2. Associate Professor, School Of Education, Machakos University
3. Associate Professor, School Of Education, Machakos University

Abstract
The objective of the study was to assess the influence of logarithmic calculation errors on students' performance in mathematics tests, in public secondary schools in mwala sub-county, machakos county, kenya. *The specific objectives were to assess the influence of language and conceptual errors in the logarithms of numbers on students' performance in mathematics tests in mwala sub-county. Newman's theory of problem-solving processes guided this study. A quantitative research methodology was adapted for the study. The study employed a descriptive research design. The target population of the study was 6720 students and 192 mathematics teachers. Using simple random sampling, the study utilized a sample size of 40 mathematics teachers and 672 students. The study collected data from the teachers using questionnaires while the students provided data in completed tests. To check their reliability, the research instruments were piloted using two schools in the yatta sub-county, which borders the mwala sub-county. Data was analyzed with descriptive and inferential statistics using spss version 25. The anova results on the language and conceptual errors had statistically significant p-values ($p=.012^b$ and $p=.004^b$), respectively, indicating their influence on solving logarithm questions in students' mathematics performance from the mwala sub-county. The study recommends that teachers develop a varying approach to teaching that matches the cognitive schema of the students.*

Keywords: *language errors, conceptual error, performance, logarithms, mathematics*

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I. Background Of The Study

Mathematics profoundly impacts all facets of our existence, including social, cultural, interpersonal, economic, and physical lives. Logarithms form the foundation of other topics, such as algebra within mathematics subjects. The application of algebra is quite useful and wide in fields such as engineering and medicine. It has several potential uses, including making comparisons, measuring, making predictions, and providing explanations, illustrations, and interpretations. The measures of how well a student performs are relative to his test-taking efforts. A student's performance is the ratio of his actual output to his benchmark and his absorption of that material.

Conceptual comprehension is "the capacity to know both the facts and the why" (ansah, 2016). Knowing the concepts is more than merely answering the questions on the examination. Developing conceptual comprehension is possible when students can connect what they already know and what they are learning. Conceptual understanding is supposedly fostered via discovery learning. Ansah (2016), explains that conceptual knowledge includes (a) understanding mathematical ideas, (b) performing mathematical operations or processes, and (c) relating mathematical concepts.

Liang & wood (2005) examined how the students understood logic and misconceptions that arose in their work in singapore. The study showed that students were comfortable solving routine calculations but had challenges in problems that required cognitive thinking. Further, the study showed that students' inadequacy in knowledge of logarithms led to over-generalization of algebraic rules.

García flores et al. (2022) investigated how verbal language effectively solved inverse function problems compared to algebraic problems in mexico. The study showed that verbal language was more useful in solving the inverse function than algebraic problems. The study indicated that the language teachers use influences students' performance in the classroom. García flores et al. (2022) acknowledge that using a second language in teaching mathematics makes the students struggle to understand the language itself while simultaneously being required to understand the mathematical problems. This is the case in schools in remote areas where the local language is the common mode of communication. Students tend to read the task and translate it to their mother tongue for encoding, which becomes difficult.

According to bukhari & yakubu (2018), in ghana, students have difficulty solving problems related to the topic of logarithms in mathematics. One of the significant challenges is understanding the concepts of logarithms. Students lack interest in learning the topic as it is deemed not useful. Teachers try to write the problems on the whiteboards to encourage students to go and solve them, but very few are ready to volunteer to work on them. Thus, the topic of logarithms has fallen back to the teacher, who expects students to write, memorize, and solve problems. This has led institutions such as the national council of curriculum and assessment in ghana (ncca) to develop frameworks that take a constructivist approach to teaching logarithm concepts.

Rukangu (2000) investigated the spatial ability development of students in mathematics in nairobi and eastern regions of kenya. The findings showed that language contributes significantly to the performance of students in mathematics. Many students generalize concepts and thus apply them where they are inappropriate, making errors. A case in point is where learners understand that multiplication makes numbers bigger while division makes them smaller; they may choose erroneously to multiply or divide numbers according to their perception of whether the numbers need to get smaller or bigger. The above sources, symbolism, language, and overgeneralization were used to analyze these calculation errors. A student committing a procedural error may do so due to the overgeneralization of rules.

Misconceptions in algebra have been found to impact students' performance. Mulungye (2023) conducted a study in machakos county, kenya, to examine the learning challenges in mathematics. The study highlighted that students were prone to using incorrect rules in solving logarithm problems. This arose mainly due to the generalization in the application of rules. A closer look at the mathematics syllabus in kenya indicates that the topic of logarithms of numbers is delayed until the second year in secondary school (form 2) before it is introduced. The traditional method of teaching logarithms is tough, contributing to the delay. One component is the relevant prior knowledge needed to understand the idea. Several studies, such as kastberg (2002) and liang & wood (2005), have pointed out that students lack the understanding of logarithms due to poor scores in both classes and mathematics as a whole subject.

Therefore, understanding logarithms is quite important as it forms a gateway to other topics in mathematics, such as algebra and arithmetic. The research in machakos county-kenya provided empirical evidence that errors in the logarithm of numbers have contributed to poor performance in mathematics tests.

Hypothesis

H_1 conceptual errors in the logarithm of numbers do not significantly impact students' performance in mathematics tests in public secondary schools in mwala-sub county.

H_2 language errors in the logarithm of numbers do not significantly influence the students' performance of mathematics tests in public secondary schools in mwala sub-county.

Theoretical framework

This study was guided by newman's theory of the problem-solving process (novia & malasari (2023). In this theory, newman proposes the problem-solving process that is classified into five steps: (a) reading- in this stage, the students are required to identify the problem and the symbols used to express it; (b) comprehension- this step involves understanding what has been expressed; (c) transform- in this stage, students are required to find approaches that can solve the expressed problem; (d) processing- this is the stage students go through computing while avoiding errors and (e)encoding- requires the students to write the answer as per the solutions expressed. The study highlighted students' challenges in solving logarithm problems using newman's error analysis. This will give teachers direction on where students' sources of errors are coming from and help them determine how to mitigate them.

II. Review Of Literature

The concept that logarithms are objects that students should grasp first came from (berezovski & zazskis, 2006). Logarithms are just numbers; they don't have to "be finished." this was one of the key points of their framework for teaching the concept to students. For some students, the ability to simplify logarithms has led them to believe that logarithms cannot be anything other than whole numbers or fractions. Hence, they reject this solution as unacceptable when solving equations. Many logarithmic expressions contain irrational numbers, which students do not easily grasp.

Further, williams (2011) found that students are challenged with a notational convention of logarithms and how they should relate to their operation orders (i.e., writing $3\log_3 4$ instead of $\log_3 3x4$) as they possess different meanings. Williams (2011) highlights that the poor performance of students on the topic of logarithms has been the reason students view logarithms as something to "figure out." A logarithm is just a number; for instance, $\log_2 5$ is just an irrational number, not something to "figure out." Thus, errors arise when students try to convert the expression to presentations such as decimal approximations, which do not exist.

Mathematics, in its very nature, is full of abstract representations; therefore, skills, concepts, and facts need to be constructed step by step for successful learning. The study of angle (2007) indicates that a concept is understood when students apply procedural skills correctly. However, this does not mean that a student must memorize formulas and rules; learners should grasp the right concepts. Rules are reinforced through rote learning and practices, and in doing this, teachers move away from the importance of teaching mathematics and leave the students to commit conceptual errors in their mathematical practices.

Conceptual errors involve using incorrect principles/concepts when solving a mathematical problem (ben-hur, 2006). If a student fails to understand a mathematical concept, it becomes obvious that he will be committing errors in performing the operations. For instance, yodiatmana & kartini (2022) noted that students are unable to use the logarithm properties well as follows;

Solve:2log 9+ 4log 3

$=2\log(3^2) + 4 \log 3$

=4log3+4 log 3

= 8 log 3

The above problem indicated that students could not use y= $a\log b^n$ = na logb. The errors will occur continually due to a misunderstanding of the concepts. Similarly, sarwadi & shahrill (2014) echo that misconceptions are systematic errors, not careless ones. With a better understanding of these misconceptions, teachers can understand what challenges our learners face and how we can develop better intervention strategies and appropriate programs to help them.

The study by valero (2010) highlights that a significant number of students never deeply understand a mathematical concept. To understand a concept and apply it, abstraction is related to the intuition and concrete experience of the students. A student must understand the concept's structure and logical relationship for it to be applied correctly. A student should be able to add, subtract, multiply, and divide indices to carry out calculations involving logarithms. For instance, the students have challenges integrating the concept of bodmas in logarithms as follows;

Solve: 2 log 10 − 2 log 5 + log 25

Step (i) = $\log (10)^2 - \log (5)^2 + \log 25$

Step (ii) = $\log = \frac{100*25}{25}$

Step (iii) = log 100

=2

The problem in the above expression is that the student will see the surface structure of the problem without grasping the concept and principles being tested. The study solves the problem of how it fits their cognitive schema, hence the errors (wan bakar & mohd kanafiah, 2020).

When students learn the topic of logarithms, they are expected to broaden the scope of the previously learned concepts into other classes of problems with similar properties and structures. Therefore, the baseline concepts are used on narrow problems, as learners must use the previously abstractions to solve more complex and higher-order logarithm problems. In other words, students should relate the concepts and how differently they are applied to solve a problem. Mathematical concepts in logarithms are similar and require the students to note the key differences in their applications. The findings from valero (2010) indicated that secondary school students need to understand the logarithm of numbers and its related concepts to be ready to solve more complex logarithmic problems at a higher level or subsequent subtopics.

The study by kut (2018) indicated that language errors occur due to various factors. The significant reason for language has been the development of intra-lingual factors. These errors come from overgeneralization, hypothesizing false concepts, simplifying the stated problem, and hypercorrection. These errors indicate the competence of a student at a particular time. Intra-lingual errors have been hugely attributed to mother tongue interference. The errors develop from the students observing "a logarithm problem structure to a semantically similar phrase in their first language." Therefore, language errors come from transferring the first language to another language, such as english, which is dominantly used in mathematics instructions. Transfer of the first language occurs frequently musafir & susiswo (2021); when the student observes a problem in the second language, it manifests similarly to the first language.

Archer et al. (2020) tested the effects of changing six linguistic features of mathematics problems on accuracy and reaction time: sentence consistency, problem theme, word acronym, number of sentences, and pronouns. In their analysis of the general student population's verbal mathematics problem-solving abilities, they failed to find any indication that specific language traits significantly impacted the results. The response time was lowered, nevertheless, because the sentences were consistent.

Rafi & retnawati (2018) tested three groups of students—those with language impairments, those with learning disabilities, and those without special educational needs—on their mathematical competence, linguistic maturity, and capacity to articulate mathematical ideas orally. There were noticeable disparities in quantitative and language-communication skills among the three categories. Nonetheless, the results indicated that those who excelled in mathematics also tended to excel in language arts. Recent reports indicate that socio-emotional skills impact mathematical learning. However, which of these abilities is causal or only a consequence is unclear. Singh & mishra (2015) examined children's growth by testing their mathematical, arithmetic, social-emotional, and linguistic abilities. Language proficiency is a robust indicator of future success in mathematics, science, and the arts, as well as social and emotional development.

Wanjala (1996), in his study of errors in algebra by secondary school students, identified three sources of errors, namely: - symbolism, where there is a fundamental conflict in the teaching of mathematics between a focus on understanding and that involving fluency in symbol manipulation, leading to numerous calculation errors, such as the use of alphabets in place of numbers. Using symbols effectively increases the understanding of mathematics, but most students will likely experience immense difficulties using symbols. Sometimes, learners fail to comprehend the importance of using the right symbols. The language problem is a source of difficulty, and it is related to symbols. According to bruce et al. (2016), errors, especially language problems, are due to reading, comprehension, translation, processing skills, and encoding. A student who fails to comprehend a problem due to communication problems or lack of proper language to translate the text will make errors.

III. Methodology

A quantitative research methodology was adapted for the study. The study employed a descriptive research design. The design was useful in describing the patterns of the data collected following the study objectives. This is similar to mugenda and mugenda (2003), who noted that descriptive research seeks to find information that describes an existing phenomenon.

The target population of the study was 6720 students and 192 mathematics teachers. Using simple random sampling, the study utilized a sample size of 40 mathematics teachers and 672 students. Data from the selected teachers was collected using questionnaires. At the same time, the students took a test on the logarithm of numbers, which was used to evaluate the frequency of commission of the errors and how the different kinds of errors manifest themselves. Piloting of the research instruments was conducted in yatta sub-county, which borders mwala sub-county, and the students' learning conditions and performance on the national examinations were similar. The data was collected and coded in excel software for analysis. The research utilized a descriptive approach in analyzing the data collected to give the variables' patterns under investigation. The data was analyzed using both inferential statistics and descriptive statistics. The findings were presented using statistical approaches such as frequency tables, charts, and anova analysis.

IV. Findings

This section highlights the results from the analysis of the data variables.

Conceptual errors and their influence on students' performance in mathematics

The following results indicate how conceptual errors have influenced students' mathematics performance in tests on the logarithm of numbers across the sampled schools in mwala-sub county.

Table 1. Conceptual errors influence students' performance in mathematics

Response	Frequency	Percent
Disagree	1	2.5
Agree	18	45.0
Strongly agree	17	42.5
Undecided	4	10.0
Total	**40**	**100.0**

Source: field data (2022)

The results in table 1 indicated that 45.0 % of the teachers agreed that conceptual errors affect the students' mathematics performance. This was also supported by 42.5% who strongly agreed that conceptual errors affect students’ performance. There was a significant response from both, who strongly agreed and agreed with the statement indicating that conceptual errors influenced the performance of mathematics.

Hypothesis test

The following hypothesis was tested;

H_1 conceptual errors do not significantly impact students’ performance in mathematics tests on the logarithm of numbers in public secondary schools in mwala-sub county.

Table 2. Anova analysis						
Model		Sum of squares	Df	Mean square	F	Sig.
1	Regression	.778	1	.778	1.340	.004[b]
	Residual	21.472	37	.580		
	Total	22.250	38			
A. Dependent variable: mathematical performance						
B. Predictors: (constant), [conceptual errors influence students' performance in mathematics tests]						

The results in table 2 above showed a p-value of 0.04[b], which is statistically significant. This implies that we reject the null hypothesis and conclude that conceptual errors significantly influence the performance of mathematics tests in public secondary schools in mwala-sub county.

Table 3. Students' findings on conceptual errors

The following were the findings on the commission of the conceptual errors from the students' tests. The students were tested on two questions as follows;

Question 1: use mathematical tables to evaluate;

Wrong workings;

$$\sqrt{\frac{0.0645}{0.0082}} = \sqrt{\frac{6.45 \times 10^{-2}}{8.2 \times 10^{-3}}}$$

$$= \frac{0.8096 \times 10^{-2}}{0.9138 \times 10^{-3}}$$

Correct workings;

No	Log
$0.0645 = 6.46\times10^{-2}$ $0.0082 = 8.2\times10^{-3}$ $\sqrt{\frac{0.0645}{0.0082}}$	$\bar{2}.8096$ $\underline{\bar{3}.9138}$ $\underline{0.8959}$ $\frac{0.8958}{2}$ 0.4979 $\underline{= 2.8}$

In the above example, the students committed a conceptual error by using an inappropriate concept of characteristics. In solving the questions above, the students showed inappropriate subtraction concepts and added negative characteristics. This part of the logarithm $\frac{0.8096 \times 10^{-2}}{0.9138 \times 10^{-3}}$ should be subtracted, not divided. Thus, it should have been -2.8096(-)-3.9138=0.8958. At this stage, the students should multiply the 0.8958 with power ½ to give the square root of the number. This would result in 0.4479, in which the students would find the antilog of the number leading to the correct answer as 2.804 $\times 10^0$=2.8. A significant number of students committed errors by using wrong concept characteristics. The frequency of the conceptual error in question 1 is shown in table 3.

Table 3. How conceptual errors affected students' performance (question 1)

Category	Frequency	Percentage
Committed errors	390	58.04 %
Completed successfully	282	41.96%
Totals	**672**	**100**

Source: field data (2022).

In question 1, the frequency of those who committed the errors was 58.04%, as seen in table 3. This implies that the population of the students committing the error above is high; hence, they are likely to record poor scores in logarithmic tests.

Question 2: simplify;

$$\frac{4^3 \times 4^{-4}}{4^{-1}}$$

$$= \frac{4^{-1}}{4^{-1}}$$

$=4^{-1+1}$

$$4^0 = 0$$

The above showed an inappropriate concept of powers because any number raised to power 0 is 1. This implies that the students have not grasped the concepts underlying the powers of logarithms. The frequency of committing the above errors was indicated as follows:

Table 4. How conceptual errors affected students' performance (question 2)

Category	Frequency	Percentage
Committed errors	423	62.95%
Completed successfully	249	37.05%
Totals	**672**	**100**

Source: field data (2022)

The results in table 4 show that more than half of the selected population (423 of 672) could not compute the concept of powers. Additionally, the following was tested.

Expand the expression; log a (xy)

Student working;

$$Log_a(xy) = xy = Log_a \times +Log_a x$$

Correct working;

$$\text{Log a (xy)} = \log_a x + \log_a y$$

On the expansion concept, the students removed the brackets {ie. Log (xy) =xy} instead of maintaining them and expanding as required (log (xy) =log x+ log y). This implied that the students did not correctly understand the concepts linked to multiplication and division properties in logarithms.

Errors due to language problems and their influence on students' performance in mathematics

The following section indicates the errors that arose from the student's inability to read and understand the mathematical problems correctly.

Table 5. Errors due to language problems influence students' performance in mathematics

Response	Frequency	Percent
Agree	12	30.0
Disagree	3	7.5
Strongly agree	21	52.5
Strongly disagree	1	2.5
Undecided	3	7.5
Total	**40**	**100.0**

Source: field data (2022)

From the results in table 5, the teachers strongly agreed with a response of 52.5% that language errors affect the students' performance. This was also supported by 30% of the teachers who agreed with the question. This implies that many teachers in mwala sub-county agree to a huge extent that the logarithm errors being committed due to language inefficiencies affect mathematics performance. This can be attributed to the location of the schools in areas where the local language is dominantly used.

Hypothesis analysis

The following hypothesis was tested;

H_2 language errors do not significantly influence the performance of students in mathematics tests regarding logarithms of numbers in public secondary schools in mwala-sub county.

Table 6.anova[a]

Model		Sum of squares	Df	Mean square	F	Sig.
1	Regression	.110	1	.110	.182	.012[b]
	Residual	22.986	38	.605		
	Total	23.096	39			
A. Dependent variable: mathematical performance						
B. Predictors: (constant), [errors due to language problems influence students' performance in mathematics]						

From the hypothesis test above, the p-value was p=.012b, which was statistically significant. This implies that we reject the null hypothesis and conclude that language errors influence the performance of mathematics tests in the topic of logarithm of numbers in public secondary schools in mwala-sub county.

Student tests' findings on language errors

The following question was tested to assess students' difficulties in solving the logarithm function.

Find the value of x that satisfies the equation

Question: log (2x-11) - log 2 = log 3-log x

$$\frac{\text{Log } 2x - 11}{\log 2} = \frac{\text{Log } 3}{Log\ x}$$

$$\frac{2x - 11}{2} = \frac{3}{x}$$

$2x^2 -11x=6$

$2x^2 -11x-6=0$

$2x^2 + x -12x - 6 = 0$

X = 6 or x = -½

In the question above, the students did not understand the need to form an "equation" from the test asked. They just canceled the logs, hence leading to the formation of an equation that would help them solve for the value of "x." Further, it can be assumed that the students did not understand "satisfies" when solving the equation.

Table 7, below, indicates the frequency at which the student outcome errors were recorded due to inefficiencies in language.

Table 7. Errors in language committed by students in solving logarithms

Category	Frequency	Percentage
Committed errors	397	59.08 %
Completed successfully	275	40.92%
Totals	**672**	**100**

Source: field data (2022)

From the sample size of 672 students, 397 committed language errors, while 275 completed the tests. The results indicate that the students struggle to grasp concepts in logarithms in mathematics since they have relatively poor mastery of english, which serves as the basis for teaching the subject. For this problem to be alleviated, teachers must emphasize to the students that they should improve their proficiency in english to decode mathematical issues easily.

The study results indicate that conceptual errors greatly influenced mathematics performance in public secondary schools in mwala sub-county. On the other hand, 52.5% of the teachers strongly agree that language errors influence the performance of logarithms. The anova results on the language and conceptual errors had statistically significant p-values ($p=.012^b$ and $p=.004^b$), respectively, indicating their influence on performance in mathematics tests on the topic of logarithm of numbers in public secondary schools in mwala sub-county.

V. Conclusions

The study's findings show that conceptual errors and errors due to language problems significantly influence students' performance in mathematics tests in public secondary schools in mwala sub-county on the topic of logarithm of numbers. Anova analysis for conceptual errors has a p-value of 0.04^b. In comparison, errors due to language problems have a p-value of $0.12^{b,}$ which is statistically significant, showing that they influence students' performance in mathematics.

VI. Recommendations

The study recommended that teachers prepare well and develop a varying approach to teaching concepts of logarithms that match the students' cognitive schema. The study also recommended that student's use of english should be greatly emphasized in school as a means of communication to enhance their comprehension levels when solving mathematics problems. Teachers should conduct more lessons on the topic to assist weak learners and do more practice on the topic to reduce/minimize the errors committed.

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