Fractal Kinetic Study of Septic Wastewater Utilization in Packed Bed Microbial Fuel Cell (MFC)

Akatah, B.M. & Yusuf M.
University of Port Harcourt

Abstract: The use of packed bed microbial fuel cell (MFC) for septic waste water remediation and electricity generation was carried out for 18 days retention period. In the study, the multistage microbial fuel cells were designed. Septic wastewater qualities such as Biochemical Oxygen Demand (BOD), nitrate, TSS, and pH of both the raw and treated wastewater were determined in the laboratory on three days basis for the MFC loaded with external resistors ranging from 50 -1000Ω. Also Fractal Model for the prediction of substrate utilization in the microbial fuel cells was developed and used to predict the substrate in the cells. The fractal model is 

\[ S_t = S_o \exp \left( -\frac{K_i}{\mu_o} \left[ \exp \frac{\mu_o}{-h+1} \left( (t+1)^{-h+1} - 1 \right) \right] \right) \]

can predict accurately the percentage BOD removal rate and the efficiency of the cell (reactor) can be predicted using 

\[ E = 1 - \exp \left( -\frac{K_i}{\mu_o} \left[ \exp \frac{\mu_o}{-h+1} \left( (t+1)^{-h+1} - 1 \right) \right] \right) \]

I. Introduction

Wastewater can be described as water that the quality has been severely affected by human impact or activities. It is a by-product of water used for operations in homes, industries, etc. Wastewater may be municipal or industrial. Most municipal wastewater contains sewage sludge. In homes, the wastewater generated are channelled into septic tanks that are air tight chamber used for anaerobic treatment of sludge in order to reduce the volume and cost of wastewater disposal (Kargi, 2009). Treatment of waste water from homes, industries and commercial centres is a serious problem. This is because treatment of wastewater uses a significant amount of energy in the world. It is a fact that waste water influent has higher energy value in the form of organics (He et al., 2006; Chang et al., 1983). The first order kinetics (Nakhla et al., 2004; Schmidt et al., 1985) have been utilized by many researchers for the treatment of wastewater, raw sludge, primary effluent, etc. He et al. (2013) used MFCs to treat raw sludge and primary effluent. Also, Yazdi et al. (2007) and Huggins et al. (2014) utilized pluggable Microbial Fuel Cell stacks to remediate septic wastewater and produce electricity. Hence, utilization of MFCs is not a new method or system for wastewater treatment but is an emerging technology in the world presently.

The knowledge of the kinetics of substrate utilization is an essential component of wastewater treatment and waste water system design. In most wastewater treatment systems, the popular Monod model and the first order kinetics have been used for the prediction of bacteria population and the substrate utilization kinetics (Nakhlha et al., 2006; Chang et al., 1983). The first order kinetics has been widely reported in literature for the prediction of bacteria growth and substrate utilization. Many wastewaters that undergo hydrolysis prior to biodegradation contain colloidal and particulate organics. Also many microbial growth and biodegradation kinetic models have been developed, proposed and used by researchers (Simpkins & Alexander, 1984; Schmidt et al., 1985; Okpokwasili & Nweke, 2005). Such models allow prediction of chemicals and substrate that remain at a certain time, calculation of time required to reduce substrate to a certain amount, estimation of the duration it take before a certain amount of substrate will be attained at a certain point. Some of these models can also be used to predict the amount of biomass production achievable at a given time. Most of these models do not consider the surface and shape of bed upon which the substrate.

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Kinetics of Substrate Utilization in Microbial Fuel Cell

Substrate performs a vital role in the efficient treatment of wastewater and electricity generation in a Microbial Fuel Cell. The substrate varies from simple to complex mixture of organic food present in wastewater. These substrates are utilized by microbes in the reactor (Microbial Fuel Cell). The kinetics of the substrate utilization in a reactor has been described by the Monod Model. The Monod Model was developed from the Michaelis-Menten equation (Aiba et al, 1965; Zielke, 2006). The Monod Model used to best describe the kinetics of substrate (food) utilization is as follows:

\[ \frac{dx}{dt} = \mu x \]  

(1)

\[ \frac{dx}{dt} / Y_{xs} = Y_{xs} \]  

(2)

\[ \frac{dx}{ds} = Y_{xs} \]  

(3)

Rewriting Eq. (2) above in terms of substrate (food) utilization, we have

\[ -\frac{ds}{dt} = \frac{dx}{dt} / Y_{xs} \]  

(4)

Rearranging Eq. (4) one obtains,

\[ \frac{-ds}{dt} = \mu x / Y_{xs} \]  

(5)

but \( \mu = S(K_s+S) \)

\[ \frac{-ds}{dt} = \frac{\mu_{max} S X}{K_s + S} \]  

(6)

Rearranging, Eq. (6), one obtains

\[ \frac{ds}{dt} = -\frac{\mu_{max} S X}{K_s + S} \]  

(7)

Let \( \frac{\mu_{max} X}{Y_{xs}} = K_m \).

\[ \frac{ds}{dt} = -\frac{K_m S}{K_s + S} \]  

(8)

Eq.(8) is the Monod’s substrate utilization model. The Monod’s substrate utilization equation can be modified by taking the inverse of \( \mu \).

\[ \frac{1}{\mu} = \frac{K_s}{K_{max} S} + \frac{1}{K_{max} S} \]  

(9)

\[ \frac{1}{\mu} = \frac{K_s}{K_{max} S} + \frac{1}{K_{max}} \]  

(10)

Plotting \( \frac{1}{\mu} \) versus \( 1/S \) would produce a slope of \( \frac{K_s}{K_{max}} \) and int ercept of \( \frac{1}{K_{max}} \)

This model is used to understand, how treatment of wastewater is achieved in a reactor. Considering the reactor as a batch reactor, the retention time (batch time of digestion) can be determined. This is achievable by taking the inverse of the Monod model. Therefore,

\[ -\int_{t=0}^{t} \frac{dt}{\mu} = \frac{K_s}{K_{m1} S} + \frac{S}{K_{m1}} \]  

\[ -\int_{t=0}^{t} \frac{dt}{\mu} = \frac{K_s}{K_m} \int_{s=30}^{s=10} \frac{1}{S} ds + \frac{I}{K_m} \int_{s=30}^{s=10} ds \]  

Fractal Kinetic Study of Septic Wastewater Utilization in Packed Bed Microbial Fuel Cell (MFC)

\[ t = - \frac{K_s}{K_m} \ln S_t^{1st} + \frac{I}{K_m} S_t^{1st} \]

\[ t = - \left[ \frac{K_s}{K_m} \ln S_t - \ln S_o + \frac{I}{K_m} (S_t - S_o) \right] \]

\[ t = - \left[ \frac{K_s}{K_m} \ln \left( \frac{S_t}{S_o} \right) + \frac{S_t - S_o}{K_m} \right] \]  \( (11) \)

\[ t = \frac{K_s}{K_m} \ln \left( \frac{S_o}{S_t} \right) + \left( \frac{S_o - S_t}{K_m} \right) \]

\[ t = \frac{I}{K_m} \left[ K_s \ln \left( \frac{S_o}{S_t} \right) + \left( S_o - S_t \right) \right] \]

\( (12) \)

\( (13) \)

\( (14) \)

where \( t \) = Retention time in day

\( S_o \) = Initial concentration of substrate expressed in term of BOD

\( S_t \) = Concentration of substrate with time expressed in term of BOD

\( K_s \) = Half substrate constant (g/L)

\( K_m \) = Maximum substrate utilization rate (g/L/day)

\( - \frac{dS_t}{dt} \) = Substrate utilization rate (g/L/day)

\( \mu \) = Specific growth rate of micro-organism (day\(^{-1}\))

\( \mu_{max} \) = Maximum specific growth rate of micro-organism (day\(^{-1}\))

\( Y_{Xs} \) = Yield coefficient

II. Methodology

A multistage Microbial Fuel cells loaded with different external resistor loads ranging from 50Ω to 1000 Ω were used as batch reactors in the study for 18 days retention period. The diagram of the fuel cell is shown in Fig 1 below. The quality of the septic wastewater used in the MFC was determined before use. Subsequently, the Biochemical Oxygen Demand (BOD) of the wastewater was determined after every three days as specified by APHA, AWWA, WPCF (1998). The BOD was used to ascertain the amount of substrate present in the MFC. After the 18 days period, the fractal model developed was used to predict the BOD concentration of the septic wastewater and a comparison was drawn between the experimental results and that of the model.

Fig 1: Multistage Packed Bed Microbial Fuel Cell
Fractal Kinetic Study of Septic Wastewater Utilization in Packed Bed Microbial Fuel Cell (MFC)

The modelling approach used for this study was based on the concept of fractal kinetics which assumes that reactions occurring on heterogeneous surfaces, such as rough and irregular media do not follow the typical first order kinetics. The process of the model development is presented below.

Consider the first order equation for substrate utilization given as

$$\frac{dS}{dt} = KXS$$

But the bacteria growth rate can be represented as

$$\frac{dx}{dt} = \mu X$$

From the fractal theorem

$$\mu = \mu_0 (t + 1)^{-h}$$

Such that Eq. (16) becomes

$$\frac{dx}{dt} = \mu_0 (t + 1)^{-h} X$$

Integrating Eq.(18), one obtains

$$\int_{x_0}^{x_t} dx = \mu_0 (t + 1)^{-h} \int_0^t dt$$

$$\ln\left(\frac{x_t}{x_0}\right) = \mu_0 \frac{(t + 1)^{-h}}{-h+1} - 1$$

$$x_t = x_0 \exp\left[\frac{\mu_0 (t + 1)^{-h+1}}{-h+1} - 1\right]$$

From Eq.(15)

$$\frac{dS}{S} = KXdt$$

but $$K = K_o (t + 1)^{-h}$$

Substituting eq.(21) into eq.(20) and integrating, we obtained

$$\int \frac{dS}{S} = K_0 (t + 1)^{-h} x_0 \exp\left[\frac{\mu_0}{-h+1} (t + 1)^{-h+1} - 1\right] \int_0^t dt$$

where $$K_0 = K_o X_0$$

$$S_t = S_0 \exp\left(-\frac{K_0}{\mu_0} \left[\exp\left(\frac{\mu_0}{-h+1} (t + 1)^{-h+1} - 1\right)\right] - 1\right)$$

$$S_t = S_0 \exp\left(-\frac{K_0}{\mu_0} \left[\exp\left(\frac{\mu_0}{-h+1} (t + 1)^{-h+1} - 1\right)\right] - 1\right)$$

The efficiency of substrate utilization in the MFC can be calculated using
Fractal Kinetic Study of Septic Wastewater Utilization in Packed Bed Microbial Fuel Cell (MFC)

\[
E = \frac{S_o - S_t}{S_o}
\]  
(24)

\[
E = \frac{S_o}{\frac{S_o}{S_t} - S_t}
\]  
(25)

\[
E = 1 - \frac{S_t}{S_o}
\]  
(26)

Substituting Eq. (22) into Eq.(26), we have

\[
E = 1 - \exp \left( -\frac{K_s}{\mu_o} \left[ \exp \frac{\mu_o}{-h + 1} \left( (t + h + 1) - 1 \right) \right] \right)
\]  
(27)

where E = Efficiency of the fractal model

### III. Results

#### Characterization of septic waste water

The septic waste water used in this research was determined to possess the following physiochemical properties presented in Table 1 before the remediation process.

**Table 1**: Characteristics of septic waste water used in the study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH</td>
<td>7.5</td>
</tr>
<tr>
<td>TSS (mg/l)</td>
<td>1146</td>
</tr>
<tr>
<td>BOD (mg/l)</td>
<td>275</td>
</tr>
<tr>
<td>Nitrate (mg/l)</td>
<td>8.7</td>
</tr>
</tbody>
</table>

#### Septic waste water remediation for 50Ω resistor load

The effect of applying a 50Ω resistor across the single set-up of the MFC is presented in Table 2 for a retention time of 18 days.

**Table 2**: Septic wastewater characteristics with retention time in MFC using 50Ω resistor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Retention Time (day)</th>
<th>% removal</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOD(mg/l)</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>275</td>
<td>255.2</td>
</tr>
</tbody>
</table>

From Table 2, it was observed that the MFC with 50Ω resistor load removed 69.56% BOD and 68.97% nitrate. The BOD removal rate is shown in Figure 4.1 below.

![BOD concentration with time in MFC with a 50Ω resistor load](image)

**Fig 2**: BOD₅ concentration with time in MFC with a 50Ω resistor load

From Fig 2 above, the relationship between the BOD₅ concentration and retention time is given by

\[
\text{BOD}_5 = -12.28\text{time} + 284.3
\]

The removal efficiency of BOD₅ was predicted using the fractal model developed in Eq.(23) which was simulated using solver function in Excel. The model was calibrated at h = 1.996176, K = 1.17x 10⁻⁷ and q = 20.25898. The model was subsequently verified by comparing the predicted values with the experimental values. The comparison showed a very high correlation coefficient of about 0.98. This is shown in Figure 3 below.
Fractal Kinetic Study of Septic Wastewater Utilization in Packed Bed Microbial Fuel Cell (MFC)

Fig 3: Predicted and experimental BOD$_5$ removal rate using MFC with 50 ohms resistor load

**Septic waste water remediation and power generation at 100 $\Omega$ resistor load**

The effect of applying a 100$\Omega$ resistor across the single set-up of the MFC is presented in Table 4.3 for a retention time of 18 days.

**Table 3**: Septic wastewater characteristics with retention time in MFC using 100 $\Omega$ ohms resistor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Retention Time (day)</th>
<th>% removal</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOD (mg/l)</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>275</td>
<td>255.3</td>
<td>240.7</td>
</tr>
</tbody>
</table>

From Table 3, it was observed that the MFC with 100$\Omega$ resistor load removed 72.29% BOD. The BOD removal rate is shown in Figure 4 below.

From Figure 4 above, the relationship between the BOD$_5$ concentration and retention time is given by BOD$_5$ = $-12.25t + 285.6.$

The removal efficiency of BOD$_5$ was predicted using the fractal model developed in Eq.(23) which was simulated using solver function in Excel. The model was calibrated at $h = 1.896936$, $K = 2.54 \times 10^{-7}$ and $q = 17.85364$. The model was subsequently verified by comparing the predicted values with the experimental values. The comparison showed a very high correlation coefficient of about 0.98. This is shown in Figure 5 below.
Fractal Kinetic Study of Septic Wastewater Utilization in Packed Bed Microbial Fuel Cell (MFC)

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Fig 5: Predicted and experimental BOD$_5$ removal rate using MFC

**Septic waste water remediation and power generation at 150 Ω resistor load**

The effect of applying a 150Ω resistor across the single set-up of the MFC is presented in Table 4 for a retention time of 18 days.

**Table 4**: Septic wastewater characteristics with retention time in MFC using 150 ohms resistor

<table>
<thead>
<tr>
<th>Parameter (mg/l)</th>
<th>Retention Time (day)</th>
<th>% removal</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOD</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>275</td>
<td>255.1</td>
<td>236.5</td>
</tr>
</tbody>
</table>

From Table 4, it was observed that the MFC with 150Ω resistor load removed 75.2% BOD. The BOD removal rate is shown in Fig 6 below.

![Fig 6: BOD$_5$ concentration with time in MFC with a 150Ω resistor load](image)

From Fig 6 above, the relationship between the BOD$_5$ concentration and retention time is given by BOD$_5$ = -12.91time + 286.4.

The removal efficiency of BOD$_5$ was predicted using the fractal model developed in Eq.(23) which was simulated using solver function in Excel. The model was calibrated at $h = 1.860902$, $K = 3.5 \times 10^{-7}$ and $q = 16.9968$. The model was subsequently verified by comparing the predicted values with the experimental values. The comparison showed a very high correlation coefficient of about 0.98. This is shown in Fig 7 below.
Septic waste water remediation and power generation at 560Ω resistor load

The effect of applying a 560Ω resistor across the single set-up of the MFC is presented in Table 5 for a retention time of 18 days.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Retention Time (day)</th>
<th>% removal</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOD (mg/l)</td>
<td>0  3  6  9  12  15  18</td>
<td>275  254.0  234.2  160.1  103.2  80.4  62.5  77.3</td>
</tr>
</tbody>
</table>

From Table 5, it was observed that the MFC with 560Ω resistor load removed 77.3% BOD. The BOD removal rate is shown in Fig 8 below.

From Fig 8 above, the relationship between the BOD₅ concentration and retention time is given by

\[ \text{BOD}_5 = -12.91t + 286.4 \]

The removal efficiency of BOD₅ was predicted using the fractal model developed in Eq.(23) which was simulated using solver function in Excel. The model was calibrated at h = 1.607145, K = 4.46E×10⁻⁵ and q = 9.353562. The model was subsequently verified by comparing the predicted values with the experimental values. The comparison showed a very high correlation coefficient of about 0.98. This is shown in Fig 9 below.
Septic wastewater remediation and power generation at 1000Ω resistor load
The effect of applying a 1000Ω resistor across the single set-up of the MFC is presented in Table 6 for a retention time of 18 days.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Retention Time (day)</th>
<th>% removal</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOD (mg/l)</td>
<td>0</td>
<td>275</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>251.6</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>220.1</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>150.3</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>91.5</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>60.2</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>37.4</td>
</tr>
</tbody>
</table>

From Table 6, it was observed that the MFC with 1000Ω resistor load removed 86.4% BOD. The BOD removal rate is shown in Figure 10 below.

From Figure 4.13 above, the relationship between the BOD₅ concentration and retention time is given by

\[ \text{BOD}_5 = -13.28t + 286.6 \]

\[ R^2 = 0.960 \]

The removal efficiency of BOD₅ was predicted using the fractal model developed in Eq.(23) which was simulated using solver function in Excel. The model was calibrated at \( h = 0.619731, K = 2.17 \times 10^{-2} \) and \( q = 0.855127 \). The model was subsequently verified by comparing the predicted values with the experimental values. The comparison showed a very high correlation coefficient of about 0.98. This is shown in Fig 11 below.
Fractal Kinetic Study of Septic Wastewater Utilization in Packed Bed Microbial Fuel Cell (MFC)

IV. Conclusion

The fractal model can provide an accurate means of predicting substrate utilization in a packed bed MFC. The fractal model developed in this paper is as follows:

\[
S_t = S_o \exp \left(-\frac{K}{\mu_o} \left[ \exp \left(\frac{\mu_o}{-h+1} \left( (t+1)^{-h+1} - 1 \right) \right) - 1 \right] \right)
\]

and

\[
E = 1 - \exp \left(-\frac{K}{\mu_o} \left[ \exp \left(\frac{\mu_o}{-h+1} \left( (t+1)^{-h+1} - 1 \right) \right) - 1 \right] \right)
\]

The model can be used for the prediction of the BOD and COD removal rate in the MFC. The model is very accurate and can give about 99% of the experimental BOD removal rate value for the septic wastewater. This indicates that the BOD removal rate in a MFC can be predicted accurately using the fractal model.

References


Fig 11: Predicted and experimental BOD\textsubscript{5} removal rate using MFC