Development of Empirical Equations for Queue Management in Vegetable Oil Plants in Imo State

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Abstract: Management of queue in vegetable oil plants, in Imo State, had been going on without the established parametric relationship to enable guided and more effective management. This research paper presents the development of empirical equations for the queue management in vegetable oil plants in Imo State. The plants form an (M/M/2):(∞, FCFS) queue model with consequent assumptions. Mathematical analyses of the queue system established that arrival rate and processing rate of palm kernel in the plants follow a Poisson distribution with the plants' mean palm kernel arrival rate and mean palm kernel processing rate as \( \lambda_m = 22.57 \)T/day and \( \mu_m = 5.74 \)T/day respectively. Characterization of the plant's queue parameters were carried out. Graphically, the average empirical relationship of mass in system, \( L_s \) versus mass in queue (\( L_q \)) and waiting time in queue (\( W_q \)) in the plants are deduced to be \( L_s = L_q + 3.925 \), \( W_q = 0.444 L_q \) respectively. The empirical relation for prediction of waiting time in queue, and queue length proved valid on a test plant where \( \chi^2_{a=0.05} \geq \chi^2 \) criterion for validity holds.

Key word: Equation, management, queue, vegetable oil plant

I. Introduction

Industry is an organized economic activity connected with production, manufacturing, or construction of a particular product or range of products. It is said to be an activity that involves many people, especially one that has become commercialized or standardized

Industry has undergone series of transformation.

Early industries involved manufacturing goods for trade. In medieval Europe, industry became dominated by the guilds in cities and towns, who offered mutual support for the member’s interests, and maintained standard industry workmanship and ethical conduct. Industrial revolution led to the development of factories for large scale production with consequent changes in society. Some manufacturing industries are developed around processing the abundant natural resources harvested or extracted in the area: examples include processing of vegetable oil, canning of fish, processing of fruits and berries, etc.

Vegetable oil industry is one of the process industries. It involves the extraction and processing of oil and fats from vegetable sources. Series of activities and machine operations are involved from preparation of fruits to the packaging of the oil for sale.

When this industry carries out its various activities, that is, processing of the raw materials into finished products, either the machines (servers/customers) or the materials (customers/servers) must wait for some time due to delay in arrival or service time. This leads to queue, that is, a situation where there is idle time for the server or customer to wait for service. In production system, both production and transfer lines have a product-flow layout and are used in mass manufacturing. We denote production lines as flow lines with asynchronous part transfer, while transfer lines have synchronous part transfer. While these lines operate, queue cannot be eliminated due to the rate of processing and rate of arrival of raw materials.

There is need for proper understanding of the queue system and development of models suitable for the management of the plants for minimal loss due to queue.

Many researchers had carried out researches applying queueing theory to firms. In the work on queueing problems with two parallel servers, a group of agents waiting for their job to be processed in a facility was considered. Assuming the same amount of processing time and waiting cost, two parallel servers, will be able to serve two agents at a time. Interest was much in finding the order to serve the agents and the (positive or negative) monetary compensations they should receive. Two rules were introduced for the problem, the minimal transfer rules and the maximal transfer rule in another work, consideration of a relationship between equity and efficiency in queueing problems was made. They showed that under strategy-proofness, anonymity in welfare implies queue-efficiency. They gave a characterization of the equally distributed pair wise pivotal
rule, as the only rule that satisfies strategy-proofness, anonymity in welfare and budget- balance. in their study focused on new measures of performance in single-server Markovian queueing system. These measures depend on the moments of order statistics. The expected value and the variance of the maximum (minimum) number of customers in the system as well as the expected value and the variance of the minimum (maximum) waiting time are presented. Application to an M/M/1 model is given to illustrate the idea and the applicability of the proposed measures. in their work discussed dynamic system performance evaluation in the river port utilizing queuing models with batch arrivals. The general models of the system are developed. This system is modeled by queue with finite waiting areas and identical and independent cargo-handling capacities. The models are considered with whole and part batch acceptance (or whole and part batch rejections) and the inter-arrival and service times are exponentially distributed. Numerical results and computational experiments are reported to evaluate the efficiency of the models for the real system.

In Nigeria, vegetable oil industry’s revolution emerged as a result of 1970-1974 economic plan where industries whose raw materials can be sourced locally are encouraged by restricting the importation of finished products of such raw materials. The raw materials required for the operation of these plants (palm kernel) are sourced randomly and the arriving quantities are undefined. In time of arrival, some of these raw materials are processed and leaving the unprocessed waiting. This cumulates to queue of the raw materials which sometimes take very long time before being processed, and subsequently degenerate to other problems. Some situations occur where there is scarcity of raw materials, the machines lie idle. It is observed that the queue continues to build up, notwithstanding the effort of the operators to manage the system.

It is noted that proper management of queueing problems associated with vegetable oil plants, using empirical equations, had not been established in the State. Although, various operation and production systems are instituted in the industry with their consequent queueing arrangements, the industry is still groping with queue system whose is not clear to the investors. In effect management parameters are not available.

Following this backdrop, most of these plants experience lots of queueing problems resulting in heavy economic losses. The aim of this work is to develop the empirical equation for the queue management in the plants. To enable proper decision that will enhance cost effective operation of the system.

II. Materials And Method

Data Collection:
Five plants, producing crude vegetable oil from Palm Kernel, are considered for study following the existence of queue in them, operation age not less than ten years and expected to have attained steady state operation. They are tagged plants M, N, O, P and Q.

- The following data were sourced:
  **Mass of palm kernel arrived per day.**
  This was generated by measuring the mass of the palm kernel that arrived the company’s store per day. This is repeated for 60 days.
  **Mass of palm kernel processed per day.**
  Knowing the capacity of the reservoir/hopper, the quantity of raw material processed per day of operation is ascertained by subtracting the remaining quantity (mass) from the initial mass. This was repeated for 60 days.
  **Number of machine(s) in operation per day;** for 60 days.

Method of data analysis
The data collected, i.e., mass of palm kernel arriving per day (feed rate), mass of palm kernel processed per day and number of machines engaged per day were statistically analyzed. The mean distribution of the data were evaluated using the equation:

\[ \text{Mean, } \lambda = \frac{\sum f x}{N} \]  \hspace{1cm} 2.1

where \( f \) = frequency; \( x \) = variable; and \( N \) = sum of frequency.

Variance, \( \sigma^2 = \frac{\sum (x_i - \lambda)^2}{N} \)  \hspace{1cm} 2.2

Standard deviation, \( \sigma = \sqrt{\frac{\sum (x_i - \lambda)^2}{N}} \)  \hspace{1cm} 2.3

The mean distribution of the data was evaluated for unitized masses of arrival and processed palm kernel. This was done using the chi square equation given thus:

\[ \chi^2 = \frac{(o_i - e_i)^2}{e_i} \]  \hspace{1cm} 2.4

Where \( o_i \) = observed frequency; \( e_i \) = expected frequency; \( \alpha \) = significance level chosen = 0.05. Also, \( v \) = degree of freedom

Critical region: \( \chi^2_u \geq \chi^2 \), we cannot reject that the data is from a Poisson distribution.
Ascertaining that the data is from the presumed Poisson distribution for the process system, and examining the other queue system characteristics as well as the arrangement/capacities of production facilities (Hopper feeding the machines and the machines processing the palm kernel), the queue model was defined. With the model definition, the related queueing mathematical model was adopted for use in the prediction of the queue performance characteristics of the process system. This was established for the industry considering the average operation characteristics of the plants.

From the performance characteristics, of the queue system, obtained, graph showing the mathematical relationship between the number of palm kernel units in the queue, Lq, and number of units of palm kernel in the system, Ls, and waiting time in queue, Wq, were obtained. Considering graph of Ls against Lq:
Gradient, \( a_1 = \frac{\Delta L_s}{\Delta L_q} \).
Limit as \( \Delta L_q \) tends to zero \( \frac{\Delta L_s}{\Delta L_q} = \frac{dL_s}{dL_q} = a_1 \)
dLs = \( a_1 \)dLq

Integrating equation (3i) i.e., \( \int (dL_s) = \int a_1 dL_q + b_1 \).

Where \( b_1 \) = constant of integration = intercept on Ls-axis.

\( L_s = a_1 L_q + b_1 \) 2.5

Considering graph of Wq against Lq:
Gradient, \( a_2 = \frac{\Delta W_q}{\Delta L_q} \).
Limit as \( \Delta L_q \) tends to zero \( \frac{\Delta W_q}{\Delta L_q} = \frac{dW_q}{dL_q} = a_2 \)
dWq = \( a_2 \)dLq

Integrating equation (3ii) i.e., \( \int (dW_q) = \int a_2 dL_q + b_2 \).

\( W_q = a_2 L_q + b_2 \) 2.6

Where \( b_2 \) = constant of integration = intercept on Wq-axis.

NB: \( a_1, a_2 \) are gradients; \( b_1, b_2 \) are intercepts on vertical (Ls-, Wq- ) axis.
These are equivalent to \( y = ax + b \) (equation of a straight line).
With these, the unique relationship of Lq with Wq and Ls, for the plants, and the industry were established.

### III. Results And Discussion

#### Table 1: Mean distributions of the mass of palm kernel arriving and mass processed per day.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Mean mass of palm kernel arriving per day (( \bar{m}_\lambda ))</th>
<th>Mean mass of palm kernel processed per day (( \bar{m}_\mu ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>45.6</td>
<td>42.45</td>
</tr>
<tr>
<td>N</td>
<td>52.95</td>
<td>47.78</td>
</tr>
<tr>
<td>O</td>
<td>32.12</td>
<td>20.95</td>
</tr>
<tr>
<td>P</td>
<td>27.28</td>
<td>22.78</td>
</tr>
<tr>
<td>Q</td>
<td>25.28</td>
<td>21.78</td>
</tr>
</tbody>
</table>

The mean distributions of the data when unitized/ discretized, considering the initials as zero variable, is presented in table 2. Also, using the same equation 2.1, and table 2, the unitized industry’s mean distributions of data is presented in table 2a.

#### Table 2: Mean distributions of the mass of palm kernel arriving and mass processed when unitized

<table>
<thead>
<tr>
<th>Plant</th>
<th>Mean mass units of palm kernel arriving per day (( \bar{m}<em>{\lambda</em>{\text{unit}}} ))</th>
<th>Mean mass units of palm kernel processed per day (( \bar{m}<em>{\mu</em>{\text{unit}}} ))</th>
<th>Mean number of machines processing per day (( \bar{s} ))</th>
<th>Mean rate of processing per machine unit per day (( \bar{\mu}_{\text{unit}} ))</th>
<th>( \lambda_{\text{industry}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1.47</td>
<td>1.10</td>
<td>5</td>
<td>0.22</td>
<td>6.682</td>
</tr>
<tr>
<td>N</td>
<td>2.80</td>
<td>2.28</td>
<td>5</td>
<td>0.46</td>
<td>6.087</td>
</tr>
<tr>
<td>O</td>
<td>2.72</td>
<td>2.20</td>
<td>3</td>
<td>0.73</td>
<td>3.699</td>
</tr>
<tr>
<td>P</td>
<td>1.97</td>
<td>1.93</td>
<td>3</td>
<td>0.64</td>
<td>3.078</td>
</tr>
<tr>
<td>Q</td>
<td>2.30</td>
<td>1.63</td>
<td>2</td>
<td>0.82</td>
<td>2.805</td>
</tr>
</tbody>
</table>
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Table 2a: The industry’s mean distribution of the operation characteristics.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Mean mass units of palm kernel arriving per day ( (\lambda_b) )</th>
<th>Mean number of machines processing per day ( (s_a) )</th>
<th>Mean rate of processing per machine unit per day ( (\mu_a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vegetable oil industry in Imo state</td>
<td>2.252</td>
<td>3.6</td>
<td>0.574</td>
</tr>
</tbody>
</table>

Fitting into distribution

Using the values of the unitized mean distribution of the data presented in table 2 above, the fitting into the Poisson distribution was carried out using the chi square test. See equation 2.4 and its decision criterion. The 95 percent confidence interval was chosen (i.e., at a significance of 0.05). The table 3a and 3b show the results obtained in a plant. Similar results were obtained in other plants.

Table 3a: Fitting a Poisson distribution to the mass of palm kernel arriving per day (mean distribution, \( \lambda_b = 2.8 \)).

<table>
<thead>
<tr>
<th>Mass of kernel arriving per day (Tonnes)</th>
<th>Number of variable, ( x_i = m_i ) (new equivalence, unitized)</th>
<th>( \Pr(x_i \text{ occurring}) )</th>
<th>Observed frequency ( (o_i) )</th>
<th>Expected frequency ( (e_i) )</th>
<th>( (o_i-e_i)/e_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.0 – 29.9</td>
<td>0</td>
<td>0.061</td>
<td>4</td>
<td>4</td>
<td>0.0000</td>
</tr>
<tr>
<td>30.0 – 39.9</td>
<td>1</td>
<td>0.170</td>
<td>12</td>
<td>10</td>
<td>0.4000</td>
</tr>
<tr>
<td>40.0 – 49.9</td>
<td>2</td>
<td>0.238</td>
<td>12</td>
<td>14</td>
<td>0.2857</td>
</tr>
<tr>
<td>50.0 – 59.9</td>
<td>3</td>
<td>0.223</td>
<td>13</td>
<td>13</td>
<td>0.0000</td>
</tr>
<tr>
<td>60.0 – 69.9</td>
<td>4</td>
<td>0.156</td>
<td>9</td>
<td>9</td>
<td>0.0000</td>
</tr>
<tr>
<td>70.0 – 79.9</td>
<td>5</td>
<td>0.087</td>
<td>5</td>
<td>5</td>
<td>0.0000</td>
</tr>
<tr>
<td>80.0 – 89.9</td>
<td>6</td>
<td>0.041</td>
<td>3</td>
<td>2</td>
<td>0.5000</td>
</tr>
<tr>
<td>90.0 – 99.9</td>
<td>7</td>
<td>0.016</td>
<td>2</td>
<td>1</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

\( \chi^2 = 2.1857 \). \( \nu = k-1-m = 8-1-1 = 6 \). Therefore, \( \chi^2_{0.05} = 12.592 \)

Crt. \( \chi^2_{0.05} \geq \chi^2 \), we cannot reject that the sample is from Poisson distribution.

Table 3b: Fitting a Poisson distribution to the mass of palm kernel processed per day (mean distribution, \( \mu_{ab} = 2.3 \)).

<table>
<thead>
<tr>
<th>Mass of kernel processed per day (Tonnes)</th>
<th>Number of variable, ( x_i = m_i ) (new equivalence, unitized)</th>
<th>( \Pr(x_i \text{ occurring}) )</th>
<th>Observed frequency ( (o_i) )</th>
<th>Expected frequency ( (e_i) )</th>
<th>( (o_i-e_i)/e_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.0 – 29.9</td>
<td>0</td>
<td>0.100</td>
<td>4</td>
<td>6</td>
<td>0.6667</td>
</tr>
<tr>
<td>30.0 – 39.9</td>
<td>1</td>
<td>0.231</td>
<td>12</td>
<td>14</td>
<td>0.2857</td>
</tr>
<tr>
<td>40.0 – 49.9</td>
<td>2</td>
<td>0.265</td>
<td>18</td>
<td>16</td>
<td>0.2500</td>
</tr>
<tr>
<td>50.0 – 59.9</td>
<td>3</td>
<td>0.203</td>
<td>18</td>
<td>12</td>
<td>3.0000</td>
</tr>
<tr>
<td>60.0 – 69.9</td>
<td>4</td>
<td>0.117</td>
<td>5</td>
<td>7</td>
<td>0.5714</td>
</tr>
<tr>
<td>70.0 – 79.9</td>
<td>5</td>
<td>0.054</td>
<td>3</td>
<td>3</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

\( \chi^2 = 4.7738 \). \( \nu = k-1-m = 6-1-1 = 4 \). Therefore, \( \chi^2_{0.05} = 9.488 \)

Crt. \( \chi^2_{0.05} \geq \chi^2 \), we cannot reject that the sample is from Poisson distribution.

The mathematical model of queue analysis based on Poisson distribution adopted for the plants implied that the arrangement of machines (servers) and the mass arriving in the system, multiple but identical server arrangement exists (i.e. \( M_1/M_2/s \)). Single queue formation, constant service rate and infinite population with strict queue discipline (F-C, F-S) also prevail. Where \( M_1 = \) Marcovianarrival rate distribution, \( M_2 = \) Marcovian service rate distribution, and \( s = \) number of servers

In this model, let

- \( P_s = \) the probability that there are \( n \) units in the system,
- \( L_s = \) expected number of calling units in the system,
- \( W_q = \) expected waiting time a customer spends in the queue, \( s = \) number of servers,
- \( \lambda = \) expected arrival rates of calling units, \( \mu = \) expected service/processing rates per busy server,
- \( 1/\lambda = \) expected inter arrival time, \( 1/\mu = \) expected service/processing time,
- \( \gamma = \) utilization factor for the service facility \( = \lambda/\mu \).

With these notations and definitions of performance measures, the model equations stated below were used to predict the performance characteristics (queue characteristics).

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\[ P_0 = \left[ \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n \right]^{-1} \]

\[ L_q = \frac{1}{(1-\lambda)} \cdot \left( \frac{\lambda}{\mu} \right) \cdot P_0 \]

\[ L_s = L_q + \frac{1}{\mu} \]

\[ W_q = \frac{1}{(1-\lambda)} \cdot \left( \frac{\lambda}{\mu} \right) \cdot P_0 = \frac{L_q}{\lambda} \]

\[ W_s = W_q + \frac{1}{\mu} = \frac{L_q}{\lambda} + \frac{1}{\mu} \]

\[ Y_s = \frac{\lambda}{\mu} \]

Average performance characteristics of the industry

Considering the mean distribution of the mean arrival rate of palm kernel and processing rate of the machines in the sampled plants, as presented in table 2a, the mean performance characteristics of the industry (vegetable oil processing industry) in Imo state were evaluated and presented in table 4

<table>
<thead>
<tr>
<th>(s)</th>
<th>((\bar{\lambda} ))</th>
<th>((P_0 ))</th>
<th>((L_q ))</th>
<th>((L_s ))</th>
<th>((W_q ))</th>
<th>((W_s ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–3</td>
<td>(\frac{\lambda}{\mu} \geq 1)</td>
<td>0.0019</td>
<td>0.0019</td>
<td>0.0019</td>
<td>0.0019</td>
<td>0.0019</td>
</tr>
<tr>
<td>4</td>
<td>0.981</td>
<td>0.0146</td>
<td>1.9131</td>
<td>5.8451</td>
<td>0.850</td>
<td>2.592</td>
</tr>
<tr>
<td>5</td>
<td>0.785</td>
<td>0.0182</td>
<td>0.5030</td>
<td>4.4260</td>
<td>0.223</td>
<td>1.965</td>
</tr>
<tr>
<td>6</td>
<td>0.654</td>
<td>0.0193</td>
<td>0.1589</td>
<td>4.0819</td>
<td>0.071</td>
<td>1.813</td>
</tr>
<tr>
<td>7</td>
<td>0.560</td>
<td>0.0196</td>
<td>0.0515</td>
<td>3.9745</td>
<td>0.023</td>
<td>1.765</td>
</tr>
<tr>
<td>8</td>
<td>0.490</td>
<td>0.0198</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

Relationship between number of units of palm kernel in the queue (\(L_q\)) and: waiting time in queue (\(W_q\)), and number of units of palm kernel in the system (\(L_s\))

Using the table of the calculated performance characteristics of the industry’s average. The graphical representations are presented in figure 1

![Graph showing relationship between queue length and waiting time](image)

Figure 1: Relationship between expected queue length (\(L_q\)), and: waiting time in queue (\(W_q\))

(W\(q\)) and number of units of palm kernel in the system (\(L_s\)), for the industry.
The graph shows straight line relation, i.e., \(y = ax + b\) where \(a = \text{gradient}\) and \(b = \text{intercept on y-axis}\). Hence the empirical equivalent equations are \(L_s = a_1L_q + b_1\); \(W_q = a_2L_q + b_2\). i.e., equations 2.5 and 2.6

From the graphs, the values of “\(a\)” and “\(b\)” for industry’s average are deduced as: \(a_1 = 1\) and \(b_1 = 3.9252\); \(a_2 = 0.444\) and \(b_2 = 0.0002\).

Therefore, substituting the deduced values of “\(a\)” and “\(b\)” into the empirical relationship of \(L_q\) with \(W_q\) and \(L_s\) for the industry, the following empirical equations result:

\(L_s = L_q + 3.9252\)

\(W_q = 0.444L_q + 0.0002 = 0.444L_q\)

NB: The intercept on \(W_q\) is neglected, since infinitesimal
Application of the result to plant for validation

Plant, H, was selected for the application. The data observed and measured are waiting time in queue and queue length. They are presented in table 5a below.

In comparison, the data of the observed and predicted waiting time in queue and queue length are presented in table 3b and depicted in figures 2.

<table>
<thead>
<tr>
<th>Observed queue length, Lq</th>
<th>Observed waiting time, Wqo</th>
<th>Predicted waiting time, Wqpred</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.74</td>
<td>0.888</td>
</tr>
<tr>
<td>4</td>
<td>1.55</td>
<td>1.776</td>
</tr>
<tr>
<td>4</td>
<td>1.62</td>
<td>1.776</td>
</tr>
<tr>
<td>5</td>
<td>1.98</td>
<td>2.22</td>
</tr>
<tr>
<td>6</td>
<td>2.35</td>
<td>2.664</td>
</tr>
</tbody>
</table>

The gradient of the observed and predicted relationship of the queue length and waiting time in queue are 0.404 and 0.444 respectively.

Test for correctness of result

Using the data presented in table 5b, the correlation between the predicted and observed waiting time in queue was established in figure 3 below. The coefficient of correlation, $R^2$ was found to be 0.997 (i.e., > 0.8), hence the result is correct.

For the industry (vegetable oil plants in Imo state), considering figure 1, the average relationship existing between $L_q$ and $L_s$, as well as $L_q$ and $W_q$ are straight line, that is direct and proportional. The intercept on the $L_s$-axis is 3.9252. This imply that the minimum number of processing units that must be engaged for steady state operation for the industry or the utilization factor. The gradient of $W_q$ vs. $L_q$ for the industry is 0.444days which imply the inter-arrival time. The intercept on $W_q$-axis is approximated zero since value is very small. In effect, the empirical equation relationships for the industry are thus:

\[ L_s = L_q + 3.9252 \]  
\[ W_q = 0.444L_q. \]
Generally, for the industry, Ls tend to Lq as the utilization factor tends to zero or the number of processing units for steady state tends to zero. Also, Wq proportionally varies with Lq in the industry, notwithstanding the value of the inter-arrival time. All the same, waiting time in queue and quantity of palm kernel in the system can be predicted, with knowledge of queue length.

Application of the result to plants for validation
Considering the result obtained and presented in table 5b, it is observed that there is very small variation in the waiting time predicted using the result of the evaluation and that based on observation. Also, the accuracy of prediction increases as the queue length decreases, i.e., as one moves away from exploded queue environment. Furthermore, the coefficient of correlation $R^2 = 0.997$. This implies that the result of the evaluation is correct and can be used to predict the waiting time in queue as well as queue length of the queue system of vegetable oil plants in Imo state, for management decisions.

V. Conclusion
This work is a case study of palm kernel vegetable oil producing plants in Imo State. From the equation $\frac{\lambda_m}{\mu_m} < 1$, the steady state operation zone for the industry starts with 4 machines, and minimum of 5 machines are required to operate in a stable zone of zero waiting time in queue.

The relationship of Ls versus Lq, and, Wq versus Lq are direct and proportional relations with the empirical equations as

\[ L_s = L_q + 3.9252 \]
\[ W_q = 0.444 \]

References