

Dynamic Analysis Performed On Full-Scale Empty Box Culvert Steel Formwork

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Abstract:

The importance of dynamic analysis in the solution of structural systems is increasing. Dynamic analysis is mandatory for structures other than low-rise and small-sized buildings. Especially in parallel with the advancement in computer technology, the analysis of large structures with thousands of degrees of freedom can be done in a short time with advanced software programs such as SAP2000. The aim of this study is to determine some dynamic parameters of the structural element in question. Parameters such as free vibration frequencies and mass participation ratios are important in dynamic analysis. Furthermore, Ritz vectors and Eigenvectors can be used alternatively in the Mode Combination Method. This study was executed using Finite Element Method employed in SAP2000 software.

Keywords: Prefabricated box culvert formwork, External vibrators, Vibration, Ritz vectors, Eigenvectors,

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I. Introduction

External vibrators are widely used in manufacture of prefabricated concrete members. Vibration transmitted to the concrete formwork by the vibrator's run also vibrates the fresh concrete within the mold.

Wenzel [1] noted that external vibrator vibrations employed for concrete compression in precast structural element generation generally cannot reach a penetration depth greater than 20 cm. Therefore, for sections greater than this, the vibrators must be attached to both sides. It was observed that applying a cyclic frequency of 50 Hz to the mold caused "segregation" due to the large amplitude. This led to the view that higher frequencies should be exposed to the concrete to achieve a good compression impact. Optimal frequencies for this goal were specified to be between 75 and 200 Hz.

Wilson et al. [2] used Ritz vectors and Eigenvectors in Dynamic Mode Combination analysis and applied their comparisons on some example structures.

This study is essentially a theoretical dynamics study. The aim is to find some important dynamic parameters. This theoretical study has been done with the Finite Element Method using SAP2000 [3] computer software.

II. Modeling Of Empty Box Culvert Formwork

The specifications of the external vibrators used in production are given in Table 1.

Table 1. Clamp-on vibrator's properties

| Mechanical Properties | | | | Electrical Properties | |
|-----------------------|-------------------|-------|--------|-----------------------|----------------|
| Vibrat. range | Centrifugal force | | Weight | Max. input power | |
| Vibr./min | kg | kN | kg | W | Max. current A |
| 6000 (200 Hz) | 1157 | 11.34 | 25 | 1200 | 42V 250V 23 - |

The mold is made of steel plate 5 mm in thickness. Steel sections in diverse dimension and section are attached to mold in horizontal, vertical and diagonal ways to power of system.

Mold's plan view and the coordinates of the nodes where dynamic analysis was performed are illustrated for box element in Fig. 1.

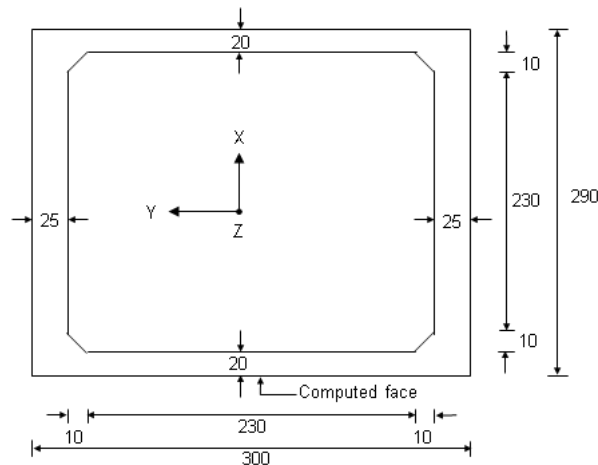


Fig. 1. Plan view of the mold for the box element (h = 97 cm)

III. Theoretical Analysis

Dynamic balance equation of a constructive tract, patterned by finite elements and lumped masses, can be stated dependent on node displacements as

$$\mathbf{M} \ddot{\mathbf{u}}(t) + \mathbf{C} \dot{\mathbf{u}}(t) + \mathbf{K} \mathbf{u}(t) = \mathbf{r}(s, t) \quad (1)$$

In Eq. (1), \mathbf{M} , \mathbf{C} and \mathbf{K} are given $N \times N$ mass, damping and rigidity matrices separately, where N is the number of degrees of freedom of the system. The time-dependent vectors $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$ and \mathbf{u} are the joint accelerations, velocities and displacements separately, \mathbf{r} is the enforced force and dot indicates differentiation with respect to time.

The standard mode-superposition technique of reaction analysis is employed to resolve the dynamic balance equations of movement for the whole construction. The Modes employed in the analysis can be undamped free-vibration modes (eigenvectors) or force-dependent Ritz-vector modes of the network differently.

The force enforced by the external vibrator to form

For most forces in Eq. 1, optional time-changing force can be more factors into a sum of space vectors multiplied by time functions:

$$\mathbf{r}(s, t) = \sum_j \mathbf{f}_j(s) g_j(t) = \mathbf{f}(s) g(t) \quad (2)$$

The arbitrary loading in Eq. 2 is going to be compatible if construction is excited by external vibrators and can be defined as

$$\mathbf{r}(s, t) = \mathbf{f}_0 \sin(\omega t) \quad (3)$$

$$\omega = 2\pi f \quad (3a)$$

$$T = 2\pi/\omega \quad (3b)$$

Where \mathbf{f}_0 , ω and f is fixed centrifugal load (the amplitude of the force), angular and cyclic frequency (100Hz) of the vibrator respectively, $\sin(\omega t)$ is the function of time, and $T(\text{sec.})$ is period. It should be noted that, force enforced by vibrator is vertical to surface of formwork.

External vibrators are connected a rigid steel sheet having sizes of 20×25cm that is constant on face of the form. Uniform pressure loads of shell elements located at this area is obtained by dividing the centrifugal force of the vibrator into this area.

Patterning of empty mold

Formwork is produced of steel plate 5 mm in thick. Steel sections in varied dimension and section are attached to the mold as horizontal, vertical and diagonal to power of network. Strengthening steel sections placed base of formwork are accepted to be simply supported in pattern. Bound requirements of the form for box culvert element are yielded in Table 2.

Table 2. The bound requirements of box culvert form

| The coordinates of nodes where $\mathbf{u} = 0$ (X-Y plane, $Z = 0$) | | | |
|---|--------|--|--|
| Box element | | | |
| X (cm) | Y (cm) | | |
| -75 | -150 | | |
| -5 | -150 | | |

| | | | |
|------|------|--|--|
| 75 | -150 | | |
| -75 | -125 | | |
| -5 | -125 | | |
| 75 | -125 | | |
| 145 | -80 | | |
| 145 | 0 | | |
| 145 | 80 | | |
| 125 | -80 | | |
| 125 | 0 | | |
| 125 | 80 | | |
| 75 | 150 | | |
| -5 | 150 | | |
| -75 | 150 | | |
| 75 | 125 | | |
| -5 | 125 | | |
| -75 | 125 | | |
| -145 | 80 | | |
| -145 | 0 | | |
| -145 | -80 | | |
| -125 | 80 | | |
| -125 | 0 | | |
| -125 | -80 | | |

Shell finite elements are created nearly in dimensions of 10x10cm as quadrilateral, determined by the four joints as square, rectangle or trapeze elements; and triangular, stated by three joints, as transition aimed triangular elements depending on the form's geometry. Frame elements are associated with the shell elements by stating them on identical nodes.

It is accepted that the movement of the form begins from the rest, and then the initial conditions for the form are $(\mathbf{u})_{t=0} = 0$, $(\partial \mathbf{u} / \partial t)_{t=0} = 0$ (4)

Eq. (1) can be solved for \mathbf{u} , exposed to bound requirements (Table 1) and initial requirements (Eq. (4)). The numerical solution of Eq. (1) is made employing structural analysis and design software called SAP2000 [3].

As mentioned previously, the dynamic analysis is carried out only while form empty in order to confirm credibility of the finite element network employed in pattern and techniques employed in analyses.

IV. Implementations

In this section, some theoretical analysis results are presented. In the theoretical analyses, damping rate is selected as %5 for entire modes. Number of modes considered in the analyses is determined so that cumulative sum of modal participating mass ratios in the global X, Y and Z directions exceed %90.

The two and three-dimensional view of box member are displayed in Fig. 2, 4. The material features of steel form are below. Young's Modulus, $E=199948 \text{ N/mm}^2$; specific weight, $\gamma=7.682 \times 10^{-5} \text{ N/mm}^3$; Poisson's ratio, $\nu=0.3$.

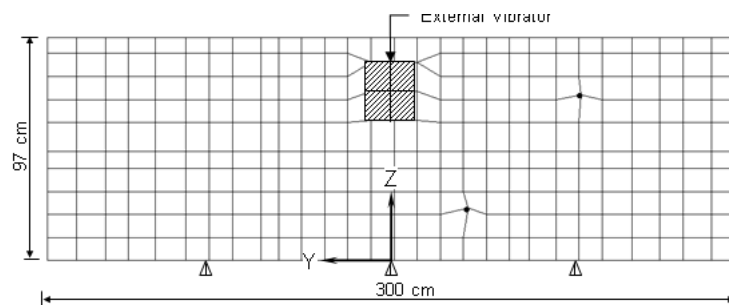


Fig. 2. The view of computed face for box culvert element (Y-Z Plane @ X = -145 cm)

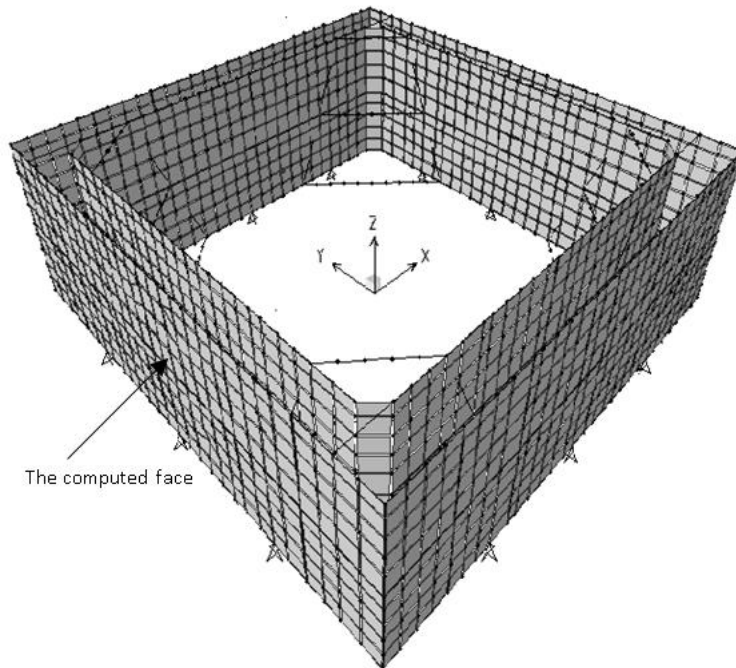


Fig. 3. Three-dimensional view of finite element network for box culvert formwork

Implementation 1

Free vibration analyses of form system for box culvert element are fulfilled. Cyclic frequencies of first six un-damped free vibration modes are shown in Table 3. As demonstrated in the Table 3, the most effective frequencies of systems are far away from cyclic frequency of vibrators used in the dynamic analysis. Therefore, no resonance situation occurs in the possible real system.

Table 3. Free vibration frequencies of the empty box culvert formwork

| Modes | Box culvert Frequency (Hz) | |
|-------|-------------------------------|--|
| 1 | 31.64 | |
| 2 | 41.90 | |
| 3 | 41.99 | |
| 4 | 45.65 | |
| 5 | 49.00 | |
| 6 | 50.89 | |

Implementation 2

In this application, the form system for box culvert element is taken into consideration. Eigenvalue and Ritz Vector analyses of systems are performed employing different numbers of eigenvectors and Ritz vectors alternatively. The modal participating mass ratios obtained from the various types of analyses are compared in Table 4. As seen in the table, the Ritz vector analysis gives more sufficient results in comparison to the eigenvalue analysis.

Table 4. Modal participating mass ratios of the box culvert element

| Number of Modes | Vector Type | Cumulative sum of modal participating mass ratios in direction, (%) | | |
|-----------------|---------------------|---|--------------|--------------|
| | | X | Y | Z |
| 50 | Eigenvectors | 27.81 | 28.94 | 0.01 |
| | Ritz vectors | 92.53 | 93.48 | 98.84 |
| 100 | Eigenvectors | 35.01 | 31.44 | 0.02 |
| | Ritz vectors | 98.30 | 98.08 | 99.63 |
| 150 | Eigenvectors | 52.35 | 50.66 | 0.18 |
| | Ritz vectors | 99.37 | 99.29 | 99.84 |
| 250 | Eigenvectors | 59.84 | 59.89 | 5.50 |
| | Ritz vectors | 99.81 | 99.82 | 99.95 |
| 500 | Eigenvectors | 69.42 | 73.74 | 86.95 |
| | Ritz vectors | ≈100 | ≈100 | ≈100 |
| 650 | Eigenvectors | 80.68 | 81.09 | 92.86 |
| | Ritz vectors | ≈100 | ≈100 | ≈100 |

Implementation 3

The time-history analysis of empty form is established employing 80 Ritz modes. In analysis, as starting Ritz vectors, Load1 (pressure load of vibrator) and acceleration loads (UX, UY, UZ) are used in the global X, Y and Z directions.

In analysis, some parameters found are provided in Table 5.

Table 5. Some vibration parameters of the box culvert element

| Number of mass degrees of freedom | Modal participating mass ratios | | |
|-----------------------------------|---------------------------------|-------|-------|
| | UX | UY | UZ |
| 7353 | 97.14 | 97.10 | 99.42 |

From the results in the table, it can be seen that, taking into account only 80 Ritz modes in the analysis are sufficient to obtain the correct behavior of the empty form system.

V. Results And Discussion

In this study, the computational result is obtained only for empty mold. In theoretical analysis, using of Ritz vectors in any number of modes not only reduces computing time significantly but also provides the greater participating mass ratios when compared to eigenvectors, provided that the same number of modes is used. Hence, for the complex structural systems, which have the mass degrees of freedom of several thousands, it is recommended to be used Ritz vectors in mode superposition method of dynamic analysis.

References

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