

Lagrangian Coherent Structure Analysis of Jellyfish Swimming Using Immersed Boundary FSI Simulations

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Abstract : In this paper, medusan swimming in a quiescent fluid flow environment is simulated via a two-way nonlinear fluid-structure interaction (FSI) technique with the aid of immersed boundary method (IBM). In this regard, incompressible Navier-Stokes equations coupled with Lagrangian interaction equations between fluid and immersed structure are solved using IB2d code. For the simulations, immersed jellyfish bell membrane is modeled in a spring fiber built-in case available in the code. Afterwards, Lagrangian coherent structures (LCS) of the flow field are extracted to capture transport barriers in the case of jellyfish swimming. These hidden structures depict formation of distinct regions in the animal wake, including starting and stopping vortices.

Keywords: LCS, Jellyfish Swimming, FSI, IBM, Bionics

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I. Introduction

Deflection is a key mechanism to efficiently generate propulsive force for animal swimming and flying, as observed in birds and fishes, like cetacean with flexible flukes and jellyfish with deformable bell, to name a few [1]. In the case of medusan (jellyfish) swimming, animal utilizes cyclic bell deformation via muscle power, which ejects propulsive vortices behind the animal to move in the fluid environment [2]. In general, motion of flexible bodies in fluid field can be mathematically formulated as a fluid-structure interaction (FSI) problem. To simulate FSI problems efficiently, immersed boundary method (IBM) is a powerful technique, in which equations of fluid flow motions, i.e. Navier-Stokes equations, are solved coupled with equation of immersed boundary in a two-way approach. In other words, solution of flow field modifies immersed structure deformation solution and vice versa as time proceeds. As an immediate result, immersed boundary deformation or structure shape in terms of time is an output of the problem (not a prescribed input). In this paper, 2D swimming of a medusan is simulated by an open-source fully-coupled IBM code, namely IB2d [3, 4]. Ejections of starting/stopping vortices in jellyfish swimming are captured by the simulation. In addition, time history of jellyfish's bell shape is obtained as an output of the adopted two-way coupled IBM simulation, and is not set a-priori. On the other hand, Lagrangian coherent structure (LCS) concept, as a useful dynamical-system concept, is utilized in this study to extract fluid flow convective barriers in the flow field [5]. Analysis depicts that transport barriers are generated in the animal wake. Details are presented in the following.

II. Numerical Immersed Boundary FSI Simulation

Flow field generated by a medusan is simulated here using a fully-coupled FSI IBM code, i.e. IB2d, developed by Batista et al. at University of North Carolina [3, 4]. In the technique, FSI problem is treated by a mixed Eulerian-Lagrangian approach; in which flow field is numerically solved in an Eulerian point of view, while the immersed boundary is described by a Lagrangian description. The code is basically based on Peskin's immersed boundary method [6]. Mathematically, 2D incompressible Navier-Stokes equations in Eulerian form are formulated by the following equations [3, 4]:

$$\rho \left(\frac{\partial u_i(x,t)}{\partial t} + u_j(x,t) \cdot \frac{\partial u_i(x,t)}{\partial x_j} \right) = - \frac{\partial p(x,t)}{\partial x_i} + \mu \frac{\partial^2 u_i(x,t)}{\partial x_j \partial x_j} + f_i(x,t) \quad (1)$$

$$\frac{\partial u_i(x,t)}{\partial x_i} = 0 \quad (2)$$

where ρ and μ is fluid density and viscosity, respectively. In addition, $f_i(x,t)$ is the body force imposed onto the fluid by the immersed boundary in Lagrangian form and stands for fluid-structure interaction. In the

technique, aforementioned term is formulated using an integral form of a force density function $F_i(\xi, t)$ that can be evaluated by fiber component modeling of the immersed structure [3, 4]. Mathematically, effect of immersed boundary on the fluid and vice versa is included using the following formula:

$$f_i(x, t) = \int F_i(\xi, t) \delta(x - X(\xi, t)) d\xi \quad (3)$$

where δ and X is a delta function and material point at time t labeled by Lagrangian parameter ξ on the curvilinear immersed boundary, respectively. Different fiber models are available in IB2d code, including: Hookean and non-Hookean springs, torsional springs, target points, mass points, porosity and muscle-fluid-structure models [4]. For jellyfish swimming here, immersed boundary is made of a set of Hookean springs which resist stretching; in this case, force density function at point k is evaluated as below:

$$F_i(\xi, t) = -\frac{\partial E_{spring}}{\partial X_k^i} = -\frac{\partial}{\partial X_k^i} \left[\frac{1}{2} k_{spring} \left(\|X_{spring}^k\| - L_R^k \right)^2 \right] \quad (4)$$

where E_{spring} , k_{spring} , X_{spring}^k and L_R^k is immersed boundary elastic deformation energy, spring stiffness, spring stretched/compressed length and spring resting length, respectively. Finally, to impose no-slip boundary condition, immersed boundary velocity at each point is set equal to the local fluid velocity [3, 4]. IB2d code also assumes periodic condition at boundaries [4]. The code has been successfully adopted in different flow simulation applications, like flexible beam [4], elastic tube [3], falling sphere under gravity in pulsatile flow [4] and idealized swimmer [4]. The code has been validated with comparison to PIV experimental data available for insect wing moving laterally in the flow field [4].

III. Jellyfish Geometry & Simulation Settings

In this paper, to extract Lagrangian coherent structures generated by jellyfish swimming in a quiescent flow environment, a parameterized built-in case in IB2d is utilized [3, 4]. Fig.1 shows geometry of the jellyfish adopted here in meter; converted from IBAMR code by Batista [3, 4] based on jellyfish geometry from courtesy of Hoover et al. [7, 8]. The jellyfish has unity fitness ration (i.e. ratio between bell height and diameter) with large size which possess high efficiency swimming via combined jet-paddling propulsion [9]. The jellyfish's immersed bell structure is constructed by 10^3 springs with spring constant 10^7 N in the case. In addition, cyclic contraction/ relaxation of the jellyfish muscles are applied via an absolute value of a sinusoidal variation for the resting length of the constructing springs as default $|\sin(2\pi t f)|$, where $f \approx 0.9$ is the frequency of oscillations [3, 4]. Domain size is set as default 2×8 m² with 128×512 grid resolution in x and y directions, respectively. The flow field is simulated for a total time equals to 2.5 seconds with a time step equals to 10^{-5} s. Results are frequently plotted and saved for post-processing purposes every 0.025 s.

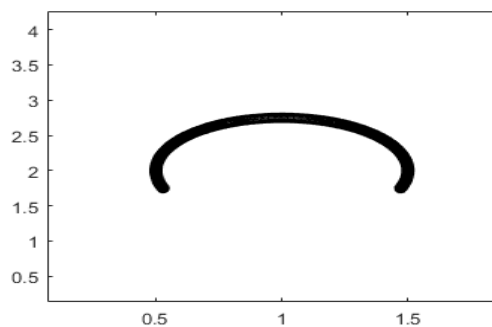


Figure 1. Jellyfish geometry adopted for simulations

IV. Jellyfish Swimming

By applying jellyfish bell muscle sinusoidal contraction/relaxation, jellyfish swims in the quiescent fluid flow environment starting from $t = 0$ s, when flow field is at rest. As time proceeds, jellyfish is propelled upward by ejecting stopping/ starting vortices behind the animal as seen in vorticity plots in Fig. 2. As mentioned earlier, deformation shape of the jellyfish immersed structure is not set a-priori in the simulation; rather resting length of the constructing springs of the jellyfish bell is only imposed as an undulatory motion. As a result, variation of the jellyfish bell shape in terms of time is obtained via purely FSI interplay between fluid and solid immersed jellyfish membrane deformation solutions. As also seen in Fig. 2, cyclic bell contraction

motion is not uniformly repeated as time proceeds to 2.5 seconds; this behavior is due to the complicated nonlinear nature of the present FSI problem.

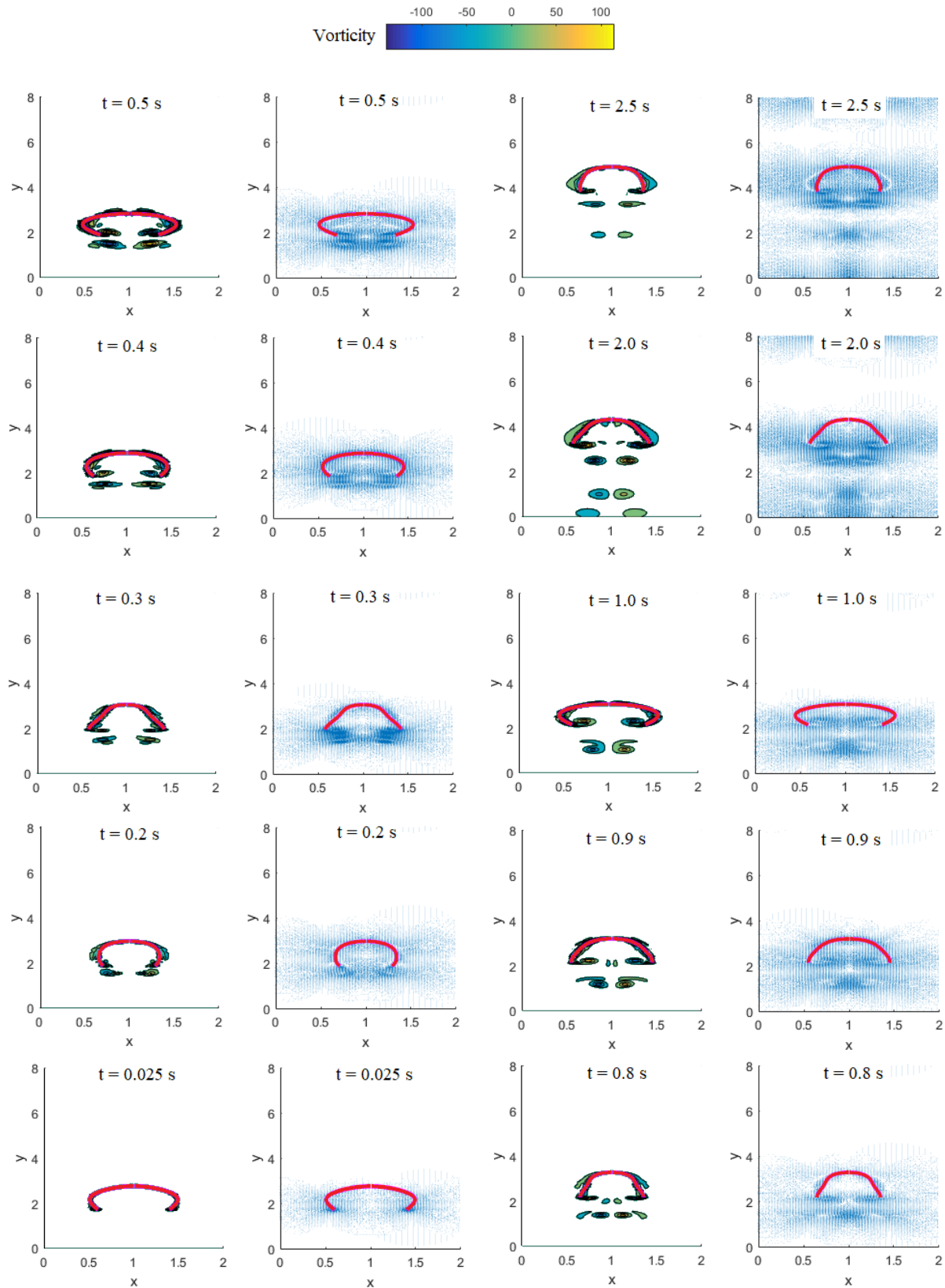


Figure 2. Velocity vector field and vorticity plots of jellyfish swimming at different time instants using IBM-FSI simulation

Fig. 2 also shows velocity vector field around the jellyfish as time proceeds; as one can see in the figure, jellyfish muscle contraction/relaxation generates induced velocity in the quiescent flow and propels the jellyfish upward. It is also clear in Fig. 2, for example at $t= 2.0$ s, sets of two vortices with different vorticity sign (positive and negative) are generated on the jellyfish immersed bell membrane wall (attached vortices) and then ejected into the downstream wake (detached vortices). Similar experimental observations have been made in the literature [2, 10]. To observe more details, Fig.3 shows vorticity field superimposed by velocity vector field in a zoomed area around the jellyfish; as one can see in the figure, vortices with positive (red) and negative (blue) vorticity are generated, detached and convected downstream consequently. It is also visible in Fig. 3 that two regions of positive and negative vorticities are generated close to the jellyfish bell surface on the top.

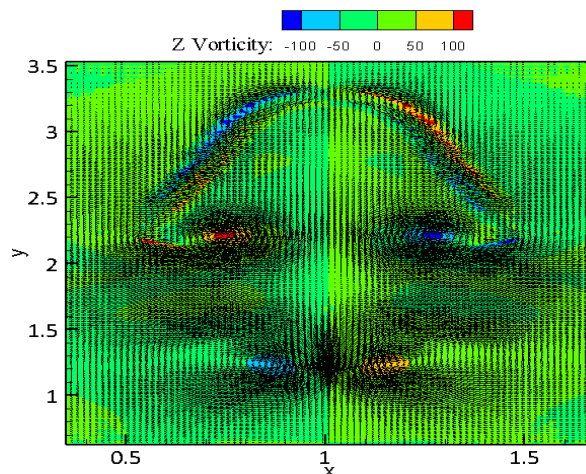


Figure 3. Velocity vector and vorticity fields of jellyfish swimming at $t= 0.875$ s

V. Lagrangian Coherent Structures

Lagrangian coherent structures (LCS) are long-lived material structures in the flow field that identify flow regions with distinct fluid particle fates. To find LCS in time-dependent fluid flow systems, finite-time Lyapunov exponent (FTLE) measure can be adopted which quantifies maximum divergence of trajectories over a given period of time [5]. LCS can be identified as ridges of FTLE field with high FTLE values. In this paper, FTLE fields are computed using a computational LCS code developed by Dabiri et al. at Caltech [11]. In this regard, 2D velocity field database for all time instants (100 time steps, with 0.025 s time interval) obtained from IB2d code in ‘*vtk*’ format [12] are restructured to a special format necessary to feed the computational LCS code by a developed in-house code here. Mathematically, FTLE is defined as below:

$$\xi_{(X,t_0,T)}^{FTLE} = \frac{1}{|T|} \ln \left\| \frac{\delta \Phi_{t_0}^{t_0+T}}{\delta X_{t_0}^{t_0+T}} \right\| \quad (5)$$

where the term $\delta \Phi_{t_0}^{t_0+T}$ stands for differential distance between stretched points in FTLE field in the time interval, T [5]. For FTLE field calculation, time integration can be performed both in forward and backward fashions, corresponding to repelling LCS (stable manifolds) and attracting LCS (unstable manifolds), respectively [5]. In the case of repelling LCS, two fluid particles close to LCS placed on different sides of the stable manifold divert from each other, in contrast to an attracting LCS, in which fluid particles approach each other. It is also worth mentioning that flux across LCS as ridges in the FTLE field is negligible, therefore they simply define convective-transport barriers in the flow field [5]. In this paper, to calculate FTLE field at $t= 1.25$ s, 50 time frames in both forward and backward directions are adopted with time step equals to 0.025 s. FTLE calculations are performed in both directions to extract both repelling and attracting LCS. Fig. 4 shows anatomy of the repelling (part a) and attracting (part b). As one can see in part (a) of the figure, an upstream separatrix forms in the upstream flow. In fact, by upward swimming of the jellyfish, flow is divided into two regions (i.e. left and right) via jellyfish bell nose as a stagnation point; therefore upstream separatrix is generated. As is also visible in the figure, another vertical separatrix forms in the core of the jellyfish downstream wake; this primarily forms by interactions of opposite sign ejected vortices in the animal wake. Fingerprints of attached and detached vortices are also visible in the repelling LCS. Close-curved regions also form by attached vortices in repelling LCS, which means low mixing of fluid particles with surrounding in these semi-isolated regions. Fig.4 part (b) shows attracting LCS pattern. Effect of jellyfish bell deformation dynamics, attached/detached ejected vortices

and internal dynamic inside the jellyfish bell due to these vortices are shown in the figure. As both repelling and attracting LCS figures depict, flow field is almost symmetric over the total integration time.

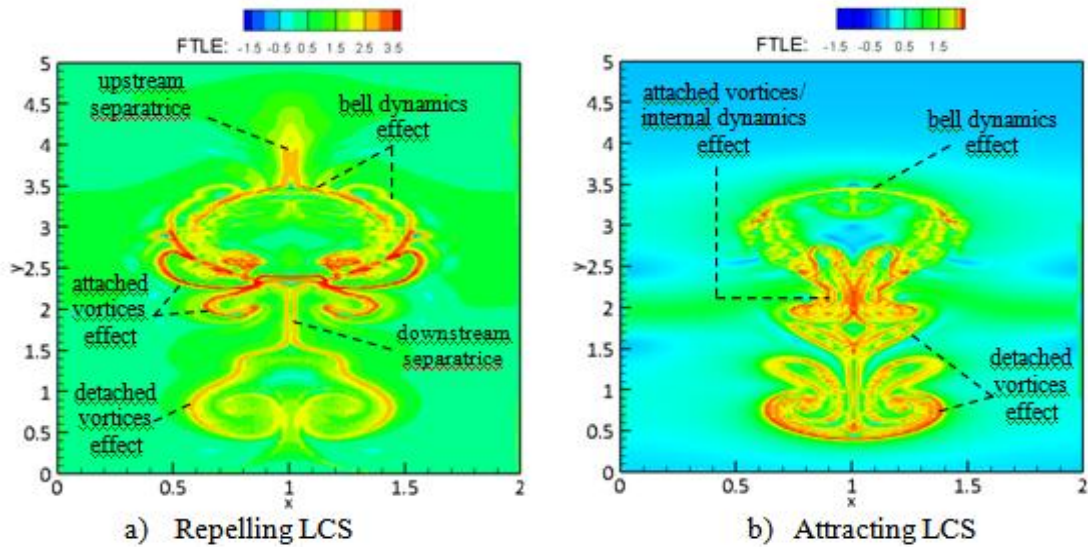


Figure 4. Lagrangian coherent structures of jellyfish swimming IBM-FSI simulation at $t= 1.25$ s

As a matter of fact, by increasing the integration time, more details of the flow field is captured by forward and backward FTLE calculations. In this regard, 100 frames of velocity field with time interval of 0.025 second are adopted to calculate FTLE here. Integrations are performed for both forward and backward directions. In Fig. 5, part (a), stable Lagrangian manifolds are shown for integration over total time of the simulation, i.e. $t= 2.5$ s. As one can see by comparison of Fig. 5 and Fig. 4 , part (a), for repelling LCS at $t= 0.025$ s, detached vortices generate a vertical separatrix downstream instead of curved LCS visible in the repelling structures at $t=1.25$ s. An upstream vertical separatrix also forms in the flow field. Effects of jellyfish bell deformation and attached and detached vortices are visible in the repelling LCS structures as shown in Fig.5, part (a); two isolated regions also form due to bell deformation dynamics/vortices effects.

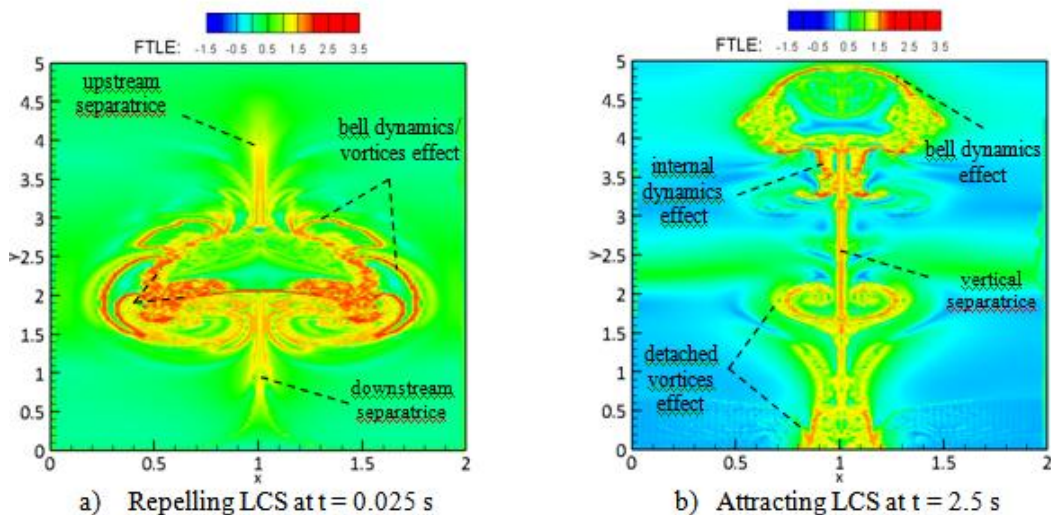


Figure 5. Lagrangian coherent structures of the jellyfish swimming IBM-FSI simulation using total time

Unstable Lagrangian manifold (attracting LCS) pattern generated by jellyfish swimming obtained by backward calculation of FTLE filed over total time of the simulation is presented in Fig.5, part (b). As one can see in the figure, effect of bell deformation and internal dynamics effects are visible on the top. A vertical attracting separatrix forms in the middle indicating formation of two separate regions on the sides of the line. Effects of detached vortices in the jellyfish wake are also captured to complete the flow skeleton picture.

VI. Conclusion

In this paper, Lagrangian coherent structures generated by a jellyfish with a flexible membrane swimming in a quiescent flow environment were extracted to obtain global key features of the flow field. In this regard, a fully-coupled IBM-FSI simulation of the jellyfish swimming was performed using IB2d code along with a built-in case. Results of the simulation showed that modeling of jellyfish muscle contraction/relaxation with spring-type element results in a good swimming performance. LCS analysis of the flow field showed that vertical separatrices are generated in upstream region and also in the wake of the animal downstream. Fingerprints of bell deformation dynamics, attached/detached vortices and internal vortical flow dynamics inside the bell were also captured.

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